



Investment in improved inventory accuracy in a decentralized supply chain

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Abstract

It is known that inaccurate inventory records can lead to profit losses in a supply chain. Inventory records may not be correct due to various reasons such as transaction errors, misplacement, shrinkage, etc. In order to eliminate inventory inaccuracy, companies may invest in new information technologies such as radio frequency identification (RFID). In this paper, we consider a supply chain consisting of a retailer (distributor) and a supplier. We assume a single-period newsvendor-type setting where the retailer purchases the items from the supplier and distributes them to the regional warehouses. The paper focuses on the problem of finding the optimal investment levels that maximize profit by decreasing inventory inaccuracy. The optimal level of investment is examined both for the centralized and the decentralized systems under two scenarios: inventory sharing between the warehouses is allowed and not allowed. The coordination problem is also considered for both scenarios. Finally, several extensions of the model are considered: asymmetric warehouse parameters, demand and inventory inaccuracy correlation and imperfect RFID implementation.

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1. Introduction

Supply chain inventory management decisions depend on inventory data gathered from automated or manual control systems. As a result of advances in information technologies, companies started to automate their inventory management processes and use inventory management software (Lee and

Ozer, 2005). Although the use of information technology (IT) has made collecting and storing data about the flow of items through supply chain easier and less expensive, the tracking of inventory remains prone to error. The data collected may not be accurate due to various reasons: incorrect product identification, transaction errors, inaccessibility of items due to improper usage of the depot, misplacements, shrinkage, etc. These may result in two problems: unplanned inventory depletion and addition. If the inventory records do not agree with the actual physical stock, either an order may not be placed in time or excessive inventory is held.

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Inventory inaccuracy appears to be a significant issue in practice as reported in a number of recent studies. Kang and Gershwin (2005) report inventory accuracies of a global retailer's stores. It is seen that the inventory accuracy is only 51% on average for 500 stores. In other words, the stores have accurate records for only about a half of the SKUs (stock keeping units). The best performing store in the study knows its actual inventory with only 75–80% accuracy. Raman et al. (2001) report similar findings for a leading retailer. Almost 370,000 SKUs are investigated for the retailer, it is concluded that more than 65% of the inventory records do not match with the physical inventory.

To cope with inventory inaccuracy, different compensation methods can be used, e.g. periodical review of inventory, tracking of items, eliminating some of its causes. In particular, RFID (radio frequency identification) technology which has received considerable attention in recent years helps to track items through the supply chain. This technology is different from bar code technology in two ways: it does not require line of sight and RFID tags have unique codes. Many companies consider investing in the RFID technology as pioneered by some major retailers such as Wal-Mart, Tesco, Marks&Spencer and by organizations such as United States Department of Defense.

The three main components of the RFID technology are: tag (transponder), antenna and reader. A tag contains a computer chip that holds data related to a product. Different types of tags are available according to their shape, size, memory properties and frequencies. The readers broadcast signals via antenna. The tags receive the signals and send the data to the readers by means of radio frequencies. The readers send the received data to the computer system for logging and processing. This identification provides tracking of items through the supply chain. However, using this technology requires a large investment. This investment consists of the cost of establishing the infrastructure as well as the costs of the tags and the readers. Tag price is one of the main issues of RFID; although pallet-level or case-level tagging is an option, tag prices are expected to be so low that they can be attached to every item. Besides tag prices, RFID implementations cost \$400,000 per distribution center and \$100,000 per store and \$35–\$40 million is required for the system integration of the entire organization (Kearney, 2004).

Motivated by the RFID investment issue, this research mainly focuses on the decision of the optimal investment levels in order to decrease the inventory inaccuracy in a two-level supply chain consisting of a supplier and a retailer. We consider both the centralized and decentralized systems. In a centralized system a central planner decides on the investment while in a decentralized system the investment decision is made either by the retailer or the supplier. Particularly, we defined the following research questions:

- What are the optimal investment levels in centralized and decentralized supply chains?
- What are the resulting benefits in terms of inventory costs?
- How does centralization affect investment decisions?
- What is the effect of inventory sharing on the investment decision?

In order to address the above questions, we analyze a supply chain consisting of a supplier and a retailer that has multiple warehouses. The demand for each warehouse is random. The model is investigated under two scenarios: (1) inventory sharing between warehouses is not allowed, (2) warehouses are able to share their inventories as needed. In addition, we address the issue of how to share the investment within a given class of contracts and investigate the related coordination aspects. Finally, several extensions of our basic model are considered: asymmetric warehouse parameters, demand and inventory inaccuracy correlation and imperfect RFID implementation.

The remainder of the paper is organized as follows. In the following section, the related literature is reviewed. Section 3 introduces the model and the underlying assumptions. The analysis of the model is presented in Section 4. Section 5 presents the numerical experiments and summarizes the main observations. Section 6 includes the extensions of our base model while the conclusion is presented in Section 7.

2. Literature review

This paper builds on three streams of literature: inventory inaccuracy, information technology investment and supply chain coordination. The literature about the RFID investment is mainly built on working papers, since the RFID technology is an

emerging technology and it has recently taken the attention of the researchers. Hence, the studies on this subject are not mature yet. The literature is given in three parts: inventory inaccuracy, the RFID investment and related literature.

Empirical studies have made clear the existence of the inventory inaccuracy problem. The first empirical study that addresses the inventory inaccuracy problem is performed by Rinehart (1960). The paper reports on a case study of discrepancies of a Federal government supply facility. Recently, Raman et al. (2001) performed an empirical analysis to reveal the inventory inaccuracy problem. They reported that 65% of nearly 370,000 inventory records from 37 stores of a large retailer are inaccurate. That is, the inventory record of an item fails to match the physical quantity found in the store. The profit lost due to inventory inaccuracy is reported to be 10%. In addition, misplacement can be observed even when the inventory records are accurate. For another leading retailer, it is reported that 16% of the items cannot be found in the store due to misplacement. It is also reported that misplaced items reduced profits by 25%. DeHoratius and Raman (2004) investigate the problem and find that the variation in inventory inaccuracy record is associated with the cost of an item, its annual selling quantity and the distribution method used to ship that product to the stores. Furthermore, Kang and Gershwin (2005) report similar findings for a global retailer's stores. The inventory accuracy is 51% on average for 500 stores and the best performing store in the study knows its actual inventory with 75–80% accuracy. Those empirical studies identify the magnitude of the inventory inaccuracy problem. Although there is a considerable amount of research that focuses on inventory management in the literature, most of this research assumes perfect knowledge of the inventory data. There are relatively few studies considering inventory error.

Iglehart and Morey (1972) is an early paper that studies the inventory inaccuracy problem. The objective of the paper is to select the proper frequency of inventory counts and additional safety stock by minimizing the sum of holding and counting costs when there is random demand and inventory inaccuracy. There are other articles studying counting frequencies and counting techniques to eliminate inventory inaccuracy: e.g. Buck and Sadowski (1983), Martin and Goodrich (1987), and Morey and Dittman (1996).

There are studies in which a specific reason of inventory inaccuracy is the focus. Camdereli and Swaminathan (2005) study the supply chain coordination issue under misplaced inventory. They analyze the effect of misplaced inventory on the ordering decision and compare the performance of the decentralized system with the centralized system and suggest coordinating the decentralized system by means of revenue sharing and buyback contracts. Kok and Shang (2005) consider the inventory inaccuracy problem. They work on finding a counting policy for an inventory replenishment problem to correct transaction errors. Inventory inaccuracy is modeled as random and additive errors as in our model. They develop a joint inspection and replenishment policy that minimizes total costs in a finite horizon and show that an inspection adjusted base-stock policy is near-optimal. Kang and Gershwin (2005) examine the shrinkage problem. They use simulation to see the effects of stock loss on stock outs and conclude that even a small rate of stock loss can create severe out-of-stocks.

Our research differs from above mentioned papers, since they do not model the investment decision in the decentralized system with multiple decision makers. Also, some of them focus on specific reasons of inventory inaccuracy. However, our model is built on a more general framework and focuses on investment decision under inventory inaccuracy.

Initial papers considering the RFID investment through the supply chain are emerging. Lee and Ozer (2005) review some of the ongoing research on RFID and suggest future research opportunities on the subject. They argue that there is a huge credibility gap of the value of RFID and call the academic community to produce models to obtain realistic estimates of the RFID value.

There are several papers specifically focusing on tag prices. Gaukler et al. (2003) study the introduction of item-level RFID in a decentralized supply chain and argue that the cost of item-level RFID should be allocated among the retailer and the supplier. de Kok et al. (2006) considers shrinkage (specifically theft) as the source of the inventory inaccuracy. By comparing shrinkage case with and without shrinkage case the break-even prices for an RFID tag is found. It is reported that the break-even prices are strongly correlated with the value of the items that are lost and the shrinkage fraction. Models of Gaukler et al. (2003) and de Kok et al.

(2006) are similar to our model, however there are some differences. Firstly, they assume that inventory inaccuracy is always negative. Secondly, they do not consider the costs of scanners, infrastructure and IT investments. Sahin (2004) studies a single-stage inventory system under inventory inaccuracy and builds several mathematical models and evaluates the value of the RFID system based on the constructed models. Sahin and Dallery (2004) explore the benefit of using the Auto-ID technology in improving the inventory accuracy in three stages including a supplier, a wholesaler and a retailer without considering the centralized case. A similar analysis to ours is performed by Rekik et al. (2004). Our research is similar in spirit but in contrast with the work of Rekik et al. (2004), we explicitly model the investment costs in the RFID infrastructure that depends on how the technology is deployed. Fleisch and Tellkamp (2005) use simulation to examine the relationship between inventory inaccuracy and performance in a three-stage supply chain. In a base model physical inventory and information system inventory differ due to low process quality, theft and items becoming unsaleables. The results of the paper show that an elimination of inventory inaccuracy can reduce supply chain costs and the out-of-stock level. More recently, Heese (2006) studied the inventory inaccuracy problem by considering RFID investment costs. Our model and focus are different in that we consider a multi-location supply chain and assume that partial investment is an option and investigate the effects of different supply chain policies (with or without inventory sharing) on the investment levels.

