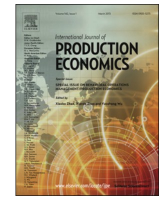
Int. J. Production Economics 154 (2014) 178–189

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe



Newsvendor model with random supply and financial hedging: Utility-based approach

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ARTICLE INFO

Article history:
Received 9 January 2013
Accepted 2 April 2014
Available online 18 April 2014

Keywords:
Newsvendor model
Utility theory
Minimum-variance portfolio
Financial hedging

ABSTRACT

This paper takes a utility-based approach to the single-period and single-item newsvendor model. Unlike most models in the literature the newsvendor is not necessarily risk-neutral and chooses the order quantity that maximizes the expected utility of the cash flow at the end of the period. We suppose that there is uncertainty in demand as well as supply. Furthermore, random demand and supply may be correlated with the financial markets. The newsvendor exploits this correlation and manages his risks by investing in a portfolio of financial instruments. The decision problem therefore includes not only the determination of the optimal ordering policy, but also the selection of the optimal portfolio at the same time. We first use a minimum-variance approach to select the portfolio. The analysis results in some interesting and explicit characterizations on the structure of the optimal policy. We also present numerical examples to illustrate the effects of the parameters on the optimal order quantity, and the importance of financial hedging on risk reduction.

1. Introduction

Inventory models, including the single-period and single-item newsvendor model, in which the decision maker needs to choose an appropriate order quantity that balances the cost of ordering too many against the cost of ordering too few, have received significant attention in the literature. Within this literature, much has been written about the newsvendor who aims to maximize expected profit or minimize expected cost. However, there is abundant evidence that decision makers are sensitive to risk and interest in risk-sensitive approaches is increasing. Expected utility theory provides by far the most widely used method for modeling risk-sensitivity in decision making. In this framework, the aim of the risk-averse decision maker is to maximize the expected utility of the cash flow. The utility function represents the risk-sensitivity of the decision maker and it has been used in financial decision making for a very long time despite its drawbacks. Our primary aim in this paper is to present a utility-based approach to the classical newsvendor problem in inventory management.

Expected utility maximization in inventory models began with Lau (1980). Bouakiz and Sobel (1992) examine the impact of exponential utility functions on optimal policies for both finite-horizon and infinite-horizon problems. Eeckhoudt et al. (1995) study a risk-averse newsvendor who is allowed to obtain additional orders if demand is higher than his initial order. Agrawal and Seshadri (2000) also consider a risk-averse newsvendor who decides not only on the order quantity, but also on the selling price which affects the demand. Agrawal and Seshadri (2000) consider the importance of intermediaries in supply chains to reduce the financial risk faced by risk-averse retailers. Schweitzer and Cachon (2000) investigate the optimal order quantity for a number of models that consider different types of risk aversion and conclude that for high-profit products the optimal order quantity is less than the order quantity maximizing the expected profit, while the opposite is true for low-profit products. Chen et al. (2007) discuss risk-aversion in a multi-period inventory model. Two problems, one where demand does not depend on price and another where demand depends on price, are considered. Keren and Pliskin (2006) consider an expected utility maximizing newsvendor who is faced with uniformly distributed demand. The objective function in Ahmed et al. (2007) involves coherent risk measures in inventory management. Wang et al. (2008) analyze how selling price affects the order quantity, while Wang and Webster (2009) consider a loss-averse newsvendor model by using a kinked piecewise-linear utility function. A model with a mean-variance objective function is discussed in Wu et al. (2009). Choi and Chiu (2012) discuss the implications of mean-downside-risk and mean-variance models for sustainable fashion retailing. Özler et al. (2009) consider a multi-product newsvendor problem under a value-at-risk constraint and review the related literature that considers downside risk. A discussion on the mean-variance approach can be found in Tekin and Özekici (2013).

Although the major source of randomness is the demand, supply may also be random in inventory models. The quantity

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http://dx.doi.org/10.1016/j.ijpe.2014.04.014
0925-5273/

received may not be equal to the quantity ordered. During production or transportation, the supply process may be disrupted because of some limitations or unforeseen events. Chopra and Sodhi (2004) state that supply failures may be caused by natural disasters, labour disputes, machine failures, economic conditions, accidents, wars, terrorism, supplier equipment malfunctions and other causes. Serel (2008) discusses issues related to inventory and pricing decisions in the presence of uncertain supply. The reader is referred to Arifoğlu and Özekici (2010) and the references cited there for an overview of the literature and recent developments on inventory models with random supply.

The randomness of both demand and supply increases the uncertainty in the model. If the decision maker is risk-sensitive, this makes the problem more challenging from the perspective of risk management. In a related line of research, Gaur and Seshadri (2005) give a very convincing argument and evidence that random demand may be highly correlated with a financial asset. Their discussion is motivated by statistical analysis that an inventory index (Redbook), that represents average sales, is highly correlated with a financial index (SP500), that represents average asset prices. This immediately leads to the conclusion that risks in an inventory model may be hedged by using a portfolio of assets in the financial markets. They show that a risk-averse newsvendor orders more inventory when hedging is applied. In our paper, we also take a look at a utility-based model where financial hedging is possible. Many financial instruments, such as options and futures, are available to hedge the inventory risks. The risk-sensitive decision maker not only tries to maximize the expected utility of the cash flow at the end of the period, but also needs to consider decreasing the risk or the variance of the cash flow by investing in a portfolio of market instruments that are correlated with the random demand and supply. An earlier paper using financial instruments to hedge the risk of inventory systems is by Anvari (1987). Caldentey and Haugh (2006) consider a non-financial corporation which simultaneously chooses an optimal operating policy and an optimal trading strategy in the financial markets. Chu et al. (2009) consider a continuously reviewed model to mitigate inventory risks when uncertain demand is correlated with the financial market. A mean-variance criterion is used to develop an effective financial hedging policy for inventory managers. Ding et al. (2007) propose a framework to combine operational and financial hedging. Chod et al. (2010) investigate the value of financial hedging with respect to operational hedging (resource flexibility) and find that financial hedging has higher value when operational hedging opportunities are low.

Our work is primarily concerned with financial hedging by using a portfolio of financial instruments in the market. One should note that there are other risk mitigation methods that include the ability to set prices, and buy/sell intermediate products in the market. Price setting, for example, allows the inventory manager to have some control on the uncertainty of the demand or risks by choosing the price as well as the order quantity. This is due to the fact that the demand is some random function of the price. In the newsvendor setting, Kocabıyıkoğlu and Popesu (2011) introduce a measure, called lost sales rate elasticity, associated with price-setting flexibility. This measures the percentage change in the rate of lost sales with respect to percentage change in the price for a given order quantity. They show that the measure can be effectively used to characterize structural results for pricing and inventory decisions. In particular, they use this measure to identify conditions for which the optimal price and quantity can be obtained as any solution to the first-order optimality condition. Kazaz and Webster (2013) provide a risk-sensitive extension by incorporating supply uncertainty as well as risk aversion. In an earlier paper involving supply uncertainty, Kazaz and Webster (2011) consider a specific problem in agriculture involving a 2 stage decision process where the manager decides on the amount of land to rent in the first stage when the product yield of the land is random. In the second stage, given the realized yield of the first stage, the problem is to find the optimal selling price, amount of the final product to be produced from internally grown and externally purchased fruit, as well as the amount of fruit to be sold in the open market without converting to the final product. This is a specific model that applies to a problem faced in agriculture where yield is the only source of randomness. Our model is the well-known newsvendor model with random demand as well as random supply where supply randomness may be due to random yield, or capacity or both. The models, analysis, results and the corresponding cash flows are completely different. Regarding risk hedging, there is a resemblance to our model in Kazaz and Webster (2011) where the authors discuss the value of using fruit futures in mitigating the supply risk in their model. Assuming that there is a futures market for the fruit, they show that fruit futures do not have an impact on firm's profitability in the risk-neutral case due to the implicit assumption on no arbitrage in the futures market. They arrive at the same conclusion under yield independent trading costs. Finally, through a numerical illustration with the exponential utility function, they illustrate that using fruit futures has an impact on the optimal decisions. The ideas presented by the numerical illustration in Kazaz and Webster (2011) are very much related to our ideas since they also address the effect of risk hedging. However, as mentioned above, the fact that the cash flows are not related is a significant difference. More importantly, risk hedging is the central theme in our paper and this constitutes our main contribution to the available literature. We present a rather general model with demand and supply risks to be hedged. We do not suppose the presence of a futures market for the commodity in question. Our analysis is based on the assumption that there are a number of derivative securities present in the financial markets and the cash flow is hedged by investing in a portfolio of these derivatives. We present a complete analysis on how to hedge optimally and discuss its impact on the optimal order quantity of the newsvendor model as well as the risk (or the variance of the cash flow). We present a computationally tractable procedure and demonstrate via a numerical illustration that it is possible to mitigate inventory risks through various instruments in the financial markets. To position our paper in the literature in comparison to those discussed above, we want to mention that our model is one where the IM first identifies the optimal risk hedging financial portfolio that minimizes the variance of the cash flow for any given order quantity. Then, he chooses the optimal order quantity that maximizes the expected utility of the hedged cash flow. This approach is at the intersection of industrial and financial management related to inventory control. In this regard, our approach is similar to Gaur and Seshadri (2005) who investigate a newsvendor problem with a similar risk hedging perspective. We show that their approach can be generalized to supply uncertainty in addition to demand uncertainty and provide an explicit solution to the problem of finding the variance minimizing portfolio and the corresponding optimal inventory decisions. We think that these are significant and nontrivial generalizations of the pioneering approach of Gaur and Seshadri (2005) and enhance the application scope of their framework considerably. Although our model uses financial market instruments for risk hedging, there may be additional benefits in using price setting flexibility as well. This will surely provide improvements in the reduction of operational risks.

