

Contents lists available at [ScienceDirect](http://www.sciencedirect.com)

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Optimizing product assortment under customer-driven demand substitution

Eda Yücel, Fikri Karaesmen, F. Sibel Salman*, Metin Türkay

College of Engineering, Department of Industrial Engineering, Koç University, 34450 Istanbul, Turkey

ARTICLE INFO

Article history:

Available online 9 August 2008

Keywords:

Retailing
Inventory
Demand substitution
Product assortment
Supplier selection

ABSTRACT

The problem of product assortment and inventory planning under customer-driven demand substitution is analyzed and a mathematical model for this problem is provided in this paper. Realistic issues in a retail context such as supplier selection, shelf space constraints, and poor quality procurement are also taken into account. The performance of three modified models, one that neglects customers' substitution behavior, another that excludes supplier selection decision, and one that ignores shelf space limitations, are analyzed separately with computational experiments. The results of the analysis demonstrate that neglecting customer-driven substitution or excluding supplier selection or ignoring shelf space limitations may lead to significantly inefficient assortments. The effects of demand variability and substitution cost on optimal assortment and supplier selection decisions as well as on the optimal revenue are also investigated. The main contribution of this paper is the development of a practical and flexible model to aid retailers in finding optimal assortments to maximize the expected profit.

© 2008 Elsevier B.V. All rights reserved.

1. Introduction

In order to survive in a competitive environment and to establish a strong position in the market, retailers should be able to manage their operational activities efficiently while providing an adequate customer service-level. Activities such as store and inventory management, establishing relationships with the suppliers, ordering and purchasing of products all contribute to operational costs, while additional costs, or rather loss of revenue, may incur due to poor quality procurement and customer dissatisfaction. An important trade-off in finding the right product assortment is that increasing variety increases customer satisfaction but has a negative effect on operational costs. As a result, when a retailer decides on which suppliers to work with and what product assortment to carry, it is important to understand the expectations and the purchasing behaviors of customers. Research on this topic shows that customers are frequently willing to buy a different color, size or brand within a product category if their favorite variant is either not offered or is temporarily out of stock, rather than going home empty handed. This behavior is indicated by the term *customer-driven demand substitution* and causes the demand for the remaining product types to increase, affecting their optimal order quantities and the product assortment decisions. In selecting what products to offer, retailers are subject to store related constraints such as shelf and storage space limitations. Therefore, maximizing profit in the existence of these operational issues is a challenging

problem for retailers as they seek to rationalize assortment and inventory decisions at the category level.

Product assortment, demand substitution, supplier selection, and inventory management have been studied extensively; yet, to the best of our knowledge, there is no previous work that considers all these aspects together in the literature. This paper provides a tool for retailers to determine the product assortment, which considers supplier selection and inventory management decisions in the presence of shelf space limitations and substitution behavior of customers. The proposed tool optimizes these inter-related decisions for each product category with the goal of maximizing the retailer's expected profits, under cost and demand parameters to be estimated by the retailer over a time horizon. Specifically, we introduce a mixed-integer programming model for the joint problem in order to determine which product types should be ordered from the suppliers, as well as the optimal ordering quantities for the offered product types. The model finds an optimal policy that maximizes expected total profit over a planning horizon for which demand and customers' substitution preferences can be forecasted. By solving this model with different parameter settings designed in our computational experiments, we analyze the importance of various aspects of the problem. We identify the effect of substitution on the product assortment and profit by varying substitution cost parameters. In addition, we show numerically that incorporating the supplier selection decision into the determination of product assortment may result in significantly increased profit, and furthermore, considering shelf space limitations in the decision process leads to more profitable assortments.

The rest of the paper is organized as follows. Section 2 provides the necessary background and literature on product assortment,

* Corresponding author. Tel.: +90 212 3381707; fax: +90 212 3381548.
E-mail address: ssalman@ku.edu.tr (F.S. Salman).

demand substitution, and supplier selection problems. The mathematical model for the problem with single-period, stochastic demand is presented in Section 3. An illustrative example is also presented in this section. In Section 4, the model is analyzed computationally. Conclusions are drawn in Section 5, where possible extensions to the proposed model are also discussed.

2. Literature review

Our focus in this paper is on product assortment under demand substitution, together with other relevant retail store management issues such as shelf space allocation and supplier selection decisions. In this section, we first review previous work on inventory management in the existence of both product assortment and demand substitution, specifically in the retailing context, and then mention some related papers from the supplier selection literature.

Several researchers considered demand substitution together with product assortment and inventory management decisions for production systems [12,26], whereas our focus here is on retail operations. The main difference between substitution in retail settings and production systems is that demand substitution decisions cannot be directly controlled by retailers, while a supplier can direct the fulfillment of the demand for one product with the inventory of another product in production systems. One can consider two forms of demand substitution: in *static* or *assortment-based substitution*, a consumer might substitute when her favorite product is not in the assortment carried by the store, whereas in *dynamic* or *stockout-based substitution*, a consumer might substitute when her favorite product is stocked-out at the moment of purchasing. Mahajan and van Ryzin provide a literature survey and an analysis on the studies related to the assortment-based and stockout-based consumer choice models [16]. Kök et al. [15] also present a recent review of assortment planning problems that covers industrial practice as well as the academic literature.

There is a significant amount of work on inventory management under demand substitution (see for example Parlar and Goyal [21], Noonan [19], Rajaram and Tang [25], Avsar and Baykal-Gursoy [1], Netessine and Rudi [18]). Here, we only review in detail the literature that addresses the joint problem of product assortment and demand substitution. In an early study, Pentico [22] develops a model to find the optimal assortment under downward substitution and stochastic demand, without considering stockout-based substitution. Later, Pentico proposes an EOQ model to formulate inventory costs [23,24]. In all of these three papers, it is shown that the problem can be approximated by using an efficient dynamic programming formulation. The product assortment problem was reconsidered in late 1990s by van Ryzin and Mahajan [28] as well as Smith and Agrawal [27]. Van Ryzin and Mahajan [28] study the assortment planning problem with a stochastic demand, single-period setting with the multinomial logit (MNL) consumer choice model. Their model allows assortment-based substitution, but does not consider stockout-based substitution. Later, Mahajan and van Ryzin [17] propose a stochastic sample-path optimization method for the same model under both assortment-based and multiple rounds of stockout-based substitution. However, resource constraints and supplier selection are not considered in these papers. Cachon et al. [6] extend the model proposed by van Ryzin and Mahajan [28] in the existence of consumer search costs and show that neglecting consumer search behavior leads to an assortment with less variety and lower expected profits. Bish and Madhah [3] consider the model of van Ryzin and Mahajan [28] but investigate pricing issues as well.