One objective of our model is to find the optimal number of warehouses where the technology is applied and the optimal order quantity, such as the problem of sharing information with customers. In those problems the variance of demand is decreased by communicating with customers.

Milgrom and Roberts (1988) investigate an information acquisition model for reducing demand uncertainty. They study the effects of communication with the customers on inventories and investigate the situation where the demand variance can be decreased by means of customer surveys. In the paper, the optimum amount of investment on obtaining demand information through customer surveys is found. Zhu and Thonemann (2004) considerably extend the framework of Milgrom and Roberts (1988) by investigating the benefits of sharing future demand information when customer

demands are correlated and the information given by the customers is imperfect. Although it is optimal to contact all or none of the customers if demand is not correlated and the information is perfect (Milgrom and Roberts, 1988), it is often optimal to share information with some customers if the demands are correlated and the information is imperfect (Zhu and Thonemann, 2004). Our model resembles the one in Zhu and Thonemann but we focus on the effect of multiple decision makers. In addition, we assume the reduction of inventory inaccuracy but not of demand variance. It will be seen later that in our models, the implementation of the technology lowers the variance of inventory inaccuracy. This makes our problem similar to the other variance reduction problems.

The literature in the field of supply chain coordination by contracts is vast. Cachon (2003) presents an extensive literature review about supply chain coordination with contracts. Cachon and Lariviere (2005) study strengths and limitations of revenue sharing contracts.

3. The model

Consider a supply chain consisting of a retailer (distributor) and a supplier. We assume a single-period newsvendor-type setting where the retailer purchases the items from the supplier and distributes them to her regional warehouses. It is assumed that the retailer sells the items to the customer at a unit price of r and the supplier's unit production cost is m . The wholesale price that the supplier charges the retailer is w , and the retailer has the chance to sell the unsold items at the end of the period, the salvage value of an unsold item is s .

More precisely, we consider a single selling period with random demand at each of the retailer's warehouses. In particular, it is assumed that the retailer has N regional warehouses and the regional demand for each warehouse has an independent normal distribution with mean μ_D and standard deviation σ_D . This assumption is made to keep the analysis tractable. In Section 6, we investigate several extensions that include correlated demand and asymmetric demand structures. There are two scenarios for our model: no inventory sharing (NIS) and inventory sharing (IS). Under the IS scenario, the warehouses are able to share their inventories as needed by lateral transshipments in order to avoid stockouts. In contrast, inventory sharing is not

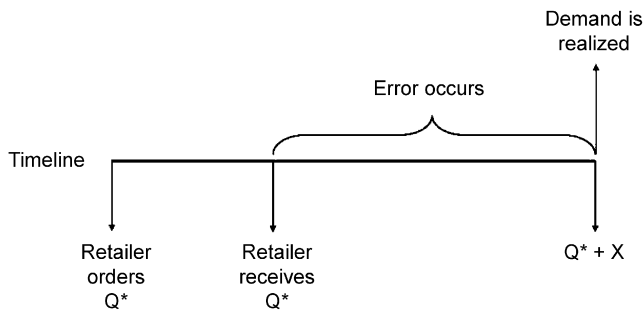


Fig. 1. The sequence of events.

allowed under the NIS scenario. The retailer decides on the total amount of inventory needed for her warehouses Q , $Q \geq 0$. At the beginning of the period, there is no inventory in the warehouses. After the retailer receives Q from the supplier, the inventory inaccuracy problem (due to shrinkage (losses), misplacement, unplanned additions, etc.) occurs and then the demand is realized. The sequence of events is represented in Fig. 1.

Clearly, under the above assumption, the total demand of the retailer is normally distributed with mean $N\mu_D$ and standard deviation $\sqrt{N}\sigma_D$ but the retailer should also take into account inventory inaccuracy to decide on the order quantity. Let us elaborate on the modeling of inventory inaccuracy which may be caused by many reasons. Those reasons can be summarized under three categories: misplacement, shrinkage (stock loss) and transaction errors (Lee and Ozer, 2005). Misplacement occurs when the products are somewhere in the facility but cannot be found. Generally, the inaccessible products eventually are found and become available for sale. The inventory misplacement can be corrected implying that the inventory can be greater than the inventory records. In our model, the misplaced products can be found or some products may be misplaced during the period. So, the inventory inaccuracy may cause an increase or a decrease in the number of products available in the warehouses. Stock loss, which is also known as shrinkage, is caused by all forms of loss of the products. The inventory records are higher than the actual inventory in case of stock loss. Finally, transaction errors may occur at the inbound or outbound of a facility during the registration of products. Such errors affect the inventory records but not physical inventory. It is stated in Lee and Ozer (2005) that shrinkage and misplacement are more challenging than transaction errors since they would remain unnoticed without tracking the items

by a technology such as RFID. As our main focus is RFID, our model considers shrinkage and misplacement as the main causes of inventory inaccuracy. So, the inventory records may be lower or higher than the actual inventory level. It should however be emphasized that negative errors are of particular concern since the products are lost for the current period and cannot be sold which is an important problem in our single-period setting.

We denote X_i as a random variable representing the inaccuracy of the inventory record. X_i represents the discrepancy between what is thought to be available and what is really available at the end of a period. In our context, X_i is the number of items that are lost or found between the reception of an order and the sales. This makes the error additive as in Kok and Shang (2005) and Sahin and Dallery (2004). More specifically, X_i is assumed to be normally distributed with mean μ_X and standard deviation σ_X for each warehouse. We assume that the retailer is aware of the inventory inaccuracy for each warehouse and gives her order by considering this in her ordering decision.

To model the above discussed structure, assume that the retailer orders the optimal order quantity Q^* from the supplier. However, due to the inaccuracy of the inventory records, the total actual quantity available for sale in the warehouses, \tilde{Q}^* , is $Q^* + X$ where $X = \sum_{i=1}^N X_i$. Therefore, the total actual inventory available to the retailer through the season is $\sum_{i=1}^N (Q_i^* + X_i)$. As a result, the system cannot satisfy the demand if $Q^* + X < D$ (equivalently $Q^* < D - X$) and has overstocked items if $Q^* + X > D$ (equivalently $Q^* > D - X$). The model with inventory inaccuracy X and demand D is then equivalent to a model with no inventory accuracy and demand $D - X$. To simplify the notation, we let $D' = D - X$. In other words, inventory inaccuracy affects the order quantity decision of the retailer in a similar way as demand uncertainty does. To summarize, with inventory inaccuracy, the total equivalent demand of the retailer is normally distributed with mean $N\mu_D - N\mu_X$ and standard deviation $\sqrt{N}(\sigma_D^2 + \sigma_X^2)$ (IS) or $N\sqrt{\sigma_D^2 + \sigma_X^2}$ (NIS). On the other hand, if RFID is applied at warehouse i then the actual inventory level at that warehouse is known with complete certainty and $\tilde{Q}_i^* = Q_i^*$. We assume that RFID technology eliminates the inventory inaccuracy problem. The random variable X_i is removed if the technology is applied in warehouse i (we consider the case of imperfect error removal in Section 6).

It is well known that RFID technology investment requires fixed and variable costs. The fixed cost includes establishing the infrastructure for the technology, whereas variable costs include cost of tags and maintenance cost. According to Kearney (2004), EPC (Electronic Product Code) and RFID implementations cost \$400,000 per distribution center and \$100,000 per store and additional costs for system integration range from \$35 to \$40 million for entire organization.

Motivated by the above structure, our model considers both the fixed and the variable investment costs. In our model, the fixed investment cost consists of the costs required to establish the infrastructure of the entire system while the variable investment costs include the costs of investment required for each warehouse to eliminate its inaccuracy. Although these costs may be very large as a one-time investment, since our model considers a single period, we interpret both fixed costs and variable costs as equivalent amortized costs per single selling season.

In short, decreasing the inventory inaccuracy of every warehouse has a cost, k (per warehouse) and making an investment requires a fixed cost, K . The fixed investment cost incurs when the technology is applied in one or more warehouses. The function $g(n)$ represents the variable cost incurred by the investment, n is the number of warehouses where the technology is applied to eliminate inventory inaccuracy. The variable investment costs depend on the number of warehouses where the new technology is used. So,

$$K_{\{n>0\}} = \begin{cases} K & \text{if } n>0 \\ 0 & \text{o.w.} \end{cases} \quad \text{and } g(n) = kn.$$

The objective of our model is to maximize the expected profit by finding the optimum number of warehouses to apply the technology. Making an investment decreases the number of warehouses that have inventory inaccuracy. The optimum number of warehouses n^* must be less than or equal to the total number of warehouses N and greater than 0. To simplify the analytical expression, we treat n as a continuous variable.

We solve the problem in two steps:

1. The optimum number of warehouses and the corresponding increase in profit are found ignoring the fixed cost.

2. If the increase in profit is greater than the fixed investment cost, then it is optimal to invest. Otherwise, the optimum solution is to make no investment.

4. Analysis of the model

In this section, we present the analysis of the model introduced. We first investigate the NIS scenario in detail and then mention the main findings for the IS scenario.

4.1. NIS scenario

Through this section, it is presumed that there is no inventory sharing between the warehouses. The retailer decides on Q_i , the amount of inventory needed for warehouse i , $i = 1, \dots, N$ and orders $\sum_{i=1}^N Q_i$ from the supplier.

4.1.1. The decentralized system

In this section, we focus on the case of a decentralized supply chain under two extreme scenarios: either the supplier makes the investment without any cost sharing support from the retailer or the retailer makes the investment without any support from the supplier.

4.1.1.1. The Retailer invests. For each warehouse, the retailer selects the optimal order quantity Q_i^* . When the investment is made at warehouse i , $Q_i^* = \mu_D + z_R \sigma_D$ otherwise $Q_i^* = \mu_D - \mu_X + z_R \sqrt{\sigma_D^2 + \sigma_X^2}$, where $z_R = \Phi^{-1}(\alpha_R)$. Φ^{-1} is the inverse cumulative distribution function of the standard normal distribution and α_R is the critical fraction for the retailer (see Zipkin, 2000) and for the decentralized system, it is given by: $\alpha_R = (r - w)/(r - s)$.