In this paper, we provide two contributions to the literature by considering a utility-based approach to the newsvendor model with random supply and by using financial hedging. The presentation and the results are given in two main parts. In the first part, the newsvendor problem with random supply is considered under the expected utility framework without financial hedging. The standard model is considered in Section 2 and the model with random supply is discussed in Section 3. The effect of risk aversion and other parameters are analyzed in Section 4. The second part considers the risk-sensitive newsvendor model with random demand and supply which are correlated with the financial market. Section 5 presents the hedging model and the main results. A number of illustrations are given in Section 6 and we make our concluding remarks in Section 7. Finally, the detailed derivations and proofs of our results are all placed in Appendix A without affecting the flow of our presentation.

2. Utility-based model

We first consider the standard newsvendor model where randomness is only due to demand. The formulation below is similar to the one in Eeckhoudt et al. (1995). The demand D during the single-period is random with a known distribution function $G_D(x) = P\{D \le x\}$ and probability density function g_D. We suppose that the newsvendor has an initial wealth z_0. He buys items at unit purchase cost c and sells at unit sale price s. Unsold items at the end of the period can be salvaged at unit salvage value v. Moreover, if demand exceeds the order quantity, the newsvendor can buy additional items at a higher cost c_h and sell them at the same price s where $c \le c_h \le s$. Therefore, we assume that there is negative unit shortage penalty $p \ge c_h - s$ for each demand that exceeds the order quantity. To avoid trivial situations, we suppose that $s > c > v \ge 0$ and $c - s \le p \le 0$. It follows from these conditions that $s + p - c \ge 0$ and $s + p - v \ge 0$.

The newsvendor is risk-sensitive and this sensitivity is represented by some utility function u that is twice differentiable. To avoid trivial situations, we suppose that u is not equal to a constant and it is strictly increasing so that its derivative $u' > 0$. Moreover, the utility function is concave with second derivative $u'' \le 0$. The risk-sensitive newsvendor chooses the order quantity y under the random demand D. The aim of the newsvendor is to maximize the expected utility of the cash flow by choosing an order quantity y, or

$$\max_{y \ge 0} H(y) = E[u(CF(D, y))] \tag{1}$$

where

$$CF(D, y) = z_0 - (c - v)y + (s + p - v) \min\{D, y\} - pD \tag{2}$$

is the random cash flow. For further analysis, let

$$CF(x, y) = \begin{cases} CF_-(x, y) = z_0 - (c - v)y + (s - v)x, & x \le y \\ CF_+(x, y) = z_0 + (s + p - c)y - px, & x \ge y. \end{cases}$$

It clearly follows that $CF(y, y) = CF_-(y, y) = CF_+(y, y) = z_0 + (s - c)y$.

Note that we can write

$$E[u(CF(D, y))] = \int_0^y u(CF_-(x, y))g_D(x)\,dx + \int_y^\infty u(CF_+(x, y))g_D(x)\,dx.$$

One can easily show that

$$\frac{d}{dy}E[u(CF(D, y))] = -(c - v)E[u'(CF(D, y))1_{\{D \le y\}}] + (s + p - c)E[u'(CF(D, y))1_{\{D > y\}}] \tag{3}$$

and

$$\frac{d}{dy}E[u'(CF(D, y))1_{\{D > y\}}] = -u'(CF(y, y))g_D(y) + (s + p - c)E[u''(CF_+(D, y))1_{\{D > y\}}]. \tag{4}$$

In order to solve (1), we set (3) equal to zero and obtain the first order optimality condition:

$$\begin{aligned} g(y) &= -(c - v)E[u'(CF(D, y))1_{\{D \le y\}}] \\ &\quad + (s + p - c)(E[u'(CF(D, y))] - E[u'(CF(D, y))1_{\{D \le y\}}]) \\ &= 0. \end{aligned} \tag{5}$$

Moreover, using (4),

$$\begin{aligned} \frac{d^2 E[u(CF(D, y))]}{dy^2} &= -(s + p - v)u'(CF(y, y))g_D(y) \\ &\quad + (c - v)^2 E[u''(CF_-(D, y))1_{\{D \le y\}}] \\ &\quad + (s + p - c)^2 E[u''(CF_+(D, y))1_{\{D > y\}}] \\ &\le 0 \end{aligned} \tag{6}$$

and the objective function is concave. This also implies that $g(y)$ is decreasing in y.

The above development follows Eeckhoudt et al. (1995). However, unlike that paper, we prefer to present the optimality condition in terms of the well-known newsvendor critical ratio that is expressed in terms of the financial parameters. From (5), we can conclude that the optimal order quantity y^* satisfies

$$\frac{E[u'(CF(D, y^*))1_{\{D \le y^*\}}]}{E[u'(CF(D, y^*))]} = \frac{s + p - c}{s + p - v} = \hat{p} \tag{7}$$

where $\hat{p}$ denotes the critical ratio which clearly satisfies $0 \le \hat{p} \le 1$. This ratio will appear throughout this paper in the characterization of the optimal order quantity y^*. Note that (7) gives the optimal solution provided that $g(0) > 0$ and $g(+\infty) < 0$. Since, $g(y)$ is decreasing in y, if $g(0) < 0$ or $g(+\infty) > 0$, there will be no solution satisfying (7). But, it is clear that the optimal solution is $y^* = 0$ if $g(0) \le 0$; or

$$P\{D = 0\} \ge \left(\frac{E[u'(z_0 - pD)]}{u'(z_0)}\right)\hat{p}. \tag{8}$$

Since $z_0 - pD \ge z_0$, we have $u'(z_0 - pD) \le u'(z_0)$, the right-hand side of (8) is clearly between 0 and 1. If $P\{D = 0\} = 1$, the decision maker trivially orders nothing and $y^* = 0$.

Moreover, the optimal solution is $y^* = \infty$ if $g(+\infty) \ge 0$; or

$$P\{D = +\infty\} \ge 1 - \hat{p}. \tag{9}$$

This argument supposes that u is bounded. If the demand is finite so that $P\{D = +\infty\} = 0$, the optimal order quantity y^* is also finite and it satisfies (7). Moreover, if $P\{D = +\infty\} = 1$, we have $y^* = +\infty$. As a special case, suppose that the decision maker is risk-neutral so that the utility function is linear with $u(x) = a + bx$. Then, the optimality condition in (7) reduces to $P\{D \le y^*\} = \hat{p}$ which is the same condition as in the standard risk-neutral newsvendor problem.

