
KOÇ UNIVERSITY

FALL 2010

FINAL EXAM

MATH 102

JANUARY 21, 2011

Duration of Exam: 105 minutes

INSTRUCTIONS:

- No calculators may be used on the test.
- No books, no notes, no questions, and talking allowed.
- You must always explain your answers and show your work to receive full credit.
- Print (use CAPITAL LETTERS) and sign your name. GOOD LUCK!

SURNAME, Name: _____

Student ID no: _____

Signature: _____

(Check One):
_____ (Barış Coşkunüzzer – TTh 14:00-15:15) : _____
_____ (Tolga Etgü – MW 14:00-15:15) : _____
_____ (Tolga Etgü – MW 17:00-18:15) : _____

PROBLEM	1	2	3	4	5	TOTAL
POINTS	20	20	15	15	41	111
SCORE						

Problem 1 (20 pts) Evaluate the limits in parts (a)-(c).

$$(a) \lim_{x \rightarrow \infty} \frac{\sqrt{x^2+2x}}{x+1}$$

$$(b) \lim_{x \rightarrow 0} \frac{\sin(3x^2)}{x^4+x^2}$$

$$(c) \lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x^2-4}$$

(d) Determine a and b which make the following function continuous everywhere.

$$f(x) = \begin{cases} e^x + a & , \text{ if } x < 0; \\ 3 & , \text{ if } x = 0; \\ x^2 + b & , \text{ if } x > 0. \end{cases}$$

Problem 2 (20 pts)

(a) Let $f(x) = \frac{1}{\sqrt{3x-2}}$. Find $f'(2)$.

(b) Let $g(x) = 2^{x \ln x}$. Find $g'(e)$.

(c) Let $h(x) = \frac{\arctan x}{x^2+1}$. Find $h'(1)$.

(d) Let $k(x) = \sqrt[3]{x} - \frac{2}{\sqrt[4]{x}}$. Find $k'(1)$.

Problem 3 (15 pts) Sketch the graph of f using the following information. Indicate the monotonicity, concavity, asymptotes, local extrema and inflection points clearly.

- The domain of f is $\mathbb{R} \setminus \{1\}$.
- $\lim_{x \rightarrow -\infty} f(x) = 2$, $\lim_{x \rightarrow \infty} f(x) = \infty$.
- $\lim_{x \rightarrow 1} f(x) = \infty$.
- $f'(x) > 0$ if x is in $(-2, 1)$ or $(4, \infty)$. $f'(x) < 0$ if x is in $(-\infty, -2)$ or $(1, 4)$.
- $f''(x) > 0$ if x is in $(-3, 1)$ or $(1, \infty)$. $f''(x) < 0$ if x is in $(-\infty, -3)$.
- $f(-3) = 0$, $f(-2) = -1$, $f(4) = -4$.

Problem 4 (15 pts) Find the area of the region between the curves

$$y = -x^2 + x + 1 \quad \text{and} \quad y = x^2 - 3x - 5 .$$

Problem 5 (41 pts) Evaluate the integrals in parts (a)-(d).

(a)(7 pts) $\int_0^{\pi/2} \sin^4 x \cos x \, dx$

(b)(7 pts) $\int_{-2}^2 \sqrt{4 - x^2} \, dx$

(c)(7 pts) $\int \frac{1}{x^2+x} \, dx$

(d)(10 pts) $\int_0^1 \arctan x \, dx$

(e)(10 pts) Determine whether the following improper integral converges or not.

$$\int_0^{\infty} x e^{-x} \, dx$$