To incorporate inventory inaccuracy to our model, let us define δ_i such that:

$$\delta_i = \begin{cases} 1 & \text{if an investment is not made at warehouse } i, \\ 0 & \text{o.w.} \end{cases}$$

The expected profit of each warehouse is found under investment decision (using Zipkin, 2000).

$$E(\Pi_{W_i}) = (r - w)\mu_D - (r - s)\phi(z_R)\sqrt{\sigma_D^2 + \delta_i\sigma_X^2} + \delta_i w \mu_X - K_{\{n>0\}} - k,$$

where $\phi(z_R)$ denotes the standard normal density function for the decentralized system.

To find the expected profit of the retailer, we sum the expected profits of N warehouses. This gives:

$$E(\Pi_R) = \sum_{i=1}^N [(r-w)\mu_D - (r-s)\phi(z_R)\sqrt{\sigma_D^2 + \delta_i\sigma_X^2} + \delta_i w\mu_X] - K_{\{n>0\}} - kn.$$

The number of warehouses where the technology is applied is denoted by n . Then, the expected profit function of the retailer is written as:

$$E(\Pi_R) = (r-w)N\mu_D - (r-s)\phi(z_R)[n\sigma_D + (N-n)\sqrt{\sigma_D^2 + \sigma_X^2}] + w(N-n)\mu_X - K_{\{n>0\}} - kn.$$

The expected profit consists of five terms. The first term, $(r-w)N\mu_D$, is the sure profit. The second term, $(r-s)\phi(z_R)[n\sigma_D + (N-n)\sqrt{\sigma_D^2 + \sigma_X^2}]$, represents the cost associated with demand uncertainty (underage and overage costs). The cost of the items which are lost or found during the period, $w(N-n)\mu_X$, is added to the expected profit function, since the cost of those items are not paid in the current period. The last two terms, $K_{\{n>0\}}$ and kn are the fixed and variable investment costs of implementing the technology, respectively.

The retailer's expected profit function is linear in n . So, if the function is increasing, the optimal investment decision is making the full investment, otherwise the optimal decision is making no investment. The first derivative of the expected profit function is

$$\frac{\partial E(\Pi_R)}{\partial n} = (r-s)\phi(z_R)(\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D) - w\mu_X - k.$$

We observe that there is a threshold for the variable investment cost k such that making an investment becomes beneficial for the retailer. We define k_R^T as the variable investment threshold value where the retailer starts making a positive profit from making an investment. k_R^T for NIS scenario is equal to:

$$k_R^T = (r-s)\phi(z_R)[\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D] - w\mu_X. \quad (1)$$

Proposition 1. *If $k_R^T > k$ then $n^* = N$, otherwise $n^* = 0$.*

Proof. If k_R^T is greater than k , the expected profit of the retailer increases in n . The first derivative of the expected profit function with respect to n is positive. Since the maximum feasible n for our model is N , the optimal decision is making the full investment.

Otherwise, the expected profit function of the retailer decreases in n and the minimum feasible n is 0, so the optimal decision is making no investment. \square

The optimal solution is: $n^* = N$ or $n^* = 0$ ignoring fixed costs. This is similar to the corresponding result in Milgrom and Roberts (1988) in a different context.

Corollary 1 examines the impacts of the parameters σ_X , σ_D and μ_X on the investment threshold.

Corollary 1. *The investment threshold k_R^T increases in σ_X and decreases in σ_D and μ_X .*

Proof. The effects of σ_X and μ_X follow directly from (1). As σ_X increases, the variable investment threshold increases. When μ_X gets smaller, k_R^T increases.

The claim on the demand variance follows from (1). The expression $\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D$ is decreasing in σ_D , so the increase in σ_D causes a decrease in the variable investment cost k_R^T . The first derivative of $\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D$ with respect to σ_D is equal to $(\sigma_D/\sqrt{\sigma_D^2 + \sigma_X^2}) - 1$, which is less than 0. \square

Remark. In our numerical results, k_R^T is also increasing in the sales price, r .

According to Corollary 1, the retailer is more likely to make the investment as the initial inventory inaccuracy increases and the demand variance decreases. When the inventory inaccuracy is an important problem for the retailer, the retailer is more likely to make an investment to decrease it. In contrast, if there is high demand variance in the market, the retailer does not prefer spending much to decrease inventory inaccuracy, since the demand variance behaves like the inventory inaccuracy variance. This means decreasing inventory inaccuracy variance will not help decreasing uncertainty in the system due to the demand variance. As the mean of the inventory inaccuracy decreases, the problem becomes so important for the retailer that she can pay more to make the investment.

4.1.1.2. The supplier invests. When the supplier makes the investment, the expected profit function of the supplier, found by multiplying the optimal order quantity of the retailer by the profit margin of the supplier, is given by:

$$E(\Pi_S) = [n(\mu_D + z_R\sigma_D) + (N-n)((\mu_D - \mu_X) + z_R\sqrt{\sigma_D^2 + \sigma_X^2})](w-m) - K_{\{n>0\}} - kn.$$

Similarly to the retailer's case, the first derivative of the supplier's expected profit is linear in n which implies that there is an investment threshold for the supplier, k_S^T , found as:

$$k_S^T = [z_R(\sigma_D - \sqrt{\sigma_D^2 + \sigma_X^2}) + \mu_X](w - m). \quad (2)$$

It can be observed that the effects of the parameters on the supplier's investment threshold is similar to the retailer's case.

4.1.2. The centralized system

In the centralized supply chain, it is assumed that a central planner determines the amount of investment made by the entire supply chain and the order quantity to maximize the total profit. The optimal order quantity for the centralized system is

$$Q^* = \sum_{i=1}^N (\mu_D - \delta_i \mu_X + z_C \sqrt{\sigma_D^2 + \delta_i \sigma_X^2}).$$

Similar to the decentralized case, z_C is $\Phi^{-1}(\alpha_C)$ and α_C is the critical fraction of the centralized system given by: $\alpha_C = (r - m)/(r - s)$.

The expected profit function of the centralized system is shown to be similar to the expected profit function of the retailer and it is equal to:

$$E(\Pi_C) = (r - m)N\mu_D - (r - s)\phi(z_C)[n\sigma_D + (N - n)\sqrt{\sigma_D^2 + \sigma_X^2}] + m(N - n)\mu_X - K_{\{n>0\}} - kn.$$

As in the decentralized case, it can be shown that, there is an investment threshold for the variable investment cost k_C^T given by:

$$k_C^T = (r - s)\phi(z_C)[\sqrt{\sigma_D^2 + \sigma_X^2} - \sigma_D] - m\mu_X. \quad (3)$$

Paralleling the results in Corollary 1, it is observed that the investment threshold increases as the variance of demand decreases or the variance of inventory inaccuracy increases.

4.1.2.1. The effects of centralization on investment.

Comparing the variable investment threshold values of the retailer and the centralized system gives insight about the investment decisions of the retailer and the centralized system. To make the comparison easier, the value of μ_X is assumed to be 0. In order to compare the investments made by the centralized system and retailer, we should compare (1) and (3). Since all the parameters are equal except the density functions, $\phi(z_C)$ and $\phi(z_R)$, the value of the density functions must be compared to analyze

the investment decision. Since $\phi(z_C)$ can be greater than or less than or equal to $\phi(z_R)$, the investment threshold of the centralized system can be greater than or less than or equal to the investment threshold of the retailer. The conditions for the comparison of the investment thresholds are presented in Proposition 2.

Proposition 2. *The threshold values of the centralized system and retailer differ according to following conditions:*

1. *If $r - w = m - s$, then the investment threshold values are equal,*
2. *If $r - w > m - s$, then k_C^T is less than k_R^T ,*
3. *If $r - w < m - s$, then k_C^T is greater than k_R^T .*

Proof. If the sum of α_C and α_R is equal to 1, then the density functions are equal for both systems. So, it is concluded that if $r - w = m - s$, then the investment thresholds are equal. Both the centralized system and retailer are willing to make an investment, if the variable investment cost of eliminating inventory inaccuracy for each warehouse is less than the investment threshold ignoring the fixed investment cost.

$r - w > m - s$ means that sum of α_C and α_R is greater than 1. Since $w > m$, α_C is always greater than α_R . When the density functions are compared, the centralized system has a smaller density function value if $r - w > m - s$ holds. That is $\phi(z_C) < \phi(z_R)$ and from (1) and (3), $k_C^T < k_R^T$. As a result, the retailer has a higher tendency to make an investment. The third condition can be shown similarly. \square

According to Proposition 2, the retailer has a higher tendency to make an investment to decrease demand variance when her profit margin is high. In a way, she tends to make an unnecessary investment for the supply chain in that case since making an investment may not be the optimal strategy for the centralized system. In contrast, when her profit margin is lower, she may not want to make an investment even though it is beneficial for the centralized system.

4.1.3. Coordination of the supply chain

In this subsection, we investigate investment cost sharing structures between the retailer and the supplier through simple contracts. We consider the well-known revenue sharing contracts and investigate coordination issues under this contract and

discuss a straightforward extension to buyback contracts. Throughout the coordination section, μ_X is assumed to be 0, since having positive or negative μ_X can cause problems such as: how to share the revenue generated from the items which are not purchased in the current period (under the revenue sharing contract) or how to give the unsold items to the supplier if they are not purchased in the current period (under the buyback contract). It is also assumed that when the decision is making an investment by considering the variable investment threshold, the fixed investment cost is compensated.

4.1.3.1. Revenue sharing contract. Revenue sharing contracts coordinate the supply chain by dividing the revenue according to a given proportion, β and adjusting the wholesale price w accordingly. Under these contracts, the retailer keeps β portion of all revenue while the supplier takes $(1 - \beta)$ portion. A conventional revenue sharing contract coordinates the system by forcing the retailer to give the (centralized) optimum order quantity. However, in our model the fixed and variable investment costs are also needed to be shared to coordinate the system. We assume that the retailer pays θ_1 portion of the fixed investment cost and θ_2 portion of the variable investment cost and the supplier pays $(1 - \theta_1)$ portion of the fixed investment cost and $(1 - \theta_2)$ portion of the variable investment cost.