3. Random supply models

We now focus on the extended model where supply is also random. Let $Q(y)$ be the amount received when the order quantity is y. Most of the literature on random supply models can be described by

$$Q(y) = W \min\{K, y\}. \tag{10}$$

where $K \ge 0$ and $0 \le W \le 1$ are random variables representing random capacity of the supplier and random yield respectively. This implies that once y units are ordered, the supplier can ship at most K and only a proportion W is received in good shape. The special case with $Q(y) = \min\{K, y\}$ is referred to as the random capacity model and $Q(y) = Wy$ is called the random yield model. We refer the reader to Okyay et al. (2014) and the references cited there for discussions and results on the newsvendor model with random supply.

We suppose that the random capacity K has the distribution function $P\{K \le z\} = G_K(z)$ and density function g_K. For technical reasons that will be clear shortly, we suppose that $P\{K > y\} > 0$ for all $y \ge 0$ so that there is a positive probability of fulfilling the whole order. Similarly, W has the distribution function $P\{W \le z\} = G_W(z)$ and density function g_W. We suppose that $P\{W = 0\} < 1$ so that $E[W] > 0$. Note that D, W and K are not necessarily independent and they have a joint distribution function $F_{DKW}(x,z,w) = P\{D \le x, K \le z, W \le w\}$. We also assume that all the conditional density functions $g_{K|W=w}$ and $g_{D|K=z,W=w}$ exist.

The cash flow can now be written as

$$CF(D,K,W,y) = z_0 - (c-v)W\min\{K,y\} + (s+p-v)\min\{D, W\min\{K,y\}\} - pD \tag{11}$$

after replacing y by $Q(y)$ in (2). The aim of the risk-averse newsvendor is

$$\max_{y \ge 0} H(y) = E[u(CF(D,K,W,y))].$$

Theorem 1. *The optimal order quantity y^* satisfies*

$$\frac{E[Wu'(CF(D,K,W,y^*))1_{\{D \le Wy^*, K > y^*\}}]}{E[Wu'(CF(D,K,W,y^*))1_{\{K > y^*\}}]} = \widehat{p}. \tag{12}$$

The existence and uniqueness of the optimal order quantity y^* satisfying (12) depends on the structure of $h(y)$ in (44). The objective function $H(y)$ is not necessarily concave as it was in the standard newsvendor model. Therefore, one needs to impose additional conditions to have a unique optimal solution that satisfies (12). For example, if $h(y)$ is increasing in y, then this condition indeed provides the optimal order quantity. If there is a solution y^* that satisfies $h(y^*) = \widehat{p}$ or $g(y^*) = 0$, then it follows from (43) that the derivative $g(y)$ is nonnegative on $[0, y^*)$ and nonpositive on $[y^*, \infty)$. So, the objective function $H(y)$ is increasing on $[0, y^*)$ and decreasing on $[y^*, \infty)$. Hence, $H(y)$ is quasi-concave and y^* satisfying (12) is the optimal solution.

Moreover, if $h(0) < \widehat{p} < h(+\infty)$, then there exists $0 < y^* < +\infty$ that satisfies the optimality condition $h(y^*) = \widehat{p}$ or $g(y^*) = 0$. However, the optimal order quantity is $y^* = 0$ if $h(0) \ge \widehat{p}$; or

$$P\{D = 0 | K > 0\} \ge \left(\frac{E[Wu'(z_0 - pD)]}{u'(z_0)E[W]}\right)\widehat{p}.$$

Similarly, $y^* = \infty$ if $h(+\infty) \le \widehat{p}$; or

$$P\{D = \infty | K = \infty\} \ge 1 - \widehat{p}.$$

We can also argue that if the demand is finite, the optimal order quantity is clearly finite.

As a special case, suppose that there is no capacity limitation and the only randomness in supply is due to yield uncertainty. In other words, K is infinite. Then, the optimality condition becomes

$$\frac{E[Wu'(CF(D,W,y^*))1_{\{D \le Wy^*\}}]}{E[Wu'(CF(D,W,y^*))]} = \widehat{p}. \tag{13}$$

The random capacity model with $W=1$ yields the optimality condition

$$\frac{E[u'(CF(D,K,y^*))1_{\{D \le y^*, K > y^*\}}]}{E[u'(CF(D,K,y^*))1_{\{K > y^*\}}]} = \widehat{p}. \tag{14}$$

Finally, when $W=1$ and $K = +\infty$, there is no randomness in supply and we obtain the previous result (7). If the newsvendor is risk-neutral so that the utility function is linear, then the optimality condition (12) reduces to

$$\frac{E[W1_{\{D \le Wy^*, K > y^*\}}]}{E[W1_{\{K > y^*\}}]} = \widehat{p} \tag{15}$$

which is the same condition in Okyay et al. (2014) for the newsvendor model with random yield and capacity.

4. Sensitivity analysis

In this section, we perform sensitivity analysis by analyzing the effect of risk aversion and other model parameters on the optimal order quantity and compare it with the risk-neutral order quantity y^*_{RN} satisfying (15) for the standard newsvendor model. As stated before, the objective function is not necessarily concave when there is random capacity. This imposes additional restrictions on sensitivity analysis. Therefore, we will suppose that $K = +\infty$ in this section so that there is supply randomness due to random yield only. The objective function is concave since the cash flow

$$CF(D,W,y) = z_0 - (c-v)Wy + (s+p-v)\min\{D, Wy\} - pD \tag{16}$$

is also concave in y.

Eeckhoudt et al. (1995) show that as risk-aversion increases, the optimal order quantity decreases when there is no supply randomness. They use an argument by Pratt (1964) which states that an increase in risk aversion corresponds to a concave transformation of the utility function. We will use the same approach here in order to show the effect of the risk aversion. For this purpose, we replace the utility function $u(x)$ with the new utility function $\kappa(u(x))$ where κ is a concave increasing function. Note that this implies the concavity of the new objective function with utility function $\kappa(u(x))$.

We can clearly write

$$CF_-(x_1, wy^*) \le CF(wy^*, wy^*) \le CF_+(x_2, wy^*)$$

for all $x_1 \le wy^* \le x_2$. Then,

$$u'(CF_-(x_1, wy^*)) \ge u'(CF(wy^*, wy^*)) \ge u'(CF_+(x_2, wy^*))$$

and

$$\kappa'(u(CF_-(x_1, wy^*))) \ge \kappa'(u(CF(wy^*, wy^*))) \ge \kappa'(u(CF_+(x_2, wy^*))) \tag{17}$$

since the utility functions u and $\kappa(u)$ are both concave increasing.

The aim of the more risk-averse newsvendor with utility function $\kappa(u)$ is

$$\max_{y \ge 0} \tilde{H}(y) = E[\kappa(u(CF(D,W,y)))]$$

and the derivative of the objective function (42) now becomes

$$\begin{aligned}\tilde{g}(y) = &-(c-v)E[W\kappa'(u(CF(D,W,y)))u'(CF(D,W,y))1_{\{D \le Wy\}}] \\ &+(s+p-c)E[W\kappa'(u(CF(D,W,y)))u'(CF(D,W,y))1_{\{D > Wy\}}]\end{aligned} \tag{18}$$

and the optimality condition is $\tilde{g}(y) = 0$. Moreover, when we substitute the optimal order quantity y^* for the newsvendor problem with utility function u in (18), we obtain

$$\begin{aligned}\tilde{g}(y^*) = &-(c-v)\int_0^\infty wg_W(w)\,dw \int_0^{wy^*} \kappa'(u(CF_-(x, wy^*))) \\ &\times u'(CF_-(x, wy^*))g_{D|W=w}(x)\,dx + (s+p-c)\int_0^\infty wg_W(w)\,dw \\ &\times \int_{wy^*}^\infty \kappa'(u(CF_+(x, wy^*)))u'(CF_+(x, wy^*))g_{D|W=w}(x)\,dx \\ &\le \kappa'(u(CF(wy^*, wy^*)))g(y^*) = 0.\end{aligned}$$

This follows from (17) by noting that $g(y^*) = 0, s+p-c \ge 0$, and $c - v \ge 0$. Therefore, we can conclude that $\tilde{g}(y^*) \le 0$ and the new optimal order quantity $\tilde{y}^*$ that satisfies $\tilde{g}(\tilde{y}^*)$ must also satisfy $\tilde{y}^* \le y^*$ since the derivative $\tilde{g}$ is decreasing due to the concavity of the objective function $\tilde{H}$. We can thus conclude that as risk-aversion increases, the optimal order quantity decreases.