Smith and Agrawal [27] study the assortment planning problem with multi-period base-stock inventory models under both assortment-based and stockout-based substitution, but allowed for one

substitution attempt only. They represent substitution with a general exogenous choice model specified by first-choice probabilities and a substitution matrix. They present an approximate solution approach by simplifying the problem using service-levels and showing that substitution results in bounds on individual demands for products. Kök and Fisher [14] also employ an exogenous probabilistic model of substitution and develop an estimation methodology for the substitution rates by leveraging data from stores with varying assortments. They propose an iterative optimization heuristic for the assortment planning and inventory problem with one-level, stockout-based substitution in the presence of constraints on shelf space, discrete maximum inventory levels and delivery lead times. Gaur and Honhon [10] model the consumer choice using a locational choice type model. In addition to analyzing the assortment-based-substitution case, they also propose approximations for the stockout-based substitution case. Finally, a recent paper by Fadiloglu et al. [9] investigate the assortment planning problem under an exogenous deterministic demand substitution model, focusing on shelf space allocation. In this study, ordering and supplier selection decisions are not taken into account.

In order to position the model in this paper, a classification of the above papers with respect to the assumptions on the type of consumer-choice model employed and the nature of demand substitution is provided in Table 1. The above papers model consumer choice in two broad categories: multinomial logit choice (MNL) and exogenous demand (see the review paper by Kök et al. [15] for details). In addition, even though all of these papers take assortment-based substitution into account, only some consider stockout-based substitution as well. To make this distinction, as in most other papers, we refer to the case with only assortment-based substitution as the *static* case whereas both assortment and stockout-based substitution is referred to as the *dynamic* case. In Table 1, the columns indicate the consumer-choice model employed (MNL, exogenous, or other) and the rows indicate the substitution assumption (static or dynamic).

According to the above classification, the model proposed in this paper is akin to those in [27,14]. We represent demand substitution by an exogenous and relatively simple model. Our model utilizes a representation of aggregate demand substitution behavior rather than individual consumer choice. On the substitution type, we take into account multiple levels of assortment- and stockout-based substitution. The resulting model is cruder than the ones in [27,17,14] in this aspect but has the virtue of being much simpler to analyze and requires significantly less data. In particular, we handle the single-period stochastic demand case by a scenario-based approach for representing demand randomness. Hence, we do not need to make specific assumptions on the demand distribution of individual products or the category itself. Consequently, the resulting mixed-integer programming (MIP) model can be solved quickly as long as the number of demand scenarios is bounded reasonably. We take advantage of the simplicity and flexibility of our approach to integrate several important constraints and cost items that are relevant in a retail context.

Our model can easily incorporate realistic issues such as shelf space limitations and ordering quantity quotas for suppliers and

Table 1
A classification of the papers according to consumer-choice model and substitution type

Substitution type	Consumer-choice model		
	MNL	Exogenous	Others
Static	[16,3,6]	[22–24,9]	–
Dynamic	[17]	[27,14]	[10]

supplier selection decisions together with product assortment and inventory management. It may be important to plan these issues simultaneously and our approach provides a tool for carrying out such an analysis. Let us take the supplier selection problem as an example. While this problem has been studied extensively in the literature, most of the existing research on this subject does not consider inventory management of the products in the assortment as part of the problem. In reality, the ordering policy and supplier choice affects one another. For instance, if frequent ordering is necessary due to inventory management reasons (e.g. perishable inventory), a supplier with low unit price but high ordering cost might generate a higher total cost than a supplier with a high unit price and low order cost. As another example, when suppliers offer quantity discounts, the trade-off between savings in purchasing and inventory holding costs should be considered. There are some mathematical programming formulations for the supplier selection problem such as Bender et al. [2], Buffa and Jackson [5], and Degraeve et al. [7]. However, none of these models considers demand substitution. The basic supplier selection decision is easily modeled in our approach and we present examples of the interaction of the supplier selection and assortment decisions.

To summarize, in the retailing sector, decisions on product assortment, category management, selection of suppliers, and inventory levels are closely-related with each other. However, in the open literature, to the best of our knowledge, multiple products, multi-level product substitution, inventory planning and supplier selection are not considered in an integrated model. In this paper, we formulate the multi-product inventory, product assortment and supplier selection problem with multi-level demand substitution under resource constraints with the objective of maximizing expected profit. We evaluate various scenarios by solving the proposed mixed-integer programming problem to optimality. Since the mixed-integer programming problem can be solved efficiently for realistic-sized problems by a commercial solver such as Cplex [13], the decision maker can easily modify the model and analyze different scenarios such as the introduction of a new brand into the market. In that sense, our work can be considered to be a first step in developing a tool to aid retailers in their decision making process.

3. Problem setting and the mathematical model

3.1. Problem setting: definitions and assumptions

We consider two types of important decisions that must be made by a retailer. On the tactical level, the retailer must choose which products to offer to its customers and which suppliers to work with. On the operational level, given the assortment and the suppliers, the retailer must choose how much to order from each product taking into consideration consumers' reaction to the assortment and the inventory availability. Traditionally, these two problems have been treated separately. Relatively recently, researchers have emphasized the benefits of making assortment and ordering decisions jointly. However, in addition to these, other operational or tactical factors such as which suppliers to work with, ordering quotas and shelf space limitations have an impact on the retailer's performance. The problem setting in this paper addresses all of these factors.

A single product category with several products is considered. These products can be purchased from several suppliers. It is assumed that each product is offered by a unique supplier. Often in retailing, a supplier provides a set of product variants in one or more related product categories that are targeted towards different market segments, and serve different consumer needs. Therefore,

establishing a long-term relationship with a supplier may ease some operational activities for the retailer but still the retailer may prefer to establish only a limited number of such relationships.

As in most of the literature in joint assortment and inventory planning (see the review paper by Kök et al. [15]), we consider a single inventory cycle. If inventory cycles can be assumed to be similar as is the case when the demand and the prices are stationary, this also approximates the situation over a longer-period consisting of several cycles. Alternatively, if demand/price non-stationarity between periods is more significant than the in-period random fluctuation, then a multi-period model would be more appropriate as in Yücel et al. [29]. Here, in a typical ordering cycle, the sequence of events in the decision problem is assumed to be as follows. First, the suppliers declare their products and the order quantity quotas that they can supply. Next, the retailer selects which products and how much to order considering the substitution behavior of customers and the relevant costs, revenues and the demand distribution associated with each product.

A significant factor that may impact both assortment and operational decisions is the randomness in demand. We model random demand by a joint demand distribution for all products in the category which is represented by a collection of demand scenarios with given probabilities. This distribution may be obtained, for instance, by considering the aggregate demand for a given category and by splitting it into product demands using the market share estimates of each product. The assortment and the suppliers are selected to maximize the expected profit over all scenarios. For each demand scenario, the substitution quantities and the resulting ending inventory levels at the end of the period have to be determined to calculate the expected costs and revenues.