Under the revenue sharing contract, the expected profit functions of the retailer and supplier are as follows:

$$E(\Pi_R) = (\beta r - w)N\mu_D - \beta(r - s)\phi(z_R)(n\sigma_D + (N - n)\sqrt{\sigma_D^2 + \sigma_X^2}) - \theta_1 K_{\{n>0\}} - \theta_2 kn$$

and the expected profit function of the supplier is

$$\begin{aligned} E(\Pi_S) = & [n(\mu_D + z_R\sigma_D) \\ & + (N - n)(\mu_D + z_R\sqrt{\sigma_D^2 + \sigma_X^2})](w - m) \\ & + (1 - \beta)r[n\mu_D + (N - n)\mu_X] - \varphi(z_R)(n\sigma_D \\ & + (N - n)\sqrt{\sigma_D^2 + \sigma_X^2}) \\ & + (1 - \beta)s[z_R + \varphi(z_R)(n\sigma_D + (N - n) \\ & \times \sqrt{\sigma_D^2 + \sigma_X^2})] - (1 - \theta_1)K_{\{n>0\}} - (1 - \theta_2)kn, \end{aligned}$$

where φ is the standard normal loss function, $\varphi(z_R) = -z_R[1 - \Phi(z_R)] + \phi(z_R)$.

Proposition 3 establishes the optimal retailer portions of the fixed and variable costs θ_1^* and θ_2^* that coordinate the system. For system coordination, the investment decisions for the centralized and decentralized systems must be the same and the order quantities must be equal.

Proposition 3. *The coordinating contract parameters are as follows: $\theta_1^* = \theta_2^* = \beta$ and $w = \beta m$.*

Proof. As it is seen in (1) and (3), the investment decision is only affected by the standard normal density function and the standard normal density function depends on the critical ratio, α (when μ_X is equal to 0). The optimal order quantity is also obtained by using the critical ratio. It is known that the revenue sharing contract coordinates the system if the wholesale price w is equal to β percent of the unit production cost m (see Cachon and Lariviere, 2005).

$$\begin{aligned} z_C &= \frac{r - m}{r - s} = \frac{\beta r - \beta m}{\beta r - \beta s}, \quad \Phi^{-1}(z_C) \\ &= \Phi^{-1}(z_R) \quad \text{and} \quad \phi(z_C) = \phi(z_R) \end{aligned}$$

as a result, $Q_C^* = Q_D^*$ and $k_C^T = k_R^T$

If the wholesale price w is equal to β percent of the unit production cost m , the system makes the optimum investment and gives the optimum order quantity. We also know that the revenue sharing contract shares both the profit and the revenue according to the proportion β (Cachon and Lariviere, 2005). Since, the profit function is multiplied by β , sharing the total investment cost according to the same ratio coordinates the system. So, we conclude that:

$$\theta_1^* = \theta_2^* = \beta.$$

The revenue sharing contract coordinates our system where $w = \beta m$ and $\theta_1^* = \theta_2^* = \beta$. \square

Proposition 3 implies that there are coordinating revenue sharing contracts. Interestingly, the revenue sharing contract may coordinate the supply chain, even if the investment cost is not shared according to the ratio β . Below we investigate conditions under which coordination can be achieved. Note that, the following conditions coordinate the system only if making an investment is optimal for the centralized system. The gain of the retailer and supplier without considering the investment cost when the investment is made are equal to:

$$\vartheta_R = \beta(r - s)\phi(z_R)[N\sqrt{\sigma_D^2 + \sigma_X^2} - N\sigma_D]$$

and

$$\vartheta_S = [N\sqrt{\sigma_D^2 + \sigma_X^2} - N\sigma_D][(m - w)z_R + (1 - \beta)(r - s)\varphi(z_R)].$$

Consider a case where the supplier is stronger than the retailer. In this case, the following conditions may hold.

- The supplier pays all variable cost, $\theta_2 = 0$ and θ_1 is negotiated. The system is coordinated if:

$$\vartheta_R \geq \theta_1 K \quad \text{and} \quad \vartheta_S \geq (1 - \theta_1)K + kN.$$

Since the supplier is the strongest member of the supply chain, he can own all the variable investment cost and even he can share the fixed investment cost required for the infrastructure to support the retailer to make an investment.

- The supplier pays all fixed cost, $\theta_1 = 0$ and θ_2 is negotiated. The system is coordinated if:

$$\vartheta_R \geq \theta_2 kN \quad \text{and} \quad \vartheta_S \geq K + (1 - \theta_2)kN.$$

The supplier establishes the infrastructure of the system and supports the retailer to use the technology by paying a fraction of the variable investment cost.

Now, consider a case where the retailer is stronger and benefits more from making an investment on RFID than the supplier. In such a case, the retailer may want to own the greater part of the investment cost.

- $\theta_2 = \beta$ and θ_1 is negotiated. The system is coordinated if:

$$\vartheta_R \geq \theta_1 K + \beta kN \quad \text{and} \\ \vartheta_S \geq (1 - \theta_1)K + (1 - \beta)kN.$$

Since the retailer is the strongest member of the supply chain, she can own all the variable cost and shares the fixed investment cost required for the infrastructure to force the supplier to make an investment.

- $\theta_1 = \beta$ and θ_2 is negotiated. The system is coordinated if:

$$\vartheta_R \geq \beta K + \theta_2 kN \quad \text{and} \\ \vartheta_S \geq (1 - \beta)K + (1 - \theta_2)kN.$$

The retailer establishes the infrastructure of the system and forces the supplier to use the technology by paying a fraction of the variable investment cost.

Finally, the supplier and the retailer may have equivalent strengths and both of them may benefit from making an investment. In that case, both sides may negotiate on sharing the variable investment cost or the fixed investment cost.

- The retailer pays all variable costs, $\theta_2 = 1$ and θ_1 is negotiated. The system is coordinated if:

$$\vartheta_R \geq \theta_1 K + kN \quad \text{and} \quad \vartheta_S \geq (1 - \theta_1)K.$$

- The retailer pays all fixed costs, $\theta_1 = 1$ and θ_2 is negotiated. The system is coordinated if:

$$\vartheta_R \geq K + \theta_2 kN \quad \text{and} \quad \vartheta_S \geq (1 - \theta_2)kN.$$

The supplier and retailer negotiate on the investment decision. Since we initially assumed that making an investment is profitable, they can find fractions both for the variable investment cost and fixed investment cost such that making an investment will be beneficial for both parties.

In the above conditions, the number of warehouses and the variance of inventory inaccuracy have positive effects on supply chain coordination. An increase in those parameters causes an increase in the gains of both the retailer and the supplier. On the other hand, the variance of demand has a negative effect on the gain.

Finally, it should be noted that coordination can also be achieved by a modified buyback type contract with an additional parameter for investment cost sharing. In particular, [Cachon and Lariviere \(2005\)](#) show that in the newsvendor setting with a fixed price, for any coordinating buyback contract there exists a unique revenue sharing contract $\{\beta, w\}$ that generates the same profit for the retailer and supplier. By using this property, the investment can be shared between the supplier and the retailer by adding an extra parameter to the standard buyback contract (see [Uçkun, 2006](#) for more detail).

4.2. IS scenario

In this section, it is assumed that the warehouses are able to share their inventories as needed by lateral transshipments in order to avoid stockouts. Since our aim is benchmarking this structure with the NIS situation, we ignore the additional costs that may be incurred due to transshipments. Once again, the retailer decides on the total optimal order

quantity, Q^* for N warehouses and orders Q^* from the supplier.

4.2.1. The decentralized system

As in Section 4.1.1, two cases will be analyzed for the decentralized system: the retailer makes the investment and the supplier makes the investment.

4.2.1.1. The retailer invests. When the retailer considers the investment decisions without any support from the supplier, she selects the optimal total quantity Q^* to maximize her individual profits. This quantity is given by:

$$Q^* = N\mu_D - (N - n)\mu_X + z_R\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2}.$$

When the investment decision is made by the retailer and the total equivalent demand is normally distributed with mean $N\mu_D - (N - n)\mu_X$ and standard deviation $\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2}$, the expected profit function of the retailer is found to be:

$$E(\Pi_R) = (r - w)N\mu_D - (r - s)\phi(z_R)\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2} + w(N - n)\mu_X - K_{\{n>0\}} - kn.$$

The retailer's expected profit function is convex in n since:

$$\frac{\partial E(\Pi_R)^2}{\partial^2 n} = \frac{\sigma_X^4(r - s)\phi(z_R)}{4^{3/2}\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2}} \geq 0.$$

Since the expected profit function is convex, the optimal solution is either making no investment or making an investment to eliminate the inventory inaccuracy in all warehouses. Therefore, if we ignore the fixed costs: $n^* = N$ or $n^* = 0$.

As in the previous section, we observe that there is a threshold for the variable investment cost k such that making an investment becomes beneficial for the retailer. We define \bar{k}_R^T as the investment threshold value where the retailer starts making a positive profit from making an investment.

Proposition 4. If $\bar{k}_R^T > k$ then $n^* = N$, otherwise $n^* = 0$.

Proof. Since the expected profit function is convex, it is argued that if the full investment case results in lower cost than no investment case, then the investment is made to eliminate the inventory inaccuracy in all warehouses. This corre-

sponds to:

$$(r - s)\phi(z_R)\sigma_D\sqrt{N} + kN < (r - s)\phi(z_R)\sqrt{N(\sigma_D^2 + \sigma_X^2)} - wN\mu_X.$$

According to the above comparison, the investment threshold is found to be:

$$\bar{k}_R^T = \frac{(r - s)\phi(z_R)[\sqrt{N(\sigma_D^2 + \sigma_X^2)} - \sigma_D\sqrt{N}]}{N} - w\mu_X. \tag{4}$$

If the variable investment cost k is less than \bar{k}_R^T , then the optimal solution for the retailer is making the full investment when the fixed investment cost is ignored. □

The effects of parameters σ_X , σ_D and N on the investment threshold are given in Corollary 2.