To analyze the effects of various model parameters on the optimal order quantity, we write the optimality condition (13) as

$$\frac{E[Wu'(CF(D,W,y(z_0,v,c,p)))1_{\{D \le Wy(z_0,v,c,p)\}}]}{E[Wu'(CF(D,W,y(z_0,v,c,p)))]} = \hat{p} \tag{19}$$

where $y(z_0, v, c, p)$ is the optimal order quantity for given parameters $z_0, v, c,$ and p.

By setting the derivative of the left-hand side of (19) equal to zero, one can show that $dy(z_0, v, c, p)/dv \ge 0$ and optimal order quantity increases as the salvage value v increases. Similarly, $dy(z_0, v, c, p)/dv \ge 0$ and optimal order quantity increases as the penalty cost p increases. Analyzing the effect of the selling price is much more complicated. Eeckhoudt et al. (1995) conclude that as the sale price increases, the optimal order quantity increases if the utility function is in the decreasing partial risk aversion class, and the quantity decreases if the utility function is exponential. Moreover, Wang et al. (2008) analyze the effect of sale price and conclude that a risk-averse newsvendor orders less than an arbitrarily small quantity as sale price increases if sale price is higher than a threshold value.

To obtain further sensitivity results, we focus on the exponential utility function which is commonly employed to represent the risk sensitivity of decision makers who have constant absolute risk aversion. Suppose that the utility function is exponential so that $u(z) = -Ce^{-z/\beta}$, $u'(z) = -(C/\beta)e^{-z/\beta}$ and $u''(z) = -(C/\beta^2)e^{-z/\beta}$ for some $C \ge 0$. Then, one can show that $dy(z_0, v, c, p)/dz_0 = 0$ and the optimal order quantity is independent of the initial wealth. This is an intuitive result which states that the newsvendor is "memoryless in wealth" when the utility function is exponential. In the exponential case, one can also show that $dy(z_0, v, c, p)/dc \le 0$ so that the order quantity decreases as the purchase cost increases. However, these statements are not necessarily true for other utility functions. Similarly, although the optimal order quantity increases as the purchase cost increases for the exponential utility model, this is not necessarily true for all utility functions.

5. Utility-based model with hedging

Gaur and Seshadri (2005) presented a strong case for hedging demand uncertainty in the newsvendor model using a financial portfolio. We now analyze the case when there is a financial market in which there are financial securities correlated with demand and supply. Therefore, the decision maker needs to decide not only how much to order from the supplier, but also how much to invest on a portfolio of financial securities to hedge the risks associated with the uncertainty in demand and supply. Okyay et al. (2011) consider the inventory management problem with hedging and provide a risk-sensitive solution approach to this problem by considering both the mean and the variance of cash flow. The first aim is to find an optimal portfolio of financial securities that minimizes the variance of the hedged cash flow for any possible order quantity. Then, the mean of the hedged cash flow with this optimal portfolio is maximized by choosing an optimal order quantity. In this paper, we use a similar risk-sensitive, two-step solution approach. Although the first step remains the same, as a second step we aim to maximize the expected utility of the hedged cash flow.

We assume that the length of the inventory planning period is T during which the risk-free interest rate is r. The financial parameters are same as before but to avoid trivial situations it is assumed that $s > ce^{rT} > v \ge 0$ and $ce^{rT} - s < p \le 0$. All cash flows occur at time T except for the cash payment made at time 0 to purchase inventory. Therefore, the unit purchase cost c of the previous analysis is now replaced by its compounded value ce^{rT}. In particular, the critical ratio is accordingly updated as

$$\hat{p} = \frac{s + p - ce^{rT}}{s + p - v}. \tag{20}$$

Let $\mathbf{X} = (D, K, W)$ denote the vector of random variables corresponding to demand and supply uncertainties, and S denote the price of a primary asset in the market at the end of the period. The random vector $\mathbf{X}$ and the financial variable S are correlated. Suppose that there are $n \ge 1$ derivative securities in the market where $f_i(S)$ is the net payoff of the ith derivative security of the primary asset at the end of the period. In other words, it is the payoff $\hat{f}_i(S)$ received at time T minus its investment cost f_i^T so that $f_i(S) = \hat{f}_i(S) - f_i^T$. Let f_i^0 denote the price of the ith derivative security at the beginning of the period when it is purchased. We then have $f_i^T = e^{rT} f_i^0$. If the market is complete with some risk-neutral probability measure Q, then it is well-known that $f_i^0 = e^{-rT} E_Q[\hat{f}_i(S)]$ and this will lead to $E_Q[f_i(S)] = E_Q[\hat{f}_i(S) - f_i^T] = 0$. We do not necessarily suppose that the market is complete. However, the consequences of such a market will be analyzed in our numerical illustrations in Section 6.

Let α_i denote the amount of security i in the portfolio. The total hedged cash flow at time T is given by

$$CF_{\boldsymbol{\alpha}}(\mathbf{X}, S, y) = CF(\mathbf{X}, y) + \boldsymbol{\alpha}^{\mathbf{T}} \mathbf{f}(S) \tag{21}$$

where $CF(\mathbf{X}, y)$ denotes the unhedged cash flow, $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_n)$ is a column vector representing the hedging portfolio, $\boldsymbol{\alpha}^{\mathbf{T}}$ is its transpose, and $\mathbf{f}(S)$ is another column vector representing the derivative security payoffs with entries $\mathbf{f}(S) = (f_1(S), f_2(S), \cdots, f_n(S))$. Note that we do not impose nonnegativity restrictions on the portfolio $\boldsymbol{\alpha}$ implying that short selling is possible.

We divide the risk-sensitive optimization problem into two. As is commonly done in financial portfolio optimization, we first seek the optimal portfolio $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_n)$ to minimize the variance of the total cash flow for a given order quantity y. So, the first step of the optimization problem is

$$\min_{\boldsymbol{\alpha}} \text{Var}(\text{CF}(\mathbf{X}, y) + \boldsymbol{\alpha}^{\mathbf{T}} \mathbf{f}(S)). \tag{22}$$

Once the optimal solution $\boldsymbol{\alpha}^*(y)$ is determined for any order quantity y, the risk-averse decision maker chooses the optimal order quantity in the second step by solving

$$\max_{y \ge 0} E[u(CF(\mathbf{X}, y) + \boldsymbol{\alpha}^*(y)^{\mathbf{T}} \mathbf{f}(S))]. \tag{23}$$

We can rewrite the objective function of (22) in compact matrix notation as

$$Var(CF_{\boldsymbol{\alpha}}(\mathbf{X}, S, y)) = \boldsymbol{\alpha}^T \mathbf{C} \boldsymbol{\alpha} + 2\boldsymbol{\alpha}^T \boldsymbol{\mu}(y) + Var(CF(\mathbf{X}, y)) \tag{24}$$

where $\mathbf{C}$ is the covariance matrix of the securities with entries

$$C_{ij} = Cov(f_i(S), f_j(S))$$

and $\boldsymbol{\mu}(y)$ is a column vector with entries

$$\mu_i(y) = Cov(f_i(S), CF(\mathbf{X}, y)).$$

Proposition 2. *For any order quantity y, the optimal portfolio is*

$$\boldsymbol{\alpha}^*(y) = -\mathbf{C}^{-1} \boldsymbol{\mu}(y). \tag{25}$$

By substituting $\boldsymbol{\alpha}^*(y) = -\mathbf{C}^{-1} \boldsymbol{\mu}(y)$ into the objective function (24), we can rewrite it as

$$Var(CF_{\boldsymbol{\alpha}^*}(\mathbf{X}, S, y)) = Var(CF(\mathbf{X}, y)) - \boldsymbol{\mu}(y)^{\mathbf{T}} \mathbf{C}^{-1} \boldsymbol{\mu}(y). \tag{26}$$