Another important factor in this setting is the consumer's reaction to what is available on the shelf. We assume that if a product is not available either because it is not in the assortment or because it is stocked-out, it is substituted by another product or a lost sale occurs with deterministic proportions. In this way, we do not attempt to model individual consumer choice decisions but employ an exogenous model capturing aggregate demand behavior of customers, as can be observed by the market share of each product within the category. The deterministic proportion assumption is standard for stockout-based substitution models (see Netessine and Rudi [18] and the references therein). These deterministic proportions represent the aggregate substitution rates from one product to another. In the simplest case, these rates might be obtained by market research. If retailer specific data is ample, they can be generated by the methodology proposed by Kök and Fisher [14]. Alternatively, in case of availability of point-of-sale data for analysis, the method proposed by Öztürk et al. [20] can be utilized to estimate these rates.

3.2. The model

The optimization model considers one product category in a retail store consisting of a set of products offered in the market, denoted by P and a set of suppliers offering these products, denoted by S . Each product is supplied by exactly one supplier, whereas a supplier may supply more than one product. The model determines which products should be ordered from the suppliers and the optimal ordering quantities that maximize expected profit in the existence of fixed ordering costs placed per order, fixed costs of supplier selection due to costs of establishing relations with suppliers, purchasing costs, inventory holding costs, costs incurred as a result of poor quality products received, and customer substitution costs. The constraints of the model include shelf space limitations and ordering quantity quotas of the suppliers.

The following notation is used in the model:

Parameters

- w_{ik} proportion of customers whose preference is product k that substitute product k with product i ($w_{kk} = 0$);
- c_i unit cost of purchasing plus transportation for product i ;
- oc_j cost of ordering per order placed with supplier j ;
- ssc_j cost of selecting supplier j ;
- d_{ib} random demand for product i under demand realization b ;
- OQ_i order quantity quota for product i ;
- SS_i shelf space limitation quantity for product i ;
- a_{ij} 1, if product i can be supplied by supplier j ; 0, otherwise [A: matrix representation; product-supplier availability matrix];
- h_i inventory holding cost per unit of product i ;
- pq_i unit cost due to receiving poor quality products of type i ;
- q_i percentage of poor quality products of type i ;
- p_i unit price of product i ;
- s_{mi} penalty cost of m th level substitution from product i (lost sale is represented as substitution to a dummy product with index 0.);
- $z0_i$ initial inventory position for product i .

Variables

- $z1_{ib}$ ending inventory position of product i under demand realization b ;
- x_i quantity of product i to be ordered;
- y_i 1, if product i is ordered; 0, otherwise;
- o_j 1, if an order is placed with supplier j ; 0, otherwise;
- $x0_{ib}$ amount of satisfied demand for product i under demand realization b ;
- xs_{mikb} amount of product i used to satisfy m th substitution from product k under demand realization b ($m = 1, 2, \dots, M$, where M is a constant).

Model

Max TP = TR – TCO – TCSS – TCP – TCI – TCPQ – TCS (1)

subject to TR = $\sum_{i \in P} \sum_{b=1}^B p_i \beta_b (z0_i + x_i - z1_{ib})$ (2)

TCO = $\sum_{j \in S} oc_j o_j$ (3)

TCSS = $\sum_{j \in S} ssc_j o_j$ (4)

$o_j \geq a_{ij} y_i, \forall i \in P, \forall j \in S$ (5)

TCP = $\sum_{i \in P} c_i x_i$ (6)

TCI = $\sum_{b=1}^B \sum_{i \in P} \frac{(z0_i + x_i + z1_{ib}) \beta_b}{2} h_i$ (7)

TCPQ = $\sum_{i \in P} pq_i q_i x_i$ (8)

TCS = $\sum_{b=1}^B \sum_{m=1}^M \sum_{i \in P} \sum_{k \in P} s_{mi} \beta_b xs_{mikb}$ (9)

$x0_{ib} + \sum_{m=1}^M \sum_{k \in P} xs_{mikb} = d_{ib}, \forall i \in P, \forall b = 1, \dots, B$ (10)

$x0_{ib} + \sum_{m=1}^M \sum_{k \in P} xs_{mikb} + z1_{ib} = z0_i + x_i, \forall i \in P, \forall b = 1, \dots, B$ (11)

$xs_{1ikb} \leq (d_{kb} - x0_{kb}) w_{ik}, \forall i, k \in P \setminus \{i\}, \forall b = 1, \dots, B$ (12)

$xs_{2ikb} \leq (d_{kb} - x0_{kb} - \sum_{r \in P} xs_{1rkb}) \sum_{r \in P} w_{rk} w_{ir}, \forall i \in P,$

$\forall k \in P \setminus \{i\}, \forall b = 1, \dots, B$ (13)

$z0_i + x_i \leq SS_i, \forall i \in P$ (14)

$0 \leq x_i \leq OQ_i y_i, \forall i \in P$ (15)

$y_i, o_j \in \{0, 1\}, \forall i \in P, \forall j \in S$ (16)

$z1_{ib}, x0_{ib}, xs_{mikb} \geq 0, \forall i \in P, \forall b = 1, \dots, B, \forall m = 1, \dots, M$ (17)

The objective function TP defined in Eq. (1) maximizes the total expected profit. In this equation, TR denotes the expected revenue, TCO denotes the expected cost of ordering, TCSS denotes the expected cost of supplier selection, TCP denotes the expected cost of purchasing, TCI denotes the expected cost of inventory holding, TCPQ denotes the expected cost of poor quality products, and TCS denotes the expected cost of substitution. The expected revenue is formulated in Eq. (2) and the expected cost of ordering is expressed in Eq. (3). Eqs. (4) and (5) provide the formulation for the expected cost of supplier selection, where selecting a supplier means at least one order is placed with that supplier. The expected cost of purchasing is expressed in Eq. (6) and the expected cost of inventory is formulated in Eq. (7), where expected inventory is calculated as the average of entering and leaving inventory levels for all demand realizations. Eq. (8) represents the formulation of the expected cost of poor quality products and the expected cost of substitution is given in Eq. (9). The demand for a product is the sum of first-choice demand satisfied and all substitutions from other products to that product and it is expressed in Eq. (10). Eq. (11) ensures that for each product, the sum of the beginning inventory level for that product and the quantity ordered for that product should be equal to the sum of substitutions to that product from any first-choice product including itself and the ending inventory level of that product. The substitution behavior of a customer is expressed in Eqs. (12) and (13), which use the substitution rate matrix W . The amount of any level of substitution from product k to product i cannot be more than a certain proportion of the demand incoming to product k either as a first-choice demand or via substitution from other products. The value of this proportion is obtained by multiplying the substitution rates in matrix W , which exist in that substitution chain. The substitution chain includes all the products that the customer tries to substitute from product k , up to product i . Substitution inequalities are written for each level of substitution. Because of the complexity associated with higher substitution levels, we provide only the first two of them here. Eq. (14) represents the shelf space constraints, Eq. (15) represents the ordering quantity quota limitations for suppliers, and Eqs. (16) and (17) represent integrality and nonnegativity constraints.