Corollary 2. The investment threshold \bar{k}_R^T increases in σ_X and decreases in N , σ_D and μ_X .

Proof. The effect of N follow directly from (4). The other effects follow in a straightforward manner as in Corollary 1. □

Remark. In our numerical results, k_R^T is also increasing in price r .

The parameters σ_X and σ_D affect the investment threshold in the same way as in the NIS scenario. On the other hand, if there is a large number of warehouses, to decide on the investment, lower variable investment costs are expected by the retailer, because the total amount of investment is higher when the number of warehouses is large.

4.2.1.2. The supplier invests. When the supplier considers the investment without any support from the retailer, his expected profit function is:

$$E(\Pi_S) = (N\mu_D - (N - n)\mu_X + z_R\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2})(w - m) - K_{\{n>0\}} - kn,$$

where the optimal ordering quantity is equal to:

$$Q^* = N\mu_D - (N - n)\mu_X + z_R\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2}.$$

The expected profit function of the supplier is known to be convex when z_R is negative and concave when z_R is positive, since:

$$\frac{\partial E(\Pi_S)^2}{\partial^2 n} = -\frac{\sigma_X^4(w - m)z_R}{4^{3/2}\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2}}.$$

As in the retailer's case, we find that there exists an investment threshold for the variable investment cost \bar{k}_S^T such that there is a positive benefit for the supplier for all $k < \bar{k}_S^T$. The supplier prefers making an investment if:

$$(N\mu_D + z_R\sigma_D\sqrt{N})(w - m) - kN > (N(\mu_D - \mu_X) + z_R\sqrt{N(\sigma_D^2 + \sigma_X^2)})(w - m).$$

Then, \bar{k}_S^T is found to be:

$$\bar{k}_S^T = \frac{(w - m)z_R[\sigma_D\sqrt{N} - \sqrt{N(\sigma_D^2 + \sigma_X^2)}]}{N} + (w - m)\mu_X. \quad (5)$$

4.2.2. The centralized system

The optimal order quantity for the centralized system is

$$Q^* = N\mu_D - (N - n)\mu_X + z_C\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2}.$$

As in Section 4.1.2, the expected profit function of the centralized system is shown to be similar to the retailer's expected profit function and it is equal to:

$$E(\Pi_C) = (r - m)N\mu_D - (r - s)\phi(z_C)\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2} + m(N - n)\mu_X - K_{\{n>0\}} - kn.$$

The convexity of the expected profit function of the centralized system can be verified since:

$$\frac{\partial^2 E(\Pi_C)}{\partial n^2} = \frac{\sigma_X^4(r - s)\phi(z_C)}{4^{3/2}\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2}} \geq 0.$$

Just like in the decentralized case, due to convexity, the optimal solution is either making no investment or making an investment to eliminate the inventory inaccuracy in all warehouses.

Like \bar{k}_R^T , the threshold for the centralized system \bar{k}_C^T is

$$\bar{k}_C^T = \frac{(r - s)\phi(z_C)[\sqrt{N(\sigma_D^2 + \sigma_X^2)} - \sigma_D\sqrt{N}]}{N} - m\mu_X. \quad (6)$$

Similar to the decentralized case, if the investment variable cost k is less than \bar{k}_C^T , then the optimal solution for the centralized system is making the investment when the fixed investment cost is not considered.

As proposed in Corollary 2, \bar{k}_C^T increases in the inventory inaccuracy variance, σ_X and decreases in the number of warehouses, N and the demand variance, σ_D . In contrast to the NIS scenario, the

variable investment threshold decreases in the number of warehouses for the retailer and the centralized system.

The analysis performed to compare the investment decisions of the centralized system and the retailer in Proposition 2 is valid for the IS scenario.

When NIS and IS scenarios are compared, it is concluded that k_R^T is always greater than \bar{k}_R^T and k_C^T is always greater than \bar{k}_C^T (see Eqs. (1) and (4) and (3) and (6)). Under the IS scenario, the retailer and the centralized system have less tendency to make an investment. A general conclusion is that inventory sharing decreases the need for the RFID investment.

For all the scenarios proposed, after finding the optimum number of warehouses, the second step is making a comparison of the increase in the expected profit with the fixed investment cost and to decide whether to invest or not.

4.2.3. Coordination of the supply chain

As in the NIS scenario, to coordinate the supply chain under the investment decision, a revenue sharing contract or a modified version of the buyback contract is used. The mean of inventory inaccuracy, μ_X is assumed to be 0. We present the analysis for the revenue sharing contract. The corresponding results for the buyback contract can be found in Uçkun (2006).

Under the revenue sharing contract, the expected profit functions of the retailer and supplier are as follows:

$$E(\Pi_R) = (\beta r - w)N\mu_D - \beta(r - s)\phi(z_R) \times \sqrt{N\sigma_D^2 + (N - n)\sigma_X^2} - \theta_1 K_{\{n>0\}} - \theta_2 kn$$

and

$$E(\Pi_S) = (N\mu_D + z_R\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2})(w - m) + (1 - \beta)r(N\mu_D - \phi(z_R)\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2}) + (1 - \beta)s(z_R + \phi(z_R)\sqrt{N\sigma_D^2 + (N - n)\sigma_X^2}) - (1 - \theta_1)K_{\{n>0\}} - (1 - \theta_2)kn.$$

For system coordination, the investment decisions must be the same and the order quantities must be equal for the centralized and decentralized systems. The optimum θ_1^* and θ_2^* that coordinate the system are equal to β as proposed in Proposition 3.

In addition to the above case, it may be argued that under the coordination conditions explained

for the revenue sharing contract under the NIS scenario, the coordination can be achieved even if investment cost is not shared according to the ratio β . However, the conditions coordinate the system only if making an investment is optimal for the centralized system.

5. Numerical results

In this section, the findings of our study are illustrated by numerical examples. As the base case in the numerical examples, we used the following parameters: $\mu_D = 100$, $\sigma_D = 30$, $\mu_X = -2$, $\sigma_X = 10$, $r = 10$, $w = 5$, $m = 3$, $s = 2$ and $N = 10$. (The variable investment cost, k , changes for every example, since the profit improvements are compared. A k value which results in an investment decision is chosen.) In order to evaluate the effect of any parameter, we vary the value of that parameter while keeping the others at their base values.

Let us first investigate the impacts of changes in the financial parameters which have an effect through the critical fraction α . In Fig. 2, the x -axis corresponds to the α and the y -axis corresponds to the expected profit when the investment is made. The critical fraction is equal to $(r - m)/(r - s)$ for the centralized system and $(r - w)/(r - s)$ for the decentralized system. If the critical fraction is low, it can be interpreted as a low relative profit margin and vice versa. The results are reported for the centralized and decentralized systems under NIS and IS scenarios. Our first focus is quantifying the decentralization penalty (i.e. the profit difference between the decentralized and the centralized system). In Fig. 2, it is observed that this difference can be severe when the critical fraction is low for the retailer (see for example $\alpha_R = 0.25$ and 0.375). This is the well-known effect of double marginalization. It is also observed from Fig. 2 that the decentralization penalties of the NIS scenario are worse than the

ones of the IS scenario. This is due to the fact that under the IS scenario the system requires lower inventories thanks to the demand pooling effect even before the investment. Under both scenarios, the retailer may not make any investment when her profit margin is low, even though it is optimal for the centralized system (when $\mu_X = 0$, see Proposition 2).

In Fig. 3, the effects of the standard deviation of the inventory inaccuracy are investigated. In this figure, the y -axis corresponds to the profit improvement which is the difference of the expected profits of the system that makes a full investment and the one that makes no investment. It is observed that as the standard deviation of the inventory inaccuracy increases, the system (the centralized system and the retailer) benefits more from making an investment. If the variance of demand is high, trying to decrease the variance of inventory inaccuracy may be meaningless; since the profit improvement may not compensate the fixed investment cost. It is easier to make an investment if the standard deviation of demand is low (see Corollaries 1 and 2). Fig. 3 depicts that property with the numerical examples for the NIS and IS scenarios. As the standard deviation of demand increases, the profit improvement decreases (for the centralized system and the retailer). It is also observed in examples not reported here (see Uçkun, 2006) that the effect of demand variance on the investment decision decreases if the mean of the inventory inaccuracy is negative, since under this situation the inventory inaccuracy problem is more critical.

Next, we compare in Figs. 4 and 5 the individual profit improvements of the retailer and the supplier. The figures illustrate that when the wholesale price increases (or the retailer's critical fraction decreases), the profit improvements for the supplier are higher. In this situation, either too low or too high critical fractions result in low profit

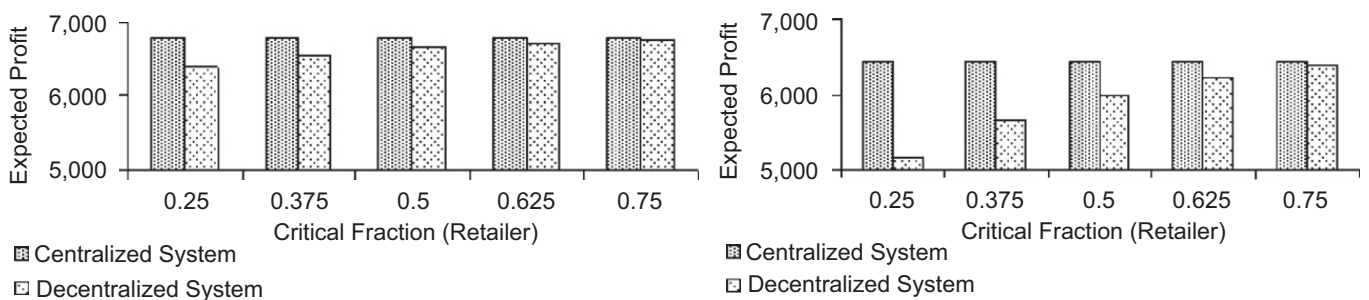


Fig. 2. The effect of investment decision on decentralization under the IS and NIS scenarios.