Therefore, this clearly shows the impact of hedging on the variance function. Since a covariance matrix is always positive definite, so is its inverse, and $\boldsymbol{\mu}(y)^{\mathbf{T}} \mathbf{C}^{-1} \boldsymbol{\mu}(y) \ge 0$ for any $y \ge 0$. This allows us to conclude that the hedged variance is always less than or equal to that of the unhedged cash flow. The amount of reduction in the variance, of course, depends on the correlation between the unhedged cash flow and payoffs of the derivative securities used for hedging. If there is no correlation and $\mu(y)=0$, then we have the same variance function and hedging has no effect since $\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\boldsymbol{\mu}(y)=0$.

When there is a single asset, it follows from Proposition 2 that

$$\alpha^*(y) = -\frac{Cov(f(S), CF(\mathbf{X}, y))}{Var(f(S))} \tag{27}$$

since $\mathbf{C}^{-1}=1/Cov(f(S),f(S))=1/Var(f(S))$.

First, suppose that there is no randomness in the supply so that $K=+\infty$ and $W=1$. Then, the hedged cash flow is

$$\begin{aligned} CF_{\alpha}(\mathbf{X}, S, y) &= CF(D, y) + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \\ &= -(ce^{rT}-v)y + (s+p-v)\min\{D, y\} - pD + \boldsymbol{\alpha}^{\mathbf{T}}\mathbf{f}(S) \end{aligned} \tag{28}$$

where $\mathbf{X}=D$.

The optimal portfolio $\boldsymbol{\alpha}^*(y)$ is used to maximize the utility of the expected cash flow. So, the new optimization problem is

$$\max_{y} E[u(CF_{\boldsymbol{\alpha}^*(y)}(D, S, y))] \tag{29}$$

and the hedged cash flow can also be represented using

$$CF_{\boldsymbol{\alpha}^*(y)}(x,t,y) = \begin{cases} CF_-(x,t,y) = -(ce^{rT}-v)y + (s-v)x - \boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\mathbf{f}(t), & x \le y \\ CF_+(x,t,y) = (s+p-ce^{rT})y - px - \boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\mathbf{f}(t), & x \ge y \end{cases}$$

where $CF_-(y,t,y) = CF_+(y,t,y) = (s-ce^{rT})y - \boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\mathbf{f}(t)$. Then, the objective function can be written as

$$\begin{aligned} E[u(CF_{\boldsymbol{\alpha}^*(y)}(x,S,y))] = &\int_0^y E_x[u(CF_-(x,S,y))]g_D(x)\,dx \\ &+ \int_y^{\infty} E_x[u(CF_+(x,S,y))]g_D(x)\,dx \end{aligned} \tag{30}$$

where E_x is the conditional expectation given $D=x$.

Theorem 3. *The optimal order quantity y^* satisfies*

$$\frac{E[u'(CF_{\alpha^*(y^*)}(D,S,y^*))1_{\{D\le y^*\}}] + Cov(\mathbf{f}^{\mathbf{T}}(S), 1_{\{D>y^*\}})\mathbf{C}^{-1}E[\mathbf{f}(S)u'(CF_{\alpha^*(y^*)}(D,S,y^*))]}{E[u'(CF_{\alpha^*(y^*)}(D,S,y^*))]} = \hat{p}. \tag{31}$$

Once again, the existence and uniqueness of the optimal order quantity depends on the structure of $h(y)$. For example, if $h(y)$ is increasing in y and $h(0)<\hat{p}<h(+\infty)$, the first order condition in (31) identifies the optimal order quantity.

As a special case when there is a single security, the optimality condition is

$$\frac{E[u'(CF(D,S,y^*))1_{\{D\le y^*\}}] + \beta'_D(y^*)E[f(S)u'(CF(D,S,y^*))]}{E[u'(CF(D,S,y^*))]} = \hat{p} \tag{32}$$

where

$$\beta'_D(y) = \frac{Cov(f(S), 1_{\{D>y\}})}{Var(f(S))}.$$

If $\boldsymbol{\alpha}^*(y)=0$, which is indeed the case if D and S are uncorrelated, the optimality condition is identical to (7). Finally, in the risk-neutral case where $u(x)=a+bx$, the optimality condition reduces to

$$P\{D\le y^*\} + Cov(\mathbf{f}^{\mathbf{T}}(S), 1_{\{D>y^*\}})\mathbf{C}^{-1}E[\mathbf{f}(S)] = \hat{p}$$

which is the same condition in Okyay et al. (2011).

We now suppose that there is also supply uncertainty. The random variables D, W and K are not necessarily independent and they have a joint distribution function $G_{DKW}(x,z,w) = P\{D\le x, K\le z, W\le w\}$. The conditional distribution function of D given $K=z$ and $W=w$ is $g_{D|K=z,W=w}$ and the conditional probability density function of K given $W=w$ is $g_{K|W=w}$. We also suppose that D, W and K are all correlated with S.

We now take $\mathbf{X}=(D,W,K)$ in the previous analysis so that we still have $\boldsymbol{\alpha}^*(y) = -\mathbf{C}^{-1}\boldsymbol{\mu}(y)$ where

$$\mu_i(y) = Cov(f_i(S_i), CF(D,K,W,y))$$

denotes the covariance between the financial securities and the unhedged cash flow for the model with random supply.

The optimization problem is (23) where the hedged cash flow is

$$\begin{aligned} CF_{\alpha^*(y)}(\mathbf{X},S,y) = &-(ce^{rT}-v)W\min\{K,y\} \\ &+ (s+p-v)\min\{D, WK, Wy\} - pD + \boldsymbol{\alpha}^*(y)^{\mathbf{T}}\mathbf{f}(S). \end{aligned}$$

Theorem 4. *The optimal order quantity y^* satisfies*

$$\begin{aligned} &\frac{E[Wu'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))1_{\{D\le Wy^*,K>y^*\}}]}{E[Wu'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y))1_{\{K>y^*\}}]} + \left(\frac{\boldsymbol{\mu}(y^*)^{\mathbf{T}}\mathbf{C}^{-1}}{(s+p-v)}\right) \\ &\times \frac{E[\mathbf{f}(S)u'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))]}{E[Wu'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))1_{\{K>y^*\}}]} = \hat{p}. \end{aligned} \tag{33}$$

As before, the existence and uniqueness of the optimal solution depends on the structure of $h(y)$. For example, if $h(y)$ is increasing in y and $h(0)<\hat{p}<h(+\infty)$, the first order condition in (33) identifies the optimal order quantity. Suppose that there is no hedging opportunity, or $\boldsymbol{\alpha}^*(y)=\mathbf{0}$, the optimality condition can now be rewritten as

$$\frac{E[Wu'(CF(\mathbf{X},S,y^*))1_{\{D\le Wy^*,K>y^*\}}]}{E[Wu'(CF(\mathbf{X},S,y^*))1_{\{K>y^*\}}]} = \hat{p}$$

which is identical to (7). If the utility function is linear $u(x)=a+bx$ so that the newsvendor is risk-neutral, then we have

$$\frac{E[W1_{\{D\le Wy^*,K>y^*\}}]}{E[W1_{\{K>y^*\}}]} + \left(\frac{\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}}{(s+p-v)}\right)\frac{E[\mathbf{f}(S)]}{E[W1_{\{K>y^*\}}]} = \hat{p}$$

which is the same condition as Okyay et al. (2011). If there is no capacity constraint and supply randomness is only due to yield so that $K=\infty$, then the condition becomes

$$\begin{aligned} &\frac{E[Wu'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))1_{\{D\le wy^*\}}]}{E[Wu'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))]} + \left(\frac{\boldsymbol{\mu}(y^*)^{\mathbf{T}}\mathbf{C}^{-1}}{(s+p-v)}\right) \\ &\times \frac{E[\mathbf{f}(S)u'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))]}{E[Wu'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))]} = \hat{p}. \end{aligned} \tag{34}$$