In the above formulation, B denotes the number of demand scenarios and β_b denotes the probability of scenario b such that $\sum_{b=1}^B \beta_b = 1$. The demand for product i under the demand scenario b is estimated as d_{ib} . Here, note that the substitution and inventory variables need to be duplicated for each demand scenario, hence, the model size increases with the number of scenarios. However, these continuous variables do not contribute as much to the problem complexity as the binary variables related to decisions on which suppliers to work with and which products to order. In fact, the number of these binary variables would be limited in a realistic problem, as a typical category would most likely consist of at most 40–50 products supplied by at most 5–10 major suppliers. We have observed in our computational experiments that the model can be solved in less than a second with a commercial MIP solver (Cplex) under default settings when 10 products, 5 suppliers and 100 demand scenarios exist.

The objective function comprises several types of costs. The retailer may decide on which of these costs to include in a particular

application depending on the circumstances. We think that the inclusion of all of these costs into the model yields a preferred solution. The supplier selection cost is a control parameter that prevents working with too many suppliers. In this regard, this cost may include the single-period amortized cost of working with the supplier but also other retailer intangible preferences. Similarly, the substitution cost may be regarded as another control parameter. It represents the cost of the loss of goodwill of customers that may reflect on the retailer at a future time point as lost sales. Clearly, it is difficult to identify when and in what magnitude the loss of goodwill will incur an actual cost. For this reason, in our model we include a penalty cost per each substitution or lost sales realized, as in most inventory models. In line with their positioning strategy, the retailers would assess whether their customers expect to find a wide product assortment, or a high quality of service at their stores. Naturally, higher consumer expectations translate to higher substitution costs, depending also on the product category under consideration. For example, since customers are typically less likely to substitute their favorite personal care items, the substitution cost parameter in the model would need to be set to a high value for such categories.

Unlike most of the previous papers in the literature, our model also handles multi-level substitutions by assuming a multi-stage flow structure. We assume that under this structure the demand-product allocation is such that the expected profit is maximized. This is a reasonable approximation of reality where customers select substitutes among available products in order to minimize their implied substitution costs.

Fig. 1 explains the role of the substitution variables in Eqs. (10)–(13). For each product i and demand scenario b , there are three sets of arcs with corresponding decision variables in the figure: (i) first-choice demand incoming to product i , x_{0ib} , (ii) substitution demand incoming to product i , one for each level of substitution, with value $\sum_j x_{S_{mij}b}$, for level m , and (iii) Substituted demand outgoing from product i , one for each level of substitution, with value $\sum_k x_{S_{mki}b}$, for level m . The dummy product corresponds to lost sale.

Retailers usually have some resource limitations, which are here collectively called *shelf space limitations*. In practice, the shelf space limitations might limit the total space that the stocked prod-

ucts in the category cover within the store, the number of products that the assortment contains, the number of suppliers that the assortment selects, and/or the number of SKUs (store keeping units) that the assortment contains. In our model, we limit the number of SKUs in the assortment. Implementation of other types of shelf space limitations can be handled with slight modifications in the model.

3.3. Illustrative example

In order to illustrate how this model can be used, we provide a small example with three products/brands: $P1$, $P2$, and $P3$; and 2 suppliers: $S1$ and $S2$, where $S1$ supplies product $P2$ and $S2$ supplies products $P1$ and $P3$. The initial inventories are assumed to be zero. Product $P4$ denotes lost sales and at most 3 levels of substitution exists ($M = 3$). The substitution matrix is provided in Table 2. The information in the table states that, for instance, among the customers whose favorite product is $P1$, 10% select $P2$ as the second choice, 20% select $P3$, and 70% leave without a purchase (lost sales), if $P1$ is not in the assortment or is stocked-out.

The parameter values for products are set as provided in Table 3 and the parameter values for suppliers are set as provided in Table 4. For this illustrative example, a single demand scenario is selected with probability one.

The model for this example is formulated as given in Table 15 in Appendix.

The optimal assortment provided by the solution of this model is composed of $P1$ with quantity 3400 and $P2$ with quantity 7000, implying that supplier $S1$ is not selected. With this assortment, all of the demand for products $P1$ and $P3$ are satisfied, whereas 40% of demand for product $P2$ is lost and 60% of demand for product $P2$ is substituted.

3.3.1. The importance of customer substitution behavior

In order to analyze the importance of customer substitution behavior, we first exclude the cost of substitution (TCS) from the objective function and solve this modified model. Next, we calculate the TCS corresponding to the optimal solution of the modified model and subtract this quantity from the optimal objective value.

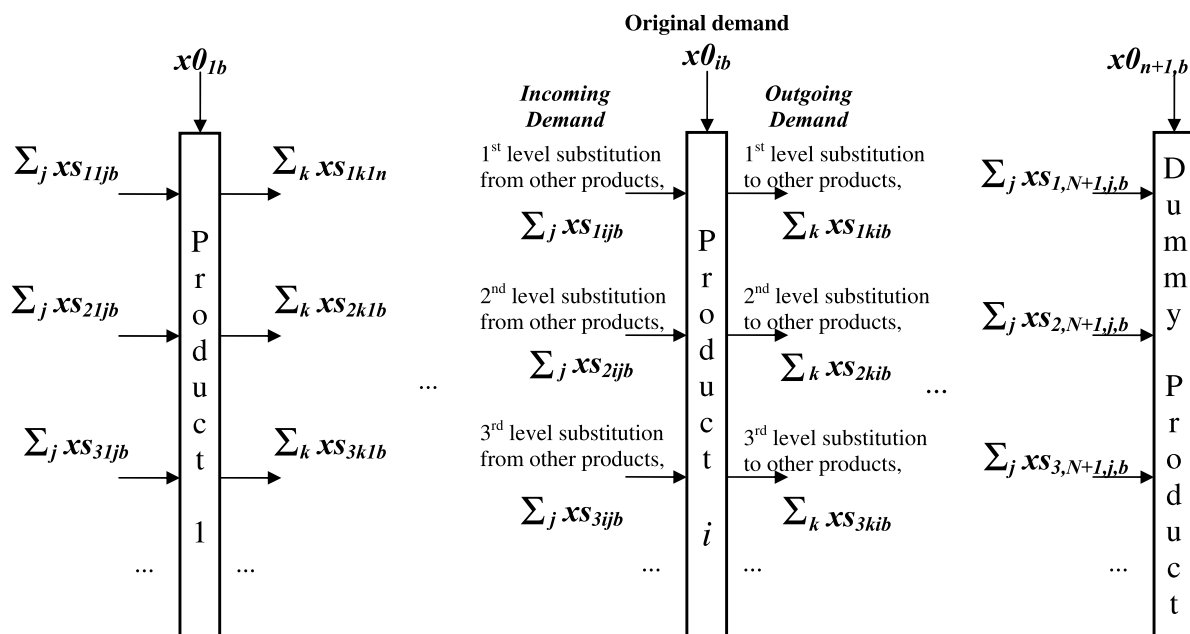


Fig. 1. An illustration of the rate of the demand substitution variables in the model for demand scenario b .