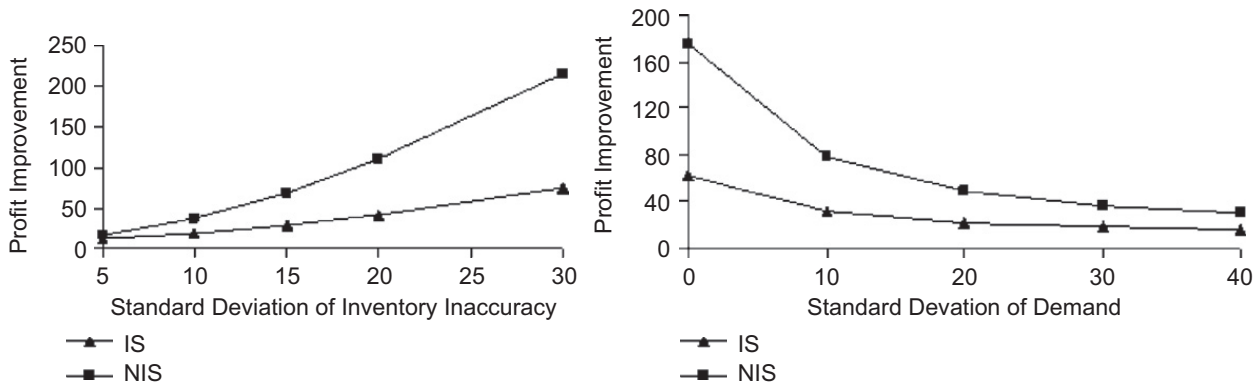


Fig. 3. The effects of the standard deviation of the inventory inaccuracy and the demand on investment decision.

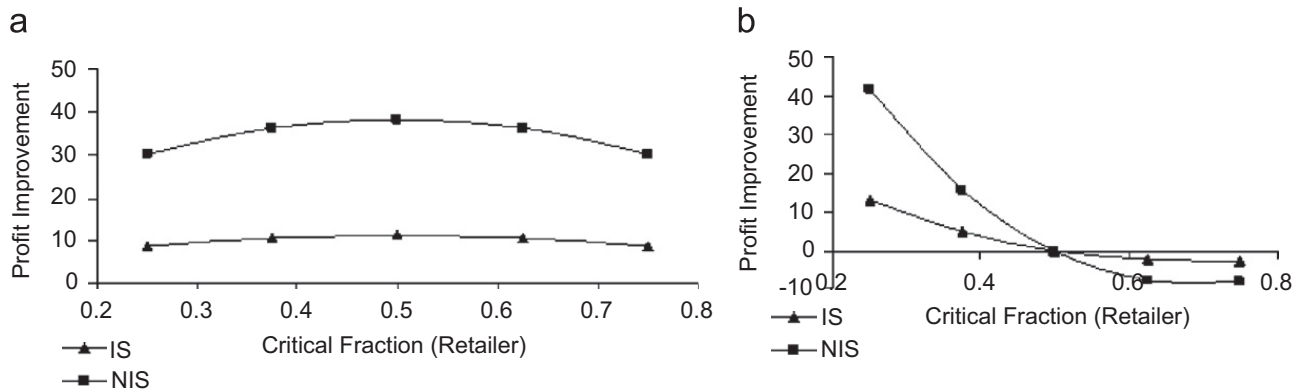


Fig. 4. The profit improvements for the retailer (left) and the supplier (right) ($\mu_x = 0$ and the retailer makes the investment decision and pays for the investment).

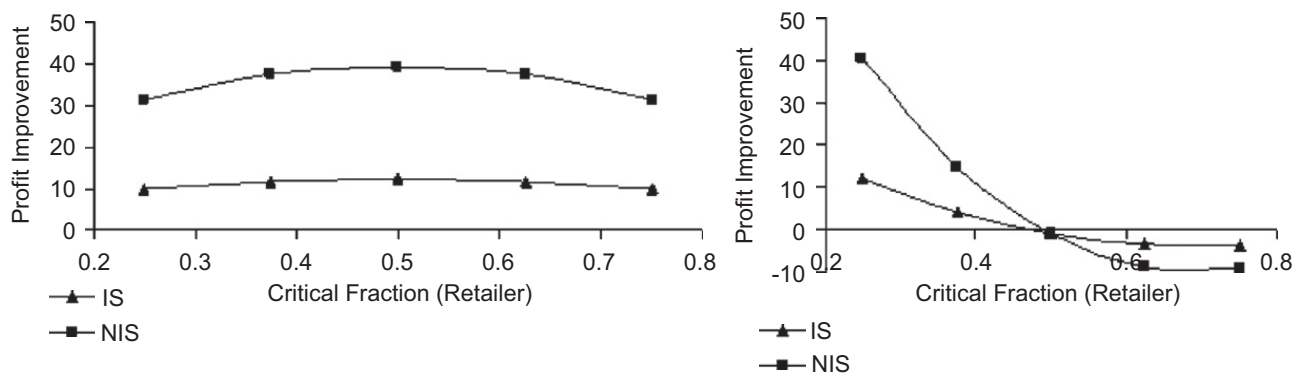


Fig. 5. The profit improvements for the retailer (left) and the supplier (right) ($\mu_x = 0$ and the supplier makes the investment decision and pays for the investment).

improvements for the retailer. If the supplier does not have a high profit margin, he may not prefer to make an investment. This can be seen, for instance, in the cases corresponding to $\alpha_R = 0.5, 0.6$ and 0.7 in Figs. 4 (right) and 5 (right). In addition, in other numerical examples, it was observed that the centralized system and the retailer prefer to make an investment if the mean of the inventory inaccuracy

is negative whereas the supplier prefers to make an investment if the mean of the inventory inaccuracy is positive (Uçkun, 2006).

In Fig. 6, the variable investment thresholds of the retailer for the IS and NIS scenarios are depicted. The retailer has a higher variable investment threshold when there is no inventory sharing. The same result is relevant for the centralized

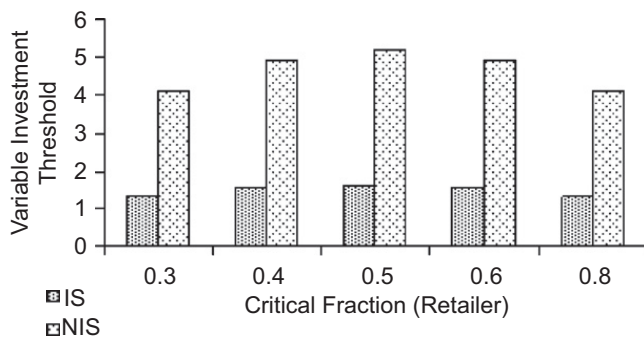


Fig. 6. Comparison of the variable investment thresholds of the retailer under IS and NIS scenarios, $\mu_X = 0$.

system. If the profit margin is either too low or too high, the variable investment threshold is low both for the centralized system and the retailer.

Finally, a common observation from our numerical studies is that if there is no inventory sharing between the warehouses, making an investment to decrease inventory inaccuracy is more beneficial.

In the following section several extensions of our model are considered and the results of relaxing some assumptions are represented.

6. Extensions

In this section, we consider some extensions of the model introduced in Section 3. We try to observe the effects of relaxing some assumptions. In Section 6.1, we analyze the case where the system parameters are asymmetric for every warehouse so that partial investment decisions can be optimal. In Section 6.2, we analyze the situation where the demands are correlated. In Section 6.3, imperfect RFID implementation is analyzed. Each section is analyzed both for NIS and IS scenarios. The analysis is performed by considering the centralized case, but the results are relevant for the decentralized case as well (the retailer makes the investment decision). Throughout this section, the fixed investment cost is not considered since it is easy to take into account and μ_X is assumed to be 0 to facilitate the comparison.

6.1. Asymmetric parameters

NIS scenario: The statistical and financial parameters for each warehouse were identical in the model in Section 3. Here, we investigate the situation with non-identical parameters. Since non-identical warehouse parameters result in different

investment thresholds, partial investment decisions can be optimal in the case of asymmetric parameters. The main results are summarized in Proposition of Appendix A.1.

In case of non-identical parameters, the decision in which warehouse to implement the technology is affected in the following ways. First, if demands for the warehouses are non-identical, the optimal policy is implementing the technology at warehouse for which the variance of the demand is the smallest. Second, if the inventory inaccuracies of warehouses are not identical, the optimal policy is implementing the technology in the warehouse for which the variance of inventory inaccuracy is the highest. The above results are obtained using similar arguments to Corollaries 1 and 2.

IS scenario: In the IS scenario, the ordering decision of a warehouse affects the other warehouses since the inventory is shared. Therefore, when deciding on investment, all investment alternatives must be considered.

Let us focus on the asymmetry of the parameters σ_X and k in order to keep the analysis tractable. If r , w and m were asymmetric for the warehouses, the expected profit function could not be written in the form that we used in the previous sections. This would bring new issues such as how to share the items that have different prices. Throughout the analysis, it is observed that the asymmetry of the mean μ_D and the standard deviation σ_D of demand do not have an effect on the investment decision.

In Appendix A.2, we show that there is a complementarity between the investments in different warehouses, which suggests that implementing the technology in a warehouse makes making the investment in other warehouses more attractive. We, therefore, argue that full investment is optimal for the system if the variable investment costs are low. On the other hand, it is found that if the variable investment costs are too large or relative values of inventory inaccuracies of the warehouses are different, making full investment may not be the optimal solution.

6.2. Demand correlation

NIS scenario: Let us relax the assumption of independent demands and consider correlated demands for the warehouses. For the NIS scenario, the demand correlation does not have an effect on the system. Under the NIS scenario, each warehouse is considered separately and the demand

correlation does not change the ordering decisions and expected profits. Since inventories of the warehouses are separate, the ordering decision is based on the marginal demand distributions.

IS scenario: The expected profit function can be written in case of demand correlation under IS scenario, the expected profit is shown to be convex in n . The optimal solution is still making the full investment or no investment. The details of the analysis can be found in Appendix B.

The variable investment threshold decreases as the correlation between warehouses increases. As the demand correlation increases, the effective demand variance also increases. Since an increase in the demand variance decreases the variable investment threshold, an increase in the demand correlation has the same effect on the variable investment threshold.

