

INVENTORY SYSTEMS WITH ADVANCE DEMAND INFORMATION AND RANDOM REPLENISHMENT TIMES

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Abstract: Advance demand information, when used effectively, improves the performance of production/inventory systems. The extent of improvement however depends on certain factors. We investigate the influence of random supply lead times on a single-stage inventory system with advance demand information. It is found that the supply lead time variability diminishes the benefits of advance demand information.

Keywords: Inventory systems, base stock policies, advance demand information

1 Introduction

This note investigates the influence of advance demand information on inventory systems with exogenous replenishment lead times. It is known that advance demand information (such as early customer orders) improves the performance of inventory systems. In one of the first papers that explicitly considers advance demand information in inventory systems, Hariharan and Zipkin [5] present a number of important properties concerning optimal replenishment policies and cost reductions. In particular, it is shown that for a system with exogenous replenishment times the cost reduction due to advance demand information can be extremely significant.

In this note, an identical system to the one in [5] is investigated under a slightly different assumption. As opposed to Hariharan and Zipkin who consider random supply lead times that are revealed at the time of order arrival, it is assumed here that random supply lead times are not known in advance (of their realization). This slight difference in the initial assumptions leads to a different analysis and different intuitions.

A number of papers investigate the effects of advance demand information on production/inventory systems. Buzacott and Shanthikumar [1], [2], [3] present a complete treatment of production control mechanisms and their performance analysis with early customer orders. Karaesmen, Buzacott and Dallery [6] explore production control issues and the potential benefits of advance demand information. Karaesmen, Liberopoulos and Dallery [7] focus on the value of advance demand information under a particular control mechanism. Other recent references that investigate the impact of advance demand information on discrete-time production/inventory systems include Gallego and Ozer [4] and Toktay and Wein [8]

The paper is structured as follows. The main investigation is presented in Section 2. Some illustrative numerical examples are presented in Section 3 and our conclusions follow in Section 4.

2 The Analysis

We present the main elements of our analysis in this section. In order to emphasize certain contrasts, some of the standard results related to inventory systems with exogenous random lead times are presented first.

2.1 Stochastic independent lead times with no advance information

Let us consider a single stage inventory system with independent stochastic lead times. Demands arrive according to a Poisson process with rate λ and the supply lead time of each order is a random variable L_S (which cannot be known in advance). We also assume linear unit holding and backorder costs of h and b (there are no fixed setup costs).

Let $X(t)$ denote the inventory level at time t . The system is run according to a base stock policy with parameter S . Let us define $N(t) = S - X(t)$, $N(t)$ measures the shortfall with respect to the base stock level S at time t .

Standard results can be used to show that, the optimal base stock policy is characterized by the parameter S^* which satisfies the optimality condition:

$$S^* = \inf\{s, F_N(s) > b/(h + b)\}$$

where $F_N()$ is the cumulative distribution of the stationary random variable N .

Let us now note that, the random variable N corresponds to the number of customers in an $M/G/\infty$ queue with service times L_S . N is known to be a Poisson distributed random variable with parameter $(\lambda E[L_S])$. This leads to the following remarks:

1. The stationary inventory level (for a given base stock level S) depends on the lead time distribution only through its mean. In other words lead time variability does not have any influence on the performance of the system.
2. In order to compute S^* , it suffices to compute the tail values of a Poisson distribution.

2.2 Deterministic Supply Lead Times with Advance Demand Information

We now assume that all orders arrive exactly τ time units before their due-dates. The parameter τ is called the demand (or customer) lead time. An order base-stock policy will be followed (which may not always be optimal, see [6] for example). At each customer order, a replenishment order is released. Because supply lead times are deterministic, order crossing does not take place.

Remark: In this special case the demand lead time τ is identical to the planned *release lead time* and we use both terms from here on. Note that, in general, the release lead time can be different from the customer lead time (see [6]) but we avoid the introduction of an extra parameter by assuming that they are equal.

By the above definition of the order base-stock policy, all demands are released τ units in advance of their due-dates. Let us also assume that $L_S > \tau$. The system follows a base stock policy with parameter S and $X_\tau(t)$ denotes the inventory position.

