KOÇ UNIVERSITY

MATH 102 - CALCULUS

Midterm Exam 1 November 7, 2006

Duration of Exam: 120 minutes

INSTRUCTIONS: No calculators may be used on the test. No books, no notes, no questions, and talking allowed. You must always **explain your answers** and **show your work** to receive **full credit**. Use the back of these pages if necessary. **Print (use CAPITAL LETTERS) and sign your name, and indicate your section below. GOOD LUCK!**

Surname, Name: ______________________________

Student ID no: ______________________________

Signature: ______________________________

Section (Check One):

Section 1 (Mehmet Sarıdereli - MW: 11-12:15) : ___
Section 2 (Mehmet Sarıdereli - TT: 15:30-16:45): ___
Section 3 (Tolga Etgü) : ___

PROBLEM	1	2	3	4	5	6	7	**TOTAL**
POINTS	20	10	25	15	25	15	10	**120**
SCORE								

Name:

Problem 1. In each of the following find the limit or explain why it does not exist.

(1.a) **(5 pts)** $\lim_{x \to \pi} \frac{|\sin x|}{\sin x \cos x}$

(1.b) **(5 pts)** $\lim_{x \to 1} \frac{\cos(\pi x/2)}{\sin(2\pi x)}$

(1.c) **(5 pts)** $\lim_{x \to \infty} \frac{\sqrt{x^2+1}}{x}$

(1.d) **(5 pts)** $\lim_{x \to 0} \frac{3x+\sin x}{2x}$

Name:

Problem 2. Let $f(x) = \begin{cases} x^2 + x + 1 & , \quad x < 2 \\ cx^2 & , \quad x \geq 2 \end{cases}$, where c is a constant.

(2.a) **(3 pts)** Explain why $f(x)$ is continuous when $x \neq 2$.

(2.b) **(3 pts)** Why is $f(x)$ not continuous at $x = 2$ if $c = 0$?

(2.c) **(4 pts)** What should c be so that f is continuous everywhere?

Name:

Problem 3.

(3.a) Find the derivatives of the following functions.

– **(5 pts)** $f(x) = (x+1)^2 \sin x$

– **(5 pts)** $f(x) = \sqrt{\cos(4x)}$

(3.b) Find the slope of the tangent line to the graph of each of the following equations at the given point (x_0, y_0).

– **(5 pts)** $y = \frac{x+1}{x-1}; \quad x_0 = -1,\ y_0 = 0$

– **(5 pts)** $y = \frac{1}{\sqrt[3]{x-1}}; \quad x_0 = 2,\ y_0 = 1$

– **(5 pts)** $\sin y = x; \quad x_0 = \frac{1}{\sqrt{2}},\ y_0 = \frac{\pi}{4}$

Name:

Problem 4.

(4.a) **(5 pts)** Explain why $f(x) = 2x^{2/3} + 1$ has absolute maximum and minimum values on the interval $[-200, 50]$.

(4.b) **(10 pts)** Find the absolute maximum and minimum values of $f(x) = 2x^{2/3} + 1$ on the interval $[-1, 8]$.

Name:

Problem 5. $f(x)$ is a continuous function on the interval $[-5, 12]$. Information about this function is given below.

when the value of x is	-5	-3	-1	1	4	7	9	11	12
value of $f(x)$ is	3	4	-1	3	2	1	0	-2	-1

when the value or interval of x is	$(-5,-3)$	-3	$(-3,-1)$	$(-1,1)$	1	$(1,7)$	7	$(7,9)$	$(9,11)$	11	$(11,12)$
$f'(x)$ is	>0	0	<0	>0	0	<0	0	<0	<0	0	>0

when the value or interval of x is	$(-5,-1)$	$(-1,4)$	4	$(4,7)$	7	$(7,9)$	$(9,12)$
$f''(x)$ is	<0	<0	0	>0	0	<0	>0

We also know that neither $f'(x)$ nor $f''(x)$ are defined at $x = -1$ and $x = 9$. Consider this function $f(x)$ in the following questions. As usual, you must justify your answers to all these questions.

(5.a) **(3 pts)** List all critical points in $[-5, 12]$.

(5.b) **(2 pts)** Identify where local maxima occur in $[-5, 12]$ using the Second Derivative Test.

(5.c) **(2 pts)** Identify where local minima occur in $[-5, 12]$ using the First Derivative Test.

(5.d) **(3 pts)** Identify the inflection points over $[-5, 12]$.

Name:

(5.e) **(3 pts)** Discuss concavity and monotonicity of $f(x)$ on $[-1, 7]$.

(5.f) **(4 pts)** In which of the following interval(s) are two further zeros of $f(x)$: $(-5, -3)$,$(-3, -1)$, $(-1, 1)$, $(7, 9)$, or $(11, 12)$? *(Note: Zeroes of $f(x)$ are those values of x for which $f(x) = 0$.)*

(5.g) **(4 pts)** Sketch the graph of $f(x)$ on $[-5, 12]$.

(5.h) **(4 pts)** There is a point on the graph of $f(x)$ in the interval $[-1, 9]$ whose tangent has a slope of $1/10$. Why? Find an interval of length 2 inside $[-1, 9]$ which is guaranteed to include a number x with $f'(x) = 1/10$.

Name:

Problem 6. (15 pts) The cost of producing and distributing 100 liters of crude oil is 20 YTL. If the selling price of 100 liters of crude oil is set at x YTL $(20 < x < 100)$, then the amount of crude oil sold is determined by the market as

$$100\left(\frac{500}{x-20}+100-x\right) \quad \text{liters.}$$

What selling price between 20 YTL and 100 YTL per 100 liters of crude oil will bring a maximum profit? (Profit is revenue minus cost!)

Name:

Problem 7. The following is the graph of $y = \tan x$. As you can see, this graph has vertical asymptotes $x = k\frac{\pi}{2}$ for every odd integer k and $\tan x$ has a period π.

(7.a) **(5 pts)** Sketch the graph $y = \cot x$ by shifting, scaling, and/or reflecting the graph of $y = \tan x$. Use the identity $\cot x = -\tan(x - \frac{\pi}{2})$. Show each step.

(7.b) **(5pts)** Verify the identity given in part (a) by using the sine and cosine addition formulas : $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and

$\cos(A + B) = \cos A \cos B - \sin A \sin B$