6.3. Imperfect RFID implementation

NIS scenario: Although RFID technology promises many benefits and most companies are ready to implement the technology, it has some shortcomings. First of all, pilot programs have shown that errors such as misread and no-read occur too often. Eighty percentage of success rate in reading is being identified in the report of [AMB Property \(2004\)](#). Secondly, radio frequencies are absorbed by liquids and reflected by metals. Such problems in implementation may result in imperfect implementation of the technology. In our initial model, we assume that when the technology is implemented in a warehouse, the inventory inaccuracy is completely eliminated. However, this may not be the case in real applications. We relax the assumption of perfect implementation in this section. Let t denote the fraction of inventory inaccuracy that is eliminated by investing on the RFID technology.

The expected profit and the variable investment threshold increase in efficiency of implementation, which means that the system can pay more for the investment if the implementation is perfect. The detailed analysis can be found in Appendix C.

IS scenario: As in the NIS scenario, the expected profit function and the investment threshold is increasing in t under the IS scenario. This is summarized in Appendix C.

7. Conclusion

One of the important premises of the RFID technology is decreasing the inventory inaccuracy.

We focused on the problem of how fixed and variable investment costs related to RFID affect a decentralized supply chain. Our model yields several insights on RFID investment cost sharing in a supply chain under different situations. Obviously, the RFID investment improves the supply chain efficiency by decreasing inventory inaccuracy under the two proposed scenarios if the per warehouse investment cost is under the threshold and the increase in expected profit compensates the fixed investment cost. The thresholds have different characterizations for the supplier and the retailer and different decisions may emerge when only one of the parties makes the investment. If the profit margin of the retailer is too low, she may not make an investment to decrease inventory inaccuracy although it is optimal for the centralized system. Also, the penalty of decentralization can be severe in cases where the profit margin of the retailer is low. Finally, the effect of the investment on supply chain efficiency is much more significant when there is no inventory sharing between the warehouses.

We can also characterize the important factors for the investment decision. Clearly, making an investment is easier when the per warehouse and the fixed investment costs are low. In addition, as the demand variance increases, the tendency of the system to make an investment decreases. If the market is characterized by highly uncertain demand, making an investment on the RFID technology to decrease inventory inaccuracy may not be reasonable. It is also observed that initiatives toward better supply chain efficiency such as increased demand pooling or inventory sharing between retailers diminishes the need for RFID investment.

Our analysis focuses on the inventory inaccuracy reduction aspect of the RFID technology. However, it is known that this technology may provide other benefits such as making warehouses smaller, improving shelf availability, decreasing out of stocks at the retail store level, etc. It would be a challenging but useful extension of our model to take into account the store-level benefits of RFID.

Appendix A. Asymmetric parameters

A.1. Asymmetric parameters in the NIS case

Let us relax the assumption of identical warehouse demands, inventory inaccuracies, prices, wholesale prices and production costs. Let μ_{D_i} and

σ_{D_i} denote the mean and standard deviation of warehouse i 's demand, σ_{X_i} denote the standard deviation of warehouse i 's inventory inaccuracy, and r_i, w_i and m_i denote price, wholesale price and production cost for warehouse i , respectively. The variable investment cost threshold for warehouse i is k_i^T . Proposition 5 identifies the optimum investment level when the parameters are non-identical for the warehouses.

Proposition 5. *If the system parameters ($\mu_D, \sigma_D, \sigma_X, r, w, m$ and s) are not identical for the warehouses, the optimal policy is ordering the warehouses according to the thresholds $k_1^T, k_1^T, \dots, k_N^T$ in decreasing order and making the investment in the warehouses whose threshold values are greater than the actual variable investment cost, k .*

Proof. In our initial model, we make the investment in warehouses for which the variable investment threshold is greater than the actual variable investment cost. (Invest if $k < k_C^T$ (without considering fixed investment cost), otherwise do not invest.)

For warehouse i :

if $k < k_C^T$, then the expected profit function increases by making an investment,
 if $k \geq k_C^T$, then the expected profit function decreases by making an investment.

If the warehouses are ordered in the decreasing order according to the their variable investment thresholds, there will be a warehouse for which making an investment becomes non-profitable. The optimal decision is investing up to this warehouse. \square

Proposition 5 defines the characteristic of optimal investment decision in case of non-identical warehouse parameters. The optimal policy is making no investment or full investment if the parameters are identical, because the investment thresholds of every warehouse are equal. In the asymmetric parameters case, the optimal solution is again implementing the technology in the warehouses that have greater variable investment thresholds than the actual variable investment cost. However, in that case the investment thresholds are not equal. So, a partial investment decision can be optimal. It should be noted that non-identical means of the warehouse demands do not affect the optimal decision.

A.2. Asymmetric parameters in the IS case

Here, we perform a marginal analysis to find out the optimal investment decision in the case of asymmetric parameters in the IS case. To this end, we use supermodularity and complementarity concepts. Complementarity suggests that having more of one variable increases the marginal returns to having more of the other variable. A function $f : E_i \times E_j \rightarrow R$ has increasing differences in (e_i, e_j) if the following inequality holds (Amir, 2005):

$$f(y + e_i + e_j) - f(y + e_i) \geq f(y + e_j) - f(y) \quad \forall y, e_i, e_j \in R. \quad (7)$$

Let us define $\Pi_C(\cdot)$ as the expected profit of the centralized system, \mathbf{y} as a vector representing the warehouses where the RFID is implemented, e_i and e_j as the unit vectors showing that the RFID technology is implemented in warehouses i and j . Please note that \mathbf{y} is a vector of 0,1 in R^N and e_i and e_j are unit vectors in R^N . In Proposition 6, inequality (7) is shown to be true for our model under some conditions.

Proposition 6.

$$\Pi_C(\mathbf{y} + e_i + e_j) - \Pi_C(\mathbf{y} + e_i) \geq \Pi_C(\mathbf{y} + e_j) - \Pi_C(\mathbf{y}) \quad \forall \mathbf{y}, e_i, e_j \in R^N. \quad (8)$$

Proof. Let us denote I as the set of warehouses where RFID is implemented and NI as the set of warehouses where RFID is not implemented and select two warehouses i and j such that $i, j \in NI$. Then, $\Pi_C(\mathbf{y} + e_i + e_j)$ is the expected profit when the technology is implemented in warehouses that belong to set I and in warehouses i and j , $\Pi_C(\mathbf{y} + e_i)$ and $\Pi_C(\mathbf{y} + e_j)$ are the expected profits when the technology is implemented in warehouses which belong to set I and additionally warehouse i and warehouse j , respectively, and $\Pi_C(\mathbf{y})$ is the expected profit when no additional investment is made (RFID is implemented in warehouses that belong to I).

To show that the inequality holds, we write the expected profits explicitly.

$$(r - m) \left(\sum_{q=1}^N \mu_{D_q} \right) - (r - s) \phi(z_C) \times \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2} - k_i - k_j$$

$$\begin{aligned}
 & - (r - m) \left(\sum_{q=1}^N \mu_{D_q} \right) + (r - s) \phi(z_C) \\
 & \times \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2 + \sigma_{X_j}^2 + k_i} \\
 & \geq (r - m) \left(\sum_{q=1}^N \mu_{D_q} \right) - (r - s) \phi(z_C) \\
 & \times \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2 + \sigma_{X_i}^2 - k_j} \\
 & - (r - m) \left(\sum_{q=1}^N \mu_{D_q} \right) + (r - s) \phi(z_C) \\
 & \times \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2 + \sigma_{X_i}^2 + \sigma_{X_j}^2}.
 \end{aligned}$$

By simplifying the above inequality, we obtain:

$$\begin{aligned}
 & \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2 + \sigma_{X_j}^2} \\
 & - \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2} \\
 & \geq \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2 + \sigma_{X_i}^2 + \sigma_{X_j}^2} \\
 & - \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2 + \sigma_{X_i}^2}.
 \end{aligned}$$

In the above form it is observed that the asymmetry of μ_D and σ_D do not affect the inequality. Multiplying and dividing both sides by their conjugates gives the following form:

$$\begin{aligned}
 & \frac{\sigma_{X_j}^2}{\sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2 + \sigma_{X_j}^2} + \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2}} \\
 & \geq \frac{\sigma_{X_j}^2}{\sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2 + \sigma_{X_i}^2 + \sigma_{X_j}^2} + \sqrt{\sum_{q=1}^N \sigma_{D_q}^2 + \sum_{p \in NI, p \neq i, j} \sigma_{X_p}^2 + \sigma_{X_i}^2}}.
 \end{aligned}$$

Looking at the above form of the inequality, we see that the inequality holds, since the denominator of the term on the left is less than the denominator of the term on the right. \square

Proposition 6 establishes that investing in warehouse j is more profitable when an investment is made at warehouse i . At first sight, Proposition 6 seems to suggest that implementing the technology at all warehouses is optimal, since implementation has a complementarity property. However, in situations where σ_X is higher for a warehouse relatively and/or the variable investment costs for the warehouses are high, the profit differences should be investigated carefully. In those situations, partial investment or no investment decisions may be optimal, since the increases in profits can be negative.