In order to analyze this system, let us define $N_\tau(t) = S - X_\tau(t)$. Some further notation is necessary for what follows:

- $A(t)$ the total number of order arrivals until time t
- $R(t)$ the total number of demand (due-date) arrivals until time t
- $D(t)$ the total number of production completions until time t

Note that we can now express $N_\tau(t) = R(t) - D(t)$

Now following Buzacott and Shanthikumar [3]:

$$\begin{aligned}
 N_\tau(t) &= R(t) - D(t) = A(t - \tau) - D(t) \\
 &= A(t - \tau) - D(t - \tau) + D(t - \tau) - D(t) \\
 &= N(t - \tau) - D(t - \tau, t)
 \end{aligned}$$

$N_\tau(t)$ is then related to $N(t - \tau)$, the (shortfall) queue length in a system without advance information (or early release). In particular, we can write the following equation for the stationary random variables N_τ and N :

$$N_\tau = N - \text{the number of production completions in an interval of } \tau \quad (1)$$

Now, noting that the number of production completions in an interval of τ is a Poisson random variable with parameter $\lambda\tau$, we find that N_τ itself is also a Poisson random variable with parameter $\lambda(L_S - \tau)$. This is precisely the result of Hariharan and Zipkin obtained from a different angle.

It is clear that, the optimal choice for the demand lead time (or the release parameter) is $\tau = L_S$. If τ is set to L_S , then the above results imply that N_τ is a Poisson process with parameter zero, In reality each order is perfectly synchronized with its supply lead time. In this ideal case, the system does not need to hold any inventory and is never backlogged, thereby achieving a cost of zero.

2.3 Independent Random Supply Lead Times with Advance Demand Information

Let us assume now that, L_S is a random variable. It is useful to start with two special cases that lead to the main general result. First, let us assume that even though L_S is random, it is bounded from below by τ . This case corresponds to one of the cases studied by Hariharan and Zipkin [5]. Their result is summarized in the following lemma:

Lemma 1 (Hariharan and Zipkin [5]) *Let $L_S > \tau$ with probability 1, then N is a Poisson random variable with parameter $\lambda E[L_S - \tau]$.*

Let us now investigate a second special case where the random variable L_S is bounded from above by τ . Equation 1 still holds, but N_τ can now take negative values (in fact, it takes only non-negative values). The stationary shortfall is then a random variable taking non-positive values following a Poisson random variable with parameter $\lambda(\tau - L_S)$. This is summarized in the next lemma.

Lemma 2 *Let $L_S < \tau$ with probability 1, then $-N$ is a Poisson random variable with parameter $\lambda E[\tau - L_S]$.*

The two lemmas present results in certain special cases but give little information on the general case where the realizations of the random variable L_S may be above or below τ . The next theorem states the general result that decomposes the system into two different subsystems corresponding each to one of the previous lemmas.

Theorem 1 *In general the shortfall process N is such that: $N = N_1 - N_2$ where N_1 and N_2 are Poisson random variables with respective parameters α_1 and α_2 . These two parameters are given by:*

$$\alpha_1 = \lambda \int_0^\tau (\tau - x) dF_{L_S}(x)$$

$$\alpha_2 = \lambda \int_\tau^\infty (x - \tau) dF_{L_S}(x)$$

The theorem says that the stationary shortfall distribution of the inventory level with random supply lead times can be expressed as the difference of two Poisson random variables. This suggests a first computational approach for optimizing the base stock level S for any given τ using the convolution of the two random variables involved. Alternatively, the stationary shortfall distribution for any (S, τ) can also be explicitly expressed in the following way .

$$P\{N = n\} = e^{-1} \sqrt{\left(\frac{\alpha_1}{\alpha_2}\right)^n} I_n(2\sqrt{\alpha_1\alpha_2})$$

where

$$I_n(x) = \sum_{k=0}^{\infty} \frac{1}{k!(k+n)!} \left(\frac{x}{2}\right)^{2k+n}$$

Unfortunately this expression still involves an infinite sum and does not seem to facilitate the computation significantly.

3 Numerical Examples

Let us consider the following example: $\lambda = 3$, $h = 1$ and $b = 10$. Figure 1 presents a comparison of two systems that have the identical parameters except for their supply lead times. The first system has a constant supply lead time of 10. The second system's supply lead time is an exponential random variable with mean 10. The figure depicts the optimal cost (for the optimal base stock level) as a function of the release (or demand) lead time τ for the two systems. It is known from the results of Hariharan and Zipkin that the cost reduction in passing from $\tau = 0$ to $\tau = 10$ (the average supply lead time) is significant. The figure shows that the same reduction is relatively modest when the supply lead time is not deterministic.

The second issue concerns the optimal release lead time τ . In Hariharan and Zipkin's model, this issue is trivial: simply follow an order base stock policy until τ reaches the constant supply lead time. With random supply lead times, setting the release lead time is less trivial. Consider the example in Figure 2, which takes the same parameters as in Figure 1 apart from the supply lead times which have the following hyper-exponential distribution (with a high coefficient of variance). :

$$f_{L_S}(x) = \frac{1}{2} \left(\frac{1}{19} e^{-1/19x} \right) + \frac{1}{2} e^{-x}$$

Note that $E[L_S] = 10$ as before.