Finally, if $W=1$, then

$$\begin{aligned} &\frac{E[u'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))1_{\{D\le y^*,K>y^*\}}]}{E[u'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))1_{\{K>y^*\}}]} \\ &+ \left(\frac{\boldsymbol{\mu}(y^*)^{\mathbf{T}}\mathbf{C}^{-1}}{(s+p-v)}\right)\frac{E[\mathbf{f}(S)u'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))]}{E[u'(CF_{\boldsymbol{\alpha}^*(y^*)}(\mathbf{X},S,y^*))1_{\{K>y^*\}}]} = \hat{p}. \end{aligned} \tag{35}$$

6. Numerical illustrations

We now illustrate how our results can be used and demonstrate how utility theory and hedging influences the optimal decisions. We will first consider a simple binary model and identify the optimal order quantity explicitly. Then, a continuous model is analyzed via simulation. The illustrations will involve the random demand case only for brevity and simplicity without loss of conceptual generality.

6.1. Analysis of a simple binomial model

In this section, to see the effects of some parameters on the optimal order quantity, we consider an example similar to the one in Eeckhoudt et al. (1995). The utility function is $u(x)=-\exp(-x/\beta)$

exponential where β represents the newsvendor's level of risk tolerance. Suppose that the newsvendor has no initial wealth ($z_0=0$) and no salvage or extra buying options exist ($v=p=0$). He purchases each item with purchase cost c and sells it at sale price $s>c$. We first analyze the problem when there is no hedging option, and then when there is hedging opportunity. Therefore, the cash flow is $CF(D,y)=-cy+s\min\{D,y\}$.

The demand D is binary and it is either 0 with probability p_1 or it is equal to some $M>0$ with probability $p_2=1-p_1$. The optimality condition for the standard newsvendor model in (7) can be written for our example as

$$h(y^*)=\frac{E[u'(CF(D,y^*))1_{\{D\le y^*\}}]}{E[u'(CF(D,y^*))]}=\frac{s-c}{s}=\widehat{p}$$

where we can explicitly obtain

$$h(y)=\begin{cases}\dfrac{p_1}{p_1+p_2\exp(-sy/\beta)}, & 0\le y<M\\ 1, & y\ge M.\end{cases}$$

It is obvious that $h(y)$ is increasing in y. If $h(0)<\widehat{p}<h(M)$, then there exists a unique y^* that satisfies the optimality condition. However, if $h(0)\ge\widehat{p}$, we have $y^*=0$; and if $h(M-)\le\widehat{p}$, we have $y^*=M$. Setting $h(y^*)=\widehat{p}$, the optimal order quantity is found to be

$$y^*=\begin{cases}0, & p_2\le c/s\\ \dfrac{\beta}{s}\ln\left(\dfrac{p_2}{p_1}\left(\dfrac{s-c}{c}\right)\right), & c/s<p_2<\dfrac{c}{c+(s-c)\exp(-sM/\beta)}\\ M, & p_2\ge\dfrac{c}{c+(s-c)\exp(-sM/\beta)}.\end{cases} \tag{36}$$

This characterization of the order quantity depends on the probability of positive demand p_2. If p_2 is less than or equal to c/s, the decision maker orders nothing. If p_2 is larger than $c/(c+(s-c)\exp(-sM/\beta))$, the decision maker orders M units. In between, the optimal order quantity is linearly increasing in β. In other words, as the risk tolerance β decreases and the newsvendor becomes more risk averse, he orders less. We observe that the optimal order quantity increases up to M as β increases and the decision maker orders at most M units which is logical because the demand can be at most M. The risk-neutral order quantity is clearly 0 if $p_2\le c/s$ or M if $p_2>c/s$. If $s=28$, $c=20, M=100$, and $p_2=0.75$, then it follows from (36) that the optimal order quantity depends on the risk tolerance β such that $y^*(\beta)=0.0065115\beta$ provided that $\beta\le 15,357.44$. Otherwise, it is 100.

Suppose now that there is a financial security with net payoff $f(S)$ which is either $-L$ or L for computational simplicity. They have a joint distribution function

$f(S)=-L$, $D=0$	with probability q_1
$f(S)=-L$, $D=M$	with probability q_2
$f(S)=L$, $D=0$	with probability q_3
$f(S)=L$, $D=M$	with probability q_4 .

Let us also assume that

$$E[f(S)]=-(q_1+q_2)L+(q_3+q_4)L=0 \tag{37}$$

so that

$$Var(f(S))=L^2. \tag{38}$$

Note that $p_2=P\{D=M\}=q_2+q_4$. The optimal portfolio can be found using (27) as

$$\alpha^*(y)=-\left(\frac{Cov(f(S),\min\{D,y\})}{Var(f(S))}\right)s.$$

One can easily show that

$$Cov(f(S),\min\{D,y\})=(q_4-q_2)Ly$$

and

$$\alpha^*(y)=\left(\frac{q_2-q_4}{L}\right)sy.$$

We observe that the sign of the optimal quantity of the derivative security in the portfolio depends on the sign of the q_2-q_4. We also have

$$Cov(f(S),D)=(q_4-q_2)LM.$$

Therefore, we can conclude that if $f(S)$ and D are positively correlated, the sign of $\alpha^*(y)$ is negative and then the optimal decision is to short sell the derivative. However, if $f(S)$ and D are negatively correlated, the sign of $\alpha^*(y)$ is positive and it is optimal to buy the derivative. Moreover, the hedged cash flow becomes

$$CF_{\alpha^*(y)}(D,S,y)=-cy+s\min\{D,y\}+\left(\frac{q_2-q_4}{L}\right)syf(S).$$

Using the exponential utility function $u(x)=-\exp(-x/\beta)$, the optimality condition in (32) can be written as

$$\begin{aligned}&((q_2-q_4)s+c)q_1\exp(-(q_4-q_2)sy^*/\beta)\\&+((q_2-q_4-1)s+c)q_2\exp(-(q_4-q_2+1)sy^*/\beta)\\&+((q_4-q_2)s+c)q_3\exp(-(q_2-q_4)sy^*/\beta)\\&+((q_4-q_2-1)s+c)q_4\exp(-(q_2-q_4+1)sy^*/\beta)=0.\end{aligned} \tag{39}$$

Letting $C^*=y^*/\beta$, (39) becomes

$$a_1e^{-b_1C^*}+a_2e^{-b_2C^*}+a_3e^{-b_3C^*}+a_4e^{-b_4C^*}=0 \tag{40}$$

where $a_1=((q_2-q_4)s+c)q_1$, $a_2=((q_2-q_4-1)s+c)q_2$, $a_3=((q_4-q_2)s+c)q_3$, $a_4=((q_4-q_2-1)s+c)q_4$, $b_1=(q_4-q_2)s$, $b_2=(q_4-q_2+1)s$, $b_3=(q_2-q_4)s$ and $b_4=(q_2-q_4+1)s$. We can easily conclude that if there exists a solution C^* to (40), it is independent of β. This further implies that the optimal order quantity $y^*=C^*\beta$ is linear in β where the slope C^* is found by solving (40).

To illustrate this numerically, recall that $s=28$, $c=20, M=100$ and suppose now that $(q_1,q_2,q_3,q_4)=(0.15,0.35,0.10,0.40)$. Then, (40) becomes

$$2.79e^{-1.4C^*}-3.29e^{-29.4C^*}+2.14e^{1.4C^*}-2.64e^{-26.6C^*}=0. \tag{41}$$

Multiplying both sides of (41) by $e^{-1.4C^*}$, we obtain

$$2.79e^{-2.8C^*}-3.29e^{-30.8C^*}-2.64e^{-28C^*}+2.14=0.$$

Moreover, by letting $x=e^{-2.8C^*}$, we have

$$r(x)=2.79x-3.29x^{11}-2.64x^{10}+2.14=0$$

where $r(x)$ is a polynomial and the problem is to find a positive root of r.