Corollary 3. *If (1) σ_{X_i} is strictly higher than σ_{X_j} and (2) k_i and k_j are sufficiently large (for any given i and j), the optimal policy may not be making the full investment.*

Proof. The values of the parameters may affect the optimal solution, since inequality (8) may hold since investing at a warehouse may not be profitable in some cases. Not to ignore those cases, the expected profit increases should be investigated carefully. To show that the optimum investment decision can be affected by the relative values of σ_X and k , let us examine the following inequality which is equal to inequality (8).

$$\begin{aligned}
 & \Pi_C(\mathbf{y} + e_i + e_j) - \Pi_C(\mathbf{y}) \geq \Pi_C(\mathbf{y} + e_i) - \Pi_C(\mathbf{y}) \\
 & + \Pi_C(\mathbf{y} + e_j) - \Pi_C(\mathbf{y}) \quad \forall \mathbf{y}, e_i, e_j \in R^N. \quad (9)
 \end{aligned}$$

In this inequality, the profit improvement when full investment is made is compared with the profit improvements of making individual investments. It is argued that inequality (9) holds since it is equivalent to inequality (8). However, by looking at this inequality, it is realized that making an investment may decrease the expected profit of

warehouse i due to high variable investment cost. Although the equation holds, making the investment in two warehouses may not be optimal. Relative values of inventory inaccuracy variances

may affect the inequality in the same way. As a result, it is certain that if all the expected profit increases are positive, the optimal investment decision is still making the full investment. Otherwise, profit improvements should be checked for each pair. \square

For example to Corollary 3, the following four cases which result in different solutions can be outlined:

- $N = 2, \sigma_{X_1} = 20, \sigma_{X_2} = 10, \sigma_{D_1} = 10, \sigma_{D_2} = 10, r = 5, m = 3, s = 0$ and $k_1 = k_2 = 10$. The difference between no investment and full investment cases is $\Pi_C(\mathbf{y} + e_1 + e_2) - \Pi_C(\mathbf{y}) = 3.8$ and investing in the first warehouse increases the profit by $\Pi_C(\mathbf{y} + e_1) - \Pi_C(\mathbf{y}) = 7.7$ and investing in the second warehouse increases the profit by $\Pi_C(\mathbf{y} + e_2) - \Pi_C(\mathbf{y}) = -6.2$. Clearly, the optimal decision is investing in warehouse 1 only. In this case, the inventory inaccuracy variance of warehouse 1 is higher than the inventory inaccuracy variance of warehouse 2.
- $N = 2, \sigma_{X_1} = 20, \sigma_{X_2} = 14, \sigma_{D_1} = 10, \sigma_{D_2} = 10, r = 5, m = 3, s = 0$ and $k_1 = k_2 = 20$. $\Pi_C(\mathbf{y} + e_1 + e_2) - \Pi_C(\mathbf{y}) = -12.8, \Pi_C(\mathbf{y} + e_1) - \Pi_C(\mathbf{y}) = -3.9$ and $\Pi_C(\mathbf{y} + e_2) - \Pi_C(\mathbf{y}) = -12.8$. As it is seen, investing in the warehouses decreases the expected profit (the variable investment costs are too large), the optimal decision is making no investment. In this case, the variable investment costs are large.
- $N = 2, \sigma_{X_1} = 20, \sigma_{X_2} = 20, \sigma_{D_1} = 10, \sigma_{D_2} = 10, r = 5, m = 3, s = 0$ and $k_1 = 5, k_2 = 15$. $\Pi_C(\mathbf{y} + e_1 + e_2) - \Pi_C(\mathbf{y}) = 13.8, \Pi_C(\mathbf{y} + e_1) - \Pi_C(\mathbf{y}) = 8.8$ and $\Pi_C(\mathbf{y} + e_2) - \Pi_C(\mathbf{y}) = -1.2$. Investing in warehouse 1 is profitable since the variable investment cost is low for warehouse 1. Although investing in warehouse 2 alone is not profitable, the optimal decision is still making the full investment.
- $N = 2, \sigma_{X_1} = 30, \sigma_{X_2} = 20, \sigma_{D_1} = 10, \sigma_{D_2} = 10, r = 5, m = 3, s = 0$ and $k_1 = k_2 = 10$. $\Pi_C(\mathbf{y} + e_1 + e_2) - \Pi_C(\mathbf{y}) = 27.5, \Pi_C(\mathbf{y} + e_1) - \Pi_C(\mathbf{y}) = 17.5$ and $\Pi_C(\mathbf{y} + e_2) - \Pi_C(\mathbf{y}) = 0.75$, the optimal decision is making the full investment.

Appendix B. Correlated demands

Since demand correlation has no effect on the decision in the NIS case, we focus only on the NIS case in this section. The demand for warehouse

i is normally distributed with mean μ_D and variance σ_D^2 and demands for any two warehouses are correlated with correlation coefficient $\rho_D, -1/(N - 1) < \rho_D < 1$.¹ By recalling that:

$$Var\left(\sum_{i=1}^N D_i\right) = \sum_{i=1}^N \sigma_{D_i} + 2 \sum_{i=1}^N \sum_{j<i}^N Cov(D_i, D_j)$$

and

$$Cov(D_i, D_j) = \rho_D \times \sigma_{D_i} \times \sigma_{D_j}$$

the total demand variance is $\tau_D^2 = [N + N(N - 1)\rho_D]\sigma_D^2$ and total variance affecting the system is $\tau_D^2 + N\sigma_X^2$.

Depending on the above structure, the optimal total order quantity of the centralized system is

$$Q^* = N\mu_D + z_C \sqrt{(N + N(N - 1)\rho_D)\sigma_D^2 + (N - n)\sigma_X^2} \tag{10}$$

If the demands for warehouses are correlated, the expected profit function of the centralized system is

$$E(\Pi_C) = (r - m)N\mu_D - (r - s)\phi(z_C) \times \sqrt{(N + N(N - 1)\rho_D)\sigma_D^2 + (N - n)\sigma_X^2} - K_{(n>0)} - kn.$$

It is seen in Eq. (10) that when z_C is positive (underage cost is higher than overage cost), as the demand correlation increases the centralized system's total order quantity increases. When z_C is negative (underage cost is lower than overage cost), as the demand correlation increases the centralized system's total order quantity decreases. On the other hand, the expected profit function decreases in ρ_D and the negative correlation is beneficial for the system, since the system has the ability to share the inventories between warehouses.

As it is seen, the demand correlation between the warehouses does not affect the structure of the expected profit function. The expected profit function is still convex in n , since:

$$\frac{\partial E(\Pi_C)^2}{\partial^2 n} = \frac{(r - s)\phi(z_C)\sigma_X^4}{4 \sqrt[3]{(N + N(N - 1)\rho_D)\sigma_D^2 + (N - n)\sigma_X^2}} \geq 0.$$

¹To ensure that the total demand variance is positive (Zhu and Thonemann, 2004).

The optimal solution is still making the full investment or no investment. However, the variable investment threshold is affected by the correlation of demands. The variable investment threshold value when demands are correlated is

$$\bar{k}_C^T = \frac{(r-s)\phi(z_C)[\sqrt{(N+N(N-1)\rho_D)\sigma_D^2 + N\sigma_X^2} - \sqrt{(N+N(N-1)\rho_D)\sigma_D^2}]}{N}$$

The variable investment threshold is decreasing in ρ_D , as can be verified by taking its first derivative:

$$\begin{aligned} \frac{\partial \bar{k}_C^T}{\partial \rho_D} &= \frac{1}{2} \phi(z_C)(N-1)(r-s)\sigma_D^2 \\ &\times \left(\frac{1}{\sqrt{(N+N(N-1)\rho_D)\sigma_D^2 + N\sigma_X^2}} \right. \\ &\left. - \frac{1}{\sqrt{(N+N(N-1)\rho_D)\sigma_D^2}} \right) \leq 0. \end{aligned}$$

As the demand correlation increases, the effective demand variance also increases. Since an increase in the demand variance decreases the variable investment threshold, an increase in the demand correlation has the same effect on the variable investment threshold.

Appendix C. Imperfect implementation

C.1. Imperfect implementation in the NIS case

Let us consider the NIS scenario first. The optimum order quantity for the warehouse in which the RFID is implemented is

$$Q^* = \mu_D + z_C \sqrt{(1-t)\sigma_X^2 + \sigma_D^2}$$

and for the warehouse in which the RFID is not implemented is

$$Q^* = \mu_D + z_C \sqrt{\sigma_X^2 + \sigma_D^2}.$$

The expected profit function is

$$\begin{aligned} E(\Pi_C) &= (r-m)N\mu_D - (r-s)\phi(z_C) \\ &\times \left[n\sqrt{\sigma_D^2 + (1-t)\sigma_X^2} + (N-n) \right. \\ &\left. \times \sqrt{\sigma_D^2 + \sigma_X^2} \right] - K_{\{n>0\}} - kn. \end{aligned}$$

It is observed that as the efficiency of implementation increases (as t increases), the expected profit increases, which is an intuitive result.

Imperfect implementation does not affect the structure of the expected profit function. The

function is increasing if:

$$\begin{aligned} \frac{\partial E(\Pi_C)}{\partial n} &= (r-s)\phi(z_C) \left(\sqrt{\sigma_D^2 + \sigma_X^2} \right. \\ &\left. - \sqrt{\sigma_D^2 + (1-t)\sigma_X^2} \right) - k \geq 0. \end{aligned}$$

The optimum solution is making the full investment, if the first derivative above is greater than 0, and making no investment otherwise.

The expected profit function is increasing when:

$$k \geq (r-s)\phi(z_C) \left[\sqrt{\sigma_D^2 + \sigma_X^2} - \sqrt{\sigma_D^2 + (1-t)\sigma_X^2} \right]. \tag{11}$$

The above expression characterizes the variable investment threshold. The variable investment threshold increases in t , which means the system can pay more for investment if the implementation is perfect.

C.2. Imperfect implementation in the IS case

For the IS scenario, similar results to the NIS scenario are obtained. The total optimum order quantity and the expected profit function in case of imperfect implementation are

$$Q^* = N\mu_D + z_C \sqrt{(N-nt)\sigma_X^2 + N\sigma_D^2}$$

and

$$\begin{aligned} E(\Pi_C) &= (r-m)N\mu_D - (r-s)\phi(z_C) \\ &\times \sqrt{(N-nt)\sigma_X^2 + N\sigma_D^2} \\ &- K_{\{n>0\}} - kn. \end{aligned}$$

Convexity of the expected profit function is verified since:

$$\frac{\partial E(\Pi_C)^2}{\partial^2 n} = \frac{\sigma_X^4 (r-s)\phi(z_C)t^2}{4^{3/2}\sqrt{N\sigma_D^2 + (N-nt)\sigma_X^2}} \geq 0.$$

In case of imperfect implementation under the IS scenario, the variable investment threshold is

$$k_C^T = \frac{(r-s)\phi(z_C)[\sqrt{N(\sigma_D^2 + \sigma_X^2)} - \sqrt{N\sigma_D^2 + (N-Nt)\sigma_X^2}]}{N}$$

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