Table 6
Experimental data

Parameter	Distribution
w_{ik}	Uniform distribution, where $\sum_{k \in P} w_{ik} = 1$ and $0 \leq w_{ik} \leq 1, \forall i, k \in P$
c_i	Uniform distribution, where $5 \leq c_i \leq 10, \forall i \in P$
oc_j	Uniform distribution, where $30 \leq oc_j \leq 50, \forall j \in S$
ssc_j	Uniform distribution, where $15,000 \leq ssc_j \leq 50,000, \forall j \in S$
d_{ib}	$d_{ib} = 10,000 * \alpha_{ib}$, where α_{ib} has uniform distribution, with $0 \leq \alpha_{ib} \leq 1, \forall i \in P, \forall b = 1, \dots, B$ and $\sum_i \alpha_i = 1$ (total demand for products is assumed to be 10,000.)
OQ_i	Uniform distribution, where $4,000 \leq OQ_i \leq 34,000, \forall i \in P$
SS_i	Uniform distribution, where $8,000 \leq SS_i \leq 40,000, \forall i \in P$
h_i	Uniform distribution, where $0.3 \leq h_i \leq 1, \forall i \in P$
pq_i	Uniform distribution, where $2 \leq pq_i \leq 4, \forall i \in P$
q_i	Uniform distribution, where $0.0 \leq q_i \leq 0.15, \forall i \in P$
p_i	$p_i = c_i + mg_i$, where mg_i has a normal distribution with mean 6 and deviation 2, $\forall i \in P$. (mg_i can be considered as the margin of product i .)

of operating costs is given under the column SOC in this table. In Table 8, the average percentage of first-choice demand satisfied is given under the column %ds, the average percentage of lost sales is provided under the column %ls, the average percentage of substituted demand to all other products in the first, second and third level of substitutions are given under the columns %f, %s, %t, respectively, and the average percentage of sum of all levels of substitutions is provided under the column %subs, where %subs = %f + %s + %t. Table 8 shows how these values change as substitution cost increases. Figs. 2 and 3 illustrate the change in the costs and demand satisfaction percentages with respect to θ .

In Fig. 2, it is seen that as substitution costs increase, even though total revenue increases, total profit decreases since the sum of operational costs increases more than the total revenue.

The optimal system carries a limited assortment and selects a subset of suppliers and favors substitution since it generates no cost when $\theta = 0$. As θ increases, the optimal system extends the assortment and increases the number of selected suppliers. Therefore, supplier selection costs increase as seen in Table 7. This shows that substitution costs alter ordering and supplier selection decisions. An increase in substitution and lost sales costs results in increased amounts of purchases, resulting in decreased amounts of substitutions as seen in Fig. 3, as well as increased inventory. Therefore, purchasing and inventory costs increase as seen in Table 7. This shows that substitution costs affect purchasing and inventory decisions as well.

4.2. The importance of substitution behavior

We compare the solution of two models, the original one and the one that neglects substitution behavior. In Table 9, the optimal profit of the original model is provided under the column [TP], the

Table 7
The effect of changing substitution cost on operating costs

θ	TP	TR	TCO	TCSS	TCP	TCl	TCPQ	TCS	SOC
0.0	199,711	978,934	835	151,027	565,876	43,609	17,876	0	779,223
0.1	190,110	1,019,464	944	153,058	605,780	44,672	17,889	7,011	829,354
0.2	185,194	1,095,064	980	165,905	669,286	45,568	17,987	10,144	909,870
0.3	182,266	1,110,036	1,050	170,896	679,810	47,005	18,003	11,006	927,770
0.4	181,089	1,139,104	1,200	178,030	700,437	48,004	18,108	12,136	958,015
0.5	180,009	1,149,834	1,261	179,999	709,976	48,112	18,171	12,306	969,825
0.6	178,716	1,174,248	1,332	184,111	732,078	48,994	18,249	10,768	995,532
0.7	176,997	1,209,345	1,488	187,687	766,116	49,123	18,296	9,644	1,032,348
0.8	175,987	1,238,356	1,549	188,037	786,175	49,356	18,348	8,904	1,062,369
0.9	173,843	1,278,200	1,660	205,789	818,875	49,985	18,459	9,589	1,104,357
1	170,990	1,300,004	1,671	214,746	835,031	50,075	18,496	8,995	1,129,014

Table 8
The effect of changing substitution cost on substitution percentages

θ	%ds	%ls	%f	%s	%t	%subs
0.0	48.3	3.7	23	16.2	8.8	48
0.1	67.5	2.6	18.3	7.9	3.7	29.9
0.2	74.3	2.0	16.0	5.6	2.7	24.3
0.3	77.0	1.6	14.8	4.2	2.4	21.4
0.4	80.2	1.2	14.0	3.4	1.2	18.6
0.5	83.1	1.0	13.1	2.1	0.7	15.9
0.6	85.1	1.0	12.0	1.4	0.5	13.9
0.7	87.6	0.9	10.2	1.0	0.3	11.5
0.8	89.2	0.9	9.0	0.7	0.2	9.9
0.9	90.4	0.7	8.3	0.6	0.2	9.1
1	91.3	0.7	7.3	0.5	0.2	8

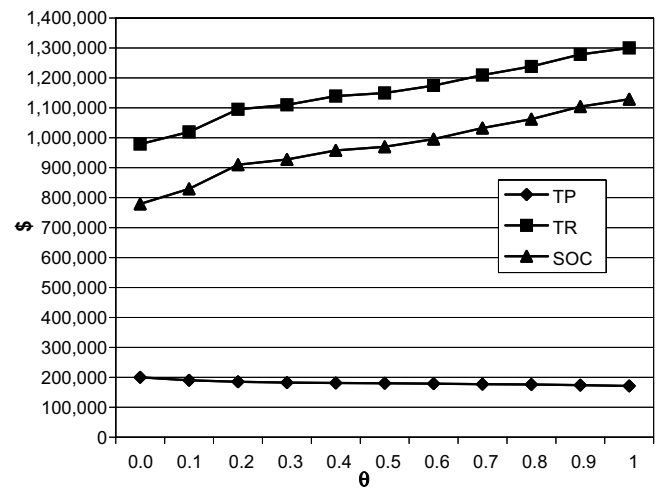


Fig. 2. Change in the profit, revenue, and operating costs as θ increases.

optimal profit of the model solved without substitution cost is given under the column [TP w/o sc], the difference between these values is provided under the column [Diff.], and the percentage of the difference compared to the optimal total profit of the original model is given under the column [%Diff.], where [%Diff.] = $100 * ([TP] - [TP w/o sc]) / [TP]$. The results show that increase in substitution costs increases profit loss if substitution behavior of customers is neglected, resulting in assortments which pay more substitution cost as shown in Fig. 4.