Note that $r'(x)=2.79-36.19x^{10}-26.4x^9$ and $r''(x)=-361.9x^9-237.6x^8\le 0$ for $x\ge 0$. Therefore, r is concave on $[0,+\infty)$. Since $r(0)=2.14>0$ and $r'(0)=2.79>0$ there may exist only one positive root x^* that satisfies $r(x^*)=0$. That value is $x^*=0.98168$ so that $C^*=-(1/2.8)\ln(x^*)=0.0066035$. Therefore, the optimal order quantity is $y^*(\beta)=0.0066035\beta$ which is clearly more than the optimal order quantity without hedging. The hedging option provides the exponential utility maximizing newsvendor the opportunity to order more.

6.2. Simulation analysis of a continuous model

In this section, a continuous demand model will be considered via simulation. Our aim is to quantify the effects of the utility framework and financial hedging to compensate for demand and supply risks. As the base scenario, we take the setting of the example in Gaur and Seshadri (2005) where the demand risk is hedged by a stock in the financial market. Let the initial stock price S_0 be \$660 and the interest rate be $r=10\%$ per year. Assume that $T=6$ months and that the return S_T/S_0 has a lognormal distribution under the risk-neutral measure with mean $(r-0.5\sigma^2)T$ and standard deviation $\sigma\sqrt{T}$ where $\sigma = 20\%$ per year. That is,

$$\ln(S_T/S_0) \sim N((r-0.5\sigma^2)T, \sigma\sqrt{T}) = N(0.04, 0.14142).$$

Let the demand be $D = b\, S_T + \epsilon$ where $b = 10$ and ϵ has a normal distribution with mean zero and standard deviation σ_ϵ. Therefore, the random demand is linearly correlated with the financial market as suggested by the statistical evidence provided by Gaur and Seshadri (2005). The financial parameters are as follows: $s = 1$, $c = 0.6$, $p = -0.3$, and $v = 0.1$. Moreover, we suppose that the utility function is $u(x) = 800 - 100e^{-x/\beta}$.

We set $S = S_T$ throughout the following and consider three types of financial portfolios. The first portfolio consists of the future on the stock only and has the net payoff $f_1(S) = S - e^{rT}S_0$, the second portfolio consists of the call option on the stock with strike price κ only and has the net payoff $f_2(S) = \max\{S_T - \kappa, 0\} - e^{rT}C$ where C is the price of the call option at time 0. Finally, the third portfolio uses both instruments jointly and has the net payoffs $f_1(S)$ and $f_2(S)$. Motivated by Gaur and Seshadri (2005) who show that the risk can be perfectly hedged when $\epsilon = 0$, we use replicating portfolios consisting of bonds, stock futures, and European call options with strike price $\kappa = y/b$. We further suppose that the call price in the market does not provide any arbitrage opportunities so that $C = E[e^{-rT} \max\{S_T - \kappa, 0\}]$ and $E[f_2(S)] = 0$.

We want to point out that all our numerical calculations are done using Monte Carlo simulations throughout the remainder of this section. We use Matlab as a simulation tool. Cash flows are generated by using the simulated values of S, D, U, and K whenever needed. The following eight scenarios are considered:

1. Newsvendor maximizes the expected cash flow.
2. Newsvendor maximizes the expected hedged cash flow using the first portfolio (futures).
3. Newsvendor maximizes the expected hedged cash flow using the second portfolio (call options).
4. Newsvendor maximizes the expected hedged cash flow using the third portfolio (futures and call options).
5. Newsvendor maximizes the expected utility of the cash flow.
6. Newsvendor maximizes the expected utility of the hedged cash flow using the first portfolio (futures).
7. Newsvendor maximizes the expected utility of the hedged cash flow using the second portfolio (call options).
8. Newsvendor maximizes the expected utility of the hedged cash flow using the third portfolio (futures and call options).

6.2.1. *Random demand model*

We will analyze various cases starting with the one where demand is the only source of uncertainty. The linear relationship $D = 10\, S_T + \epsilon$ also implies that $E[S] = 693.84, E[D] = 6938.4$, $Var(S) = 457.93 \times 10^3$, $Cov(D, S) = 10\, Var(S) = 457.93 \times 10^4$ and the coefficient of determination between D and S is

$$\rho^2 = \left(1 + \frac{\sigma_\epsilon^2}{457.93 \times 10^3}\right)^{-1}.$$

Therefore, the level of correlation increases as σ_ϵ decreases. We first suppose that the standard deviation of demand is $\sigma_\epsilon = 600$ and the risk-tolerance parameter is $\beta = 500$. We run our simulation for different order quantity values and generate 50,000 instances to calculate the optimal portfolios. In each instance, we generate the stock price and demand and determine the optimal portfolios using our results. Finally, we generate another 50,000 instances so that we obtain stock prices, demand quantities and profits. For all scenarios, we calculate the mean, the variance, and the coefficient of variation (CV) (the ratio of the standard deviation to the mean) of the cash flow for each order quantity.

Based on the mean of the cash flows, for scenarios 1–4, and the mean of the utility of the cash flows, for scenarios 5–9, we obtain the optimal order quantities approximately. Table 1 depicts the results for each scenario. Note that the means are approximately equal for scenarios 1–4 since the expected cash flow obtained from the portfolios is approximately 0. The minor differences are due to simulation error. Table 1 shows the variance reductions in the cash flows that are made possible by financial hedging. Consider, in particular, the variance reductions when both portfolios are used. The financial hedging provides variance reduction by 68.6% when we do not use the utility model and by 66% when we use the utility model. The effect of the utility model can be observed by comparing scenario 1 and scenario 5. The risk-averse decision maker orders less and so his expected gain is also less. However, the variance of the expected cash flow is reduced by 30%.

We analyzed the models by also changing the demand variability. The results are summarized in Table 2 for a perfect correlation between the demand and the stock price, in Table 3 for a high degree of a correlation between the demand and the stock price.

When the standard deviation of the demand error is zero so that there is perfect correlation between the demand and the stock price, hedging with a portfolio of futures and options eliminates the variance of the cash flow and the variance of the utility of cash flow totally. When the standard deviation of the demand error is small ($\sigma_\epsilon = 300$), indicating a high degree of correlation between the demand and the stock price, significant variance reductions are achieved, 89% for the standard model and 87% for the utility model. The reductions decrease when the correlation decreases since for $\sigma_\epsilon = 600$ the variance of the cash flow can be lowered considerably, 68.6% for the standard model and 66% for the utility model.

We also analyze the effect of the risk-tolerance parameter β on the optimal order quantity and the variance. Table 4 depicts the optimal order quantities, means of the cash flows, variances of the cash flows and the optimal portfolios. We conclude that as risk-tolerance increases, the optimal order quantity increases.

Table 1
The variances of the cash flows and the optimal investment amounts for random demand model when the standard deviation of demand error is 600.

$\sigma_\epsilon = 600$	y^*	Mean	Variance	CV	Portfolio (α)
S1	5588	2435.2	181,260	0.1748	–
S2	5587	2434.5	60,561	0.1011	−3.5191
S3	5583	2433.7	66,290	0.1058	−3.5761
S4	5587	2435.8	56,966	0.0980	−8.9780,5.7328
S5	4657	2401.4	127,480	0.1487	–
S6	5086	2422.8	42,432	0.0850	−3.1950
S7	5008	2418.9	41,358	0.0841	−3.1745
S8	5164	2427.0	43,355	0.0858	−9.8903,6.7478

Table 2
The variances of the cash flows and the optimal investment amounts for random demand model when the standard deviation of demand error is 0.