Figure 2 depicts the optimal cost as function of τ in this system. It is important to note that the optimal release lead time (that minimizes the cost) is significantly less than the average supply lead time of 10. In this case even if all customers order much earlier than their due-dates, there is little to gain by this information. The problem is the variability of the supply lead times which makes synchronization of individual demand lead times and supply lead times impossible.

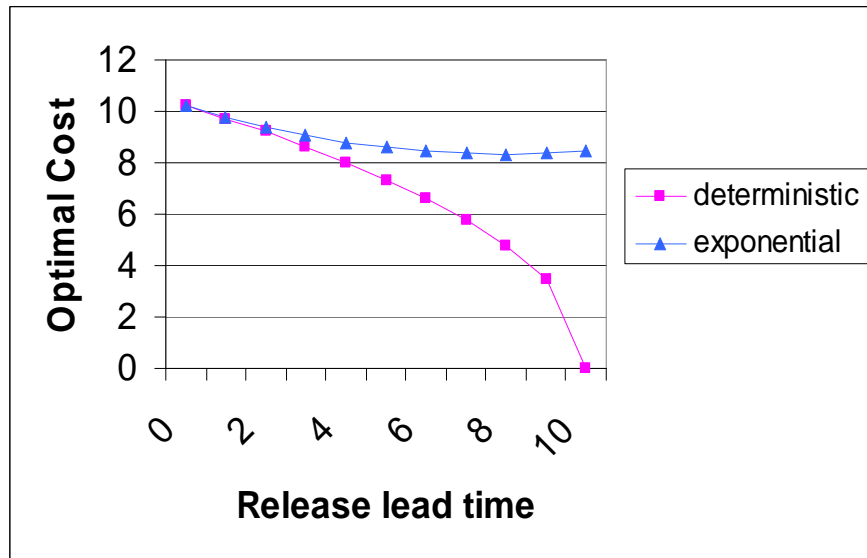


Figure 1: The Optimal Cost as a Function of the Release Lead Time τ

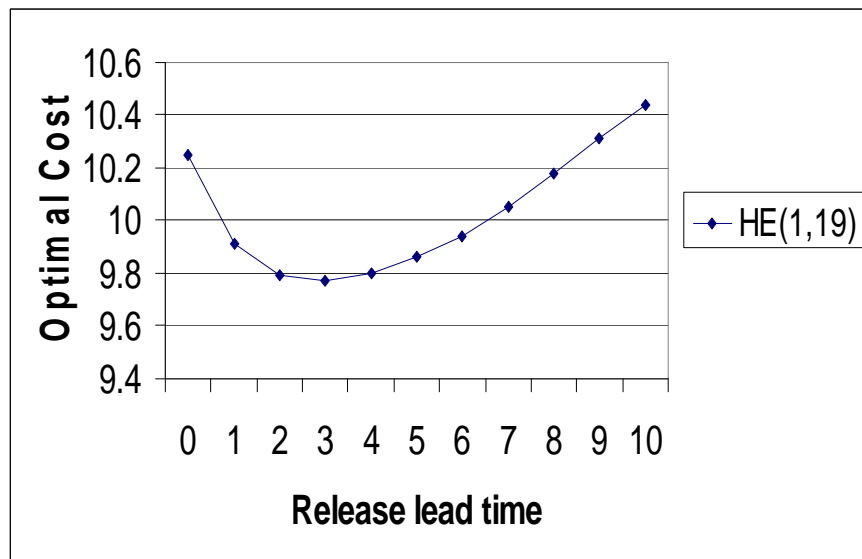


Figure 2: The Optimal Cost as a Function of the Release Lead Time τ

4 Conclusions

Our results indicate that with advance demand information, variability of supply lead times matter significantly both in terms of replenishment policies and the benefits obtained. This is in contrast with the corresponding system without advance demand information in which supply lead time variability does not have any effect on system performance. Some benefits of advance demand information are due to the synchronization of individual replenishment orders and corresponding demand lead times. This synchronization is perfect when supply lead times are constant but becomes extremely poor if supply lead times are highly variable.

For system design purposes, there is a simple implication: even though most inventory systems will benefit from advance demand information, those systems with relatively regular supply processes will benefit the most. This underlines the necessity of keeping the focus on the supply process in addition to the demand side.

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