4.3. The importance of supplier selection decision

In our experiments, excluding the supplier selection decision resulted in the loss of more than half of the profit as shown in Table 10. In this table, the optimal expected profit of the original model is

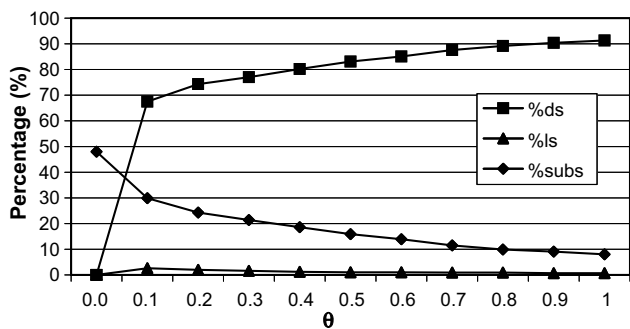


Fig. 3. Change in the percentages of satisfied demand, lost sales and substitution as θ increases.

Table 9
The effect of substitution behavior

θ	[TP]	[TP w/o sc]	[Diff.]	[%Diff.]
0	199,711	199,711	0	0
0.1	190,110	181,936	8,174	4.3
0.2	185,194	175,564	9,630	5.2
0.3	182,266	162,217	20,049	11.0
0.4	181,089	108,835	72,254	39.9
0.5	180,009	62,109	117,900	65.5
0.6	178,716	-7,668	186,384	104.3
0.7	176,997	-100,003	277,000	156.5
0.8	175,987	-196,809	373,796	212.4
0.9	173,843	-327,867	501,710	288.6
1	170,990	-376,550	547,540	320.2

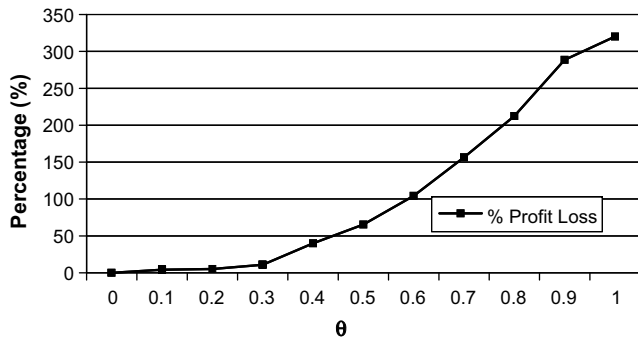


Fig. 4. Change in the percentage profit loss as θ increases in case of ignored substitution behavior.

Table 10
The effect of supplier selection decision

θ	[TP]	[TP w/o ss]	[Diff.]	[%Diff.]
0	199,711	95,662	104,049	52.1
0.1	190,110	86,125	103,985	54.7
0.2	185,194	83,337	101,857	55.0
0.3	182,266	82,936	99,330	54.5
0.4	181,089	80,947	100,142	55.3
0.5	180,009	80,364	99,645	55.3
0.6	178,716	79,388	96,328	55.5
0.7	176,997	77,012	99,985	56.4
0.8	175,987	76,946	99,041	56.2
0.9	173,843	76,266	89,877	56.1
1	170,990	75,975	95,015	55.5

provided under the column [TP], the optimal expected profit of the model excluding supplier selection decision is given under the column [TP w/o ss], the difference between these values is provided

under the column [Diff.], and the percentage of the difference compared to the optimal expected profit of the original model is given under the column [%Diff.], where $[\%Diff.] = 100 * ([TP] - [TP w/o ss]) / [TP]$. The reason for the profit loss is that if supplier selection is not included in the product assortment decision, the system offers an assortment supplied by more suppliers. Thus, a higher supplier selection cost is incurred.

4.4. The importance of shelf space limitations

As shown in the illustrative example in Section 3.3, in order to analyze the importance of the shelf space limitation in this problem, we first introduced an effective shelf space limitation of 8000 for the category and compared the optimal profit of the original model and the profit of the assortment obtained by distributing the limited shelf space proportionally among the products according to the product quantities of the assortment, which is proposed by the model without shelf space constraints. We set θ to 0.3 and generated 100 random data sets according the distributions provided in Table 6. When we look at the average expected profit over 100 data sets, we find that while the original model proposes an assortment with an average profit of 19,872, the model which excludes shelf space constraints has an average profit of 15,453. Therefore, excluding shelf space limitations results in 22.2% profit loss. When we examine the assortments proposed by both models, we observe that the model without shelf space limitation works with more suppliers paying more supplier selection costs. In order to generalize this observation we also performed experiments on the original model and observed the number of suppliers, which is denoted by $[N(S)]$. Table 11 shows the results obtained for the same experimental data except that the shelf space limitation varies. The results show that as shelf space limitations become looser, the number of selected suppliers increases though there exists a fixed cost for working with a supplier.

4.5. The impact of demand variability

In order to observe the impact of demand variability on substitution amounts as well as the optimal profit, ordering quantities, and supplier selection costs, we change demand variability and observe the corresponding parameters. We set θ to 0.3 and generated 100 random data sets according to Table 6 except that the parameters of individual demands and prices of products are generated according to Table 12. The mean of total demand over all products is assumed to be constant, namely 40,000 for all experiments, whereas total demand is uniformly distributed according to changing variances provided in Table 13. Again Monte Carlo sampling is

Table 11
The effect of shelf space limitations on the number of selected suppliers

Shelf space limitation (units)	[N(S)]
8,000	1.20
20,000	1.89
32,000	2.68

Table 12
Experimental data

Parameter	Value
d_{ib}	$d_{ib} = (\sum_i d_{ib}) * \alpha_{ib}$, where $\sum_i \alpha_{ib} = 1$ and $0 \leq \alpha_{ib} \leq 1, \forall i \in P$ (total demand distribution varies from one experiment to another according to Table 13.)
p_i	$p_i = c_i + mg_i$, where mg_i is random with $4 \leq mg_i \leq 8, \forall i \in P$

Table 13
The impact of demand variability

Uniform distribution range	Total substitution percentage	Optimal profit	Cost of ordering	Cost of supplier selection	Cost of substitution
$\sum_i d_{ib} = 40,000$	43.4%	103,259	108.2	81,414	33,254
$39,000 \leq \sum_i d_{ib} \leq 41,000$	29.6%	81,034	156.4	102,344	26,840
$35,000 \leq \sum_i d_{ib} \leq 45,000$	20.1%	66,309	203.5	102,344	22,397
$30,000 \leq \sum_i d_{ib} \leq 50,000$	11.3%	55,183	289.2	125,198	19,172

Table 14
The impact of demand variability

	Demand variability increases
Original demand satisfied	Increases
Substitution percentages	Decrease
The amount of lost sales	Decreases
Ordering quantities	Increase
Optimal profit	Decreases
Importance of substitution	Increases
Importance of supplier selection	Increases

used and for each data set, 100 demand scenarios are generated according to the provided demand distributions.

Table 13 provides the results of the experiments. Under the *Total substitution percentage* column, the sum of first, second, third levels of substitution percentages and lost sales percentages for all products is given. The substitution percentage of a product at a certain level is calculated as the ratio of the sum of substitutions from that product in that level to the demand for that product. Similarly, the lost sales percentage of a product is calculated as the ratio of lost sales amount from that product to the demand for that product.

It is observed in Table 13 that increasing demand variability decreases the substitution amounts. In addition, as the demand variability increases, the optimal profit decreases since the system orders more and pays more for ordering. By ordering more, the system makes less substitution and pays less substitution cost. However, since optimal profit decreases as demand becomes more variable, the proportion of substitution cost to total profit increases. Therefore, the importance of substitution increases as demand variability increases. Another result of increasing demand variability is increased supplier selection cost as observed in Table 13. In order to be able to satisfy the demand for products in case of variability in demand, the system extends its assortment and works with more suppliers. Therefore, as demand variability increases, the importance of integrated supplier selection decision increases. All of these observations are summarized in Table 14.

5. Conclusion

The problem of product assortment under customer-driven demand substitution in retail operations is analyzed in this paper. We developed a model for the multi-product inventory, product assortment and supplier selection problem with multi-level demand substitution. The behavior of the solution provided by the model is analyzed for the single-period problem with stochastic demand. The analysis is performed to examine the effects of three parameters, substitution cost, supplier selection cost, and shelf space limitations, separately. The results of the analysis can be summarized as follows. Varying levels of substitution costs affects purchasing, ordering, inventory management, and supplier selection decisions. In addition, high substitution and lost sales costs for retailers resulted in extended assortments and increased service-level, but reduced profit due to increase

in operational costs. This information may be useful for retailers in positioning themselves in the market and for pricing decisions. When the effect of neglecting customers' substitution behavior in the model is analyzed, it is observed that ignoring substitution in product assortment decision results in reduced profit. Therefore, retailers should try to understand customer substitution behavior and incorporate it into their operational policies. We also observed that excluding supplier selection decision may lead to significant profit loss, hence retailers need also take into account the costs associated with it. When the effect of ignoring shelf space limitations is analyzed, it is concluded that considering shelf space limitations in the model results in more profitable assortment and retailers can work with more suppliers under more space. When the impact of variability on the profit, operational costs, and substitution amounts is analyzed, it is observed that as demand variability increases, retailers should order more in order to satisfy the demand and have to pay more operational costs. In addition, they might need to extend the width of their assortment by working with more suppliers. It is seen that as demand variability increases, substitution, and supplier selection decision become more important in product assortment decision.

To the best of our knowledge, the problem of multiple products, multi-level product substitution, inventory planning, and supplier selection in the existence of shelf space limitations has not been considered in an integrated model in the literature. Therefore, the main contribution of this study is to provide an efficient tool to determine the product assortment for retailers, which considers supplier selection and inventory management decisions in the existence of shelf space limitations and customers' substitution behavior. In addition, using our model, retailers can position themselves in the market by solving the model for different customer segments that have different substitution behaviors. The model can also be used to decide on new product introduction, given the estimate of the profit margin of the product, substitution rates from/to this product, and its expected effect on the category demand. Another contribution of this paper is that the analysis of the developed model gives insights about the effect of substitution on product assortment, the importance of incorporating supplier selection decision to the product assortment problem, and the significance of shelf space limitations on determining the right assortment.

A number of modifications and extensions of the model developed in this paper are possible. For instance, in certain cases suppliers have to pay a fixed fee to the retailer in order to obtain shelf space in a given category. This situation can easily be handled by taking negative supplier selection costs. As for some extensions, first, vendor-managed inventory might be integrated into the model since it is an emerging trend in retailing that requires the rapid and accurate transfer of information between the retailer and its suppliers. Therefore, while integrating vendor-managed inventory into our model, inventory related issues should be modeled according to the agreements between the retailer and its suppliers. Second, promotional activities might be considered in the model. The studies show that promotions such as price cuts have significant effects on product choice [11] and product substitution [8].

Table 15
The model for the illustrative example

Maximize	$TP = TR - TCO - TCSS - TCP - TCI - TCPQ - TCS$
Subject to	$TR = 19x_1 + 14x_2 + 12x_3, \quad TCO = 40o_1 + 45o_2, \quad TCSS = 35000o_1 + 50000o_2,$ $TCP = 10x_1 + 8x_2 + 6x_3, \quad TCI = 0.7x_1/2 + 0.5x_2/2 + 0.4x_3/2,$ $TCP = 4 * 0.05 * x_1 + 3 * 0.1 * x_2 + 2 * 0.09 * x_3,$ $TCS = 0.3(9xs_{112} + 9xs_{113} + 6xs_{121} + 6xs_{123} + 6xs_{131} + 6xs_{132}),$ $\quad + 0.3 * 2(9xs_{212} + 9xs_{213} + 6xs_{221} + 6xs_{223} + 6xs_{231} + 6xs_{232}),$ $\quad + 0.3 * 3(9xs_{312} + 9xs_{313} + 6xs_{321} + 6xs_{323} + 6xs_{331} + 6xs_{332}),$ $xs_{112} \leq 0.1(3000 - x_{01}), \quad xs_{113} \leq 0.2(3000 - x_{01}), \quad xs_{114} \leq 0.7(3000 - x_{01}), \quad xs_{121} \leq 0.2(4000 - x_{02}),$ $xs_{123} \leq 0.5(4000 - x_{02}), \quad xs_{124} \leq 0.3(4000 - x_{02}), \quad xs_{131} \leq 0.1(5000 - x_{03}), \quad xs_{132} \leq 0.5(5000 - x_{03}),$ $xs_{134} \leq 0.4(5000 - x_{03}), \quad xs_{212} \leq 0.1(3000 - x_{01} - xs_{112} - xs_{113} - xs_{114}),$ $xs_{213} \leq 0.05(3000 - x_{01} - xs_{112} - xs_{113} - xs_{114}), \quad xs_{214} \leq 0.11(3000 - x_{01} - xs_{112} - xs_{113} - xs_{114}),$ $xs_{221} \leq 0.05(4000 - x_{02} - xs_{121} - xs_{123} - xs_{124}), \quad xs_{223} \leq 0.04(4000 - x_{02} - xs_{121} - xs_{123} - xs_{124}),$ $xs_{224} \leq 0.34(4000 - x_{02} - xs_{121} - xs_{123} - xs_{124}), \quad xs_{231} \leq 0.1(5000 - x_{03} - xs_{131} - xs_{132} - xs_{134}),$ $xs_{232} \leq 0.01(5000 - x_{03} - xs_{131} - xs_{132} - xs_{134}), \quad xs_{234} \leq 0.22(5000 - x_{03} - xs_{131} - xs_{132} - xs_{134}),$ $xs_{314} \leq 0.05(3000 - x_{01} - xs_{112} - xs_{113} - xs_{114} - xs_{212} - xs_{213} - xs_{214}),$ $xs_{324} \leq 0.051(4000 - x_{02} - xs_{121} - xs_{123} - xs_{124} - xs_{221} - xs_{223} - xs_{224}),$ $xs_{334} \leq 0.073(5000 - x_{03} - xs_{131} - xs_{132} - xs_{134} - xs_{232} - xs_{231} - xs_{234}),$ $x_1 \leq 10000, \quad x_2 \leq 12000, \quad x_3 \leq 9000, \quad x_1 \leq 12000y_1, \quad x_2 \leq 10000y_2, \quad x_3 \leq 20000y_3, \quad o_1 \geq y_2, \quad o_2 \geq y_1, \quad o_2 \geq y_3,$ $x_1, x_2, x_3, xs_{112}, xs_{113}, xs_{121}, xs_{123}, xs_{131}, xs_{132}, xs_{212}, xs_{213} \geq 0,$ $xs_{221}, xs_{223}, xs_{231}, xs_{232}, xs_{313}, xs_{321}, xs_{323}, xs_{331}, xs_{332} \geq 0,$ $y_1, y_2, y_3, o_1, o_2 \in \{0, 1\}.$

However, it will require understanding the reaction of customers to price discounts and assessing the effect of brand loyalty.

Acknowledgement

Financial support from KUMPEM-Koç University Migros Retail Education Center is gratefully acknowledged.

Appendix

See Table 15.

References

[1] Z.M. Avsar, M. Baykal-Gursoy, Inventory control under substitutable demand: A stochastic game application, *Naval Research Logistics* 49 (2002) 359–375.
 [2] P.S. Bender, R.W. Brown, M.H. Issac, J.F. Shapiro, Improving purchasing productivity at IBM with a normative decision support system, *Interfaces* 15 (1985) 106–115.
 [3] E. Bish, B. Maddah, On the interaction between variety, pricing, and inventory decisions under consumer choice, Working Paper, Virginia Tech, 2004.
 [4] A. Brooke, D. Kendrick, A. Meeraus, R. Raman, GAMS: A User's Guide, GAMS Development Co., 1998.
 [5] F.P. Buffa, W.M. Jackson, A goal programming model for purchase planning, *Journal of Purchasing and Materials Management* 19 (1983) 27–34.
 [6] G. Cachon, C. Terwiesch, Y. Xu, Retail assortment planning in the presence of consumer search, *Manufacturing and Service Operations Management* 7 (4) (2005) 330–346.
 [7] Z. Degraeve, E. Labro, F. Roodhooft, An evaluation of vendor selection models from a total cost of ownership perspective, *European Journal of Operational Research* 125 (2000) 34–59.
 [8] J.A. Donson, A. Tybout, B. Sternthal, Impact of deals and deal retraction on brand switching, *Journal of Marketing Research* 15 (February) (1978) 72–81.
 [9] M. Fadiloglu, O. Karasan, M. Pinar, A model and case study for efficient shelf usage and assortment analysis, Working Paper, Bilkent University, 2007.
 [10] V. Gaur, D. Honhon, Assortment planning and inventory decisions under a locational choice model, *Management Science* 52 (2006) 1528–1543.
 [11] P. Guadagni, J.D.C. Little, A logit model of brand choice calibrated on scanner panel data, *Marketing Science* 2 (1983) 317–328.

[12] A. Hsu, Y. Bassok, Random yield and random demand in a production system with downward substitution, *Operations Research* 47 (1999) 277–290.
 [13] ILOG, ILOG CPLEX User's Manual, 9.0 ed., 2003.
 [14] A.G. Kök, M.L. Fisher, Demand estimation and assortment optimization under substitution: methodology and application, *Operations Research* 55 (2007) 1001–1021.
 [15] A.G. Kök, M.L. Fisher, R. Vaidyanathan, Assortment planning: Review of literature and industry practice, in: N. Agrawal, S.A. Smith (Eds.), Invited chapter to appear in *Retail Supply Chain Management*, Kluwer Publishers, 2006.
 [16] S. Mahajan, G. van Ryzin, Quantitative models for supply chain management, Retail Inventories and Consumer choice, Kluwer Academic Publishers, 1999. Chapter 17, pp. 491–551.
 [17] S. Mahajan, G. van Ryzin, Stocking retail assortments under dynamic consumer substitution, *Operations Research* 49 (2001) 334–351.
 [18] S. Netessine, N. Rudi, Centralized and competitive inventory model with demand substitution, *Operational Research* 51 (2003) 329–335.
 [19] P. Noonan, When consumers choose: a multi-product, multi-location newsboy model with substitution, Technical report, Emory University, Atlanta, Georgia, 1995.
 [20] C. Öztürk, B. Tan, S. Karabati, A point-of-sale based method for estimation of product substitution rates and inventory control under stock-out based substitution, Working paper, Koç University, 2006.
 [21] M. Parlar, S.K. Goyal, Optimal ordering decisions for two substitutable products with stochastic demands, *OPSEARCH* 21 (1984) 1–15.
 [22] D.W. Pentico, The assortment problem with probabilistic demands, *Management Science* 21 (3) (1974) 286–290.
 [23] D.W. Pentico, The assortment problem with nonlinear cost functions, *Operations Research* 24 (6) (1976) 1129–1142.
 [24] D.W. Pentico, The discrete two dimensional assortment problem, *Operations Research* 36 (2) (1988) 324–332.
 [25] K. Rajaram, C.S. Tang, The impact of product substitution on retail merchandising, *European Journal of Operational Research* 135 (2001) 582–601.
 [26] U. Rao, J. Swaminathan, J. Zhang, Multi-product inventory planning with downward substitution, stochastic demand and setup costs, *IIE Transactions* 36 (2004) 59–71.
 [27] S.A. Smith, N. Agrawal, Management of multi-item retail inventory systems with demand substitution, *Operations Research* 48 (2000) 50–64.
 [28] G. van Ryzin, S. Mahajan, On the relationship between inventory costs and variety benefits in retail assortments, *Management Science* 45 (1999) 1496–1509.
 [29] E. Yücel, F. Karaesmen, F.S. Salman, M. Türkay, Product assortment under customer-driven demand substitution in retail operations, in: Proceedings of the International Conference on Information Systems, Logistics and Supply Chain, 2006.