$\sigma_\epsilon = 0$	y^*	Mean	Variance	CV	Portfolio (α)
S1	5804	2457.4	125,410	0.1441	–
S2	5802	2456.7	7,150	0.0344	−3.4823
S3	5800	2455.9	18,470	0.0553	−3.5538
S4	5804	2458.1	0	0	−9.00, 6.00
S5	5235	2440.0	94,726	0.1261	–
S6	5662	2455.4	4,914	0.0285	−3.3467
S7	5468	2449.3	6,426	0.0327	−3.2467
S8	5804	2458.1	0	0	−9.00, 6.00

Table 3
The variances of the cash flows and the optimal investment amounts for random demand model when the standard deviation of demand error is 300.

$\sigma_\epsilon = 300$	y^*	Mean	Variance	CV	Portfolio (α)
S1	5742	2451.3	139,460	0.1523	–
S2	5740	2450.6	20,553	0.0585	−3.4916
S3	5736	2449.7	29,810	0.0705	−3.5601
S4	5744	2451.9	14,855	0.0497	−8.9135, 5.8281
S5	5085	2430.1	103,180	0.1322	–
S6	5513	2447.4	15,294	0.0505	−3.2970
S7	5348	2441.3	15,089	0.0503	−3.2273
S8	5640	2451.3	13,554	0.0475	−8.9600, 5.8849

Table 4
The variances of the cash flows and the optimal investment amounts for different risk-tolerance values when the standard deviation of demand error is 600.

$\sigma_\epsilon = 600$	β	y^*	Mean	Variance	Portfolio (α)
S5	250	3850	2348.8	120,680	–
	500	4650	2401.0	127,340	–
	750	4950	2417.3	135,830	–
S6	250	4500	2391.2	34,444	−3.0469
	500	5100	2423.4	42,766	−3.2010
	750	5250	2428.8	46,896	−3.2753
S7	250	4450	2387.7	35,286	−3.0415
	500	5000	2418.6	43,532	−3.1712
	750	5200	2426.6	47,319	−3.2693
S8	250	4000	2382.9	32,700	−14.2763, 11.2262
	500	5150	2426.5	41,017	−9.9375, 6.7979
	750	5350	2432.8	43,391	−9.3728, 6.1884

Moreover, from the variances of the cash flows, we can state that hedging always reduces the variance significantly and leads to some relatively modest benefits in the expected profit. It is also observed that the variance reductions decrease slightly as β increases.

As for the optimal portfolio structure, it is always optimal to sell the future since demand and stock price are assumed to be positively correlated in the above examples. On the other hand, in the optimal portfolio, the call option is bought when used as the second instrument along with the future, but is sold when it is used as the sole instrument. It is also interesting to note that using a portfolio consisting only of the future on the stock is very effective and achieves most of the variance reduction benefits. On the other hand, the call option serves to fine tune the portfolio along with the investment in the stock but is not as effective when used alone.

6.2.2. Random yield model

To analyze the problem with random yield, we take the following plausible example where $U = 1 - e^{-(1/S_0)(\gamma + S_T)}$ and γ is normally distributed with mean zero and standard deviation σ_γ independent of S_T and ϵ. We take the same base scenario and use identical portfolio options to see the effect of financial hedging on risks. Therefore, we fix the order quantity to $y^* = 7000$ and consider only the first four scenarios. We first set $\sigma_\epsilon = 600$ and $\beta = 500$. Then, for different values of σ_γ (0, 200,400), we calculate the means, variances, coefficient of variations and the optimal portfolios. The result is presented in Table 5.

Although the variance reduction decreases when σ_γ increases, we can conclude that financial hedging provides considerable reductions in the variance for all scenarios. Then, by considering the same example, we vary the standard deviations σ_γ and σ_ϵ together. Table 6 reports the results of this experiment. We can conclude that when the standard deviations are smaller, the variance reduction is 94% for the standard models. However, when we further increase the standard deviation, the variance reduction is lower as expected due to increased uncertainty.

Table 5
The variances of the cash flows and the optimal investment amounts for different random yield models when the standard deviation of demand error is 600 and the order quantity is 7000.

σ_γ	Scenario	Mean	Variance	CV	Portfolio (α)
0	S1	2395	135,518	0.1537	–
	S2	2395	32,688	0.0755	−3.2570
	S3	2395	88,389	0.1242	−1.7302
	S4	2395	32,678	0.0755	−3.3122, 0.1032
200	S1	2384	143,524	0.1589	–
	S2	2384	37,658	0.0814	−3.3048
	S3	2384	95,544	0.1296	−1.7458
	S4	2384	37,618	0.0813	−3.4185, 0.2128
400	S1	2349	173,863	0.1775	–
	S2	2349	58,096	0.1026	−3.4558
	S3	2349	122,914	0.1492	−1.7990
	S4	2349	57,862	0.1024	−3.7315, 0.5159

Table 6
The variances of the cash flows and the optimal investment amounts when the standard deviations of demand error and yield error vary together ($y = 7000$).

$\sigma_\gamma = \sigma_\epsilon$	Scenario	Mean	Variance	CV	Portfolio (α)
200	S1	2386	110,752	0.1394	–
	S2	2386	6,807	0.0346	−3.2746
	S3	2386	63,221	0.1054	−1.7376
	S4	2386	6,793	0.0345	−3.3418, 0.1257
400	S1	2353	148,684	0.1639	–
	S2	2353	35,452	0.0800	−3.4178
	S3	2353	98,538	0.1334	−1.7847
	S4	2353	35,275	0.0798	−3.6577, 0.4490

6.2.3. Random capacity model

We also analyze a model with random capacity so that $K = kS_T + \eta$ where $k = 9$ and η has a normal distribution with mean zero and standard deviation σ_η independent of S_T and ϵ. Once more, we set $y^* = 7000$ and $\beta = 500$. Note that as σ_ϵ and σ_η increase, the correlations between the demand and the market, and the capacity and the market weaken. At the same time, the correlation between the demand and the capacity also weakens. Table 7 reports that the resulting variances as σ_ϵ and σ_η are changed together. It can be observed that, once again, significant reductions in variance can be achieved by hedging. The reductions are naturally most important when the market correlation is strong. For instance, the case $\sigma_\epsilon = \sigma_\eta = 300$ corresponds to high correlation with the market and the variance can be reduced by 92%. Even in the case when the correlations with the market are relatively low ($\sigma_\epsilon = \sigma_\eta = 900$), the variance reduction is less but considerable, 44% for the standard models.

6.2.4. Random yield and capacity model

Finally, we analyzed the combination of random yield and capacity models discussed above. We take $y^* = 7000$ as before and fix the standard deviations as $\sigma_\epsilon = 600$, $\sigma_\gamma = 200$ and $\sigma_\eta = 300$. The resulting means, variances, coefficient of variations and the optimal portfolios are summarized in Table 8.

From Table 8, we can conclude that significant variance reductions for financial hedging are achievable, 78% for the models

Table 7
The variances of the cash flows and the optimal investment amounts for random capacity models when the standard deviations of demand error and capacity error vary together ($y=7000$).

$\sigma_\epsilon=\sigma_\eta$	Scenario	Mean	Variance	CV	Portfolio (α)
300	S1	2500	129,629	0.1440	–
	S2	2500	10,959	0.0419	−3.4989
	S3	2500	77,189	0.1111	−1.8251
	S4	2500	10,756	0.0415	−3.7561, 0.4814
600	S1	2459	202,656	0.1831	–
	S2	2459	69,219	0.1070	−3.7102
	S3	2459	143,498	0.1541	−1.9385
	S4	2459	69,020	0.1068	−3.9644, 0.4757
900	S1	2403.1	339,815	0.2426	–
	S2	2402.7	189,127	0.1810	−3.9427
	S3	2402.7	272,720	0.2173	−2.0644
	S4	2402.8	188,944	0.1809	−4.1866, 0.4564

Table 8
The variances of the cash flows and the optimal investment amounts for a random yield and capacity model when the standard deviations of demand error, yield error and capacity error are 600, 200 and 300, respectively ($y=7000$).

Scenario	Mean	Variance	CV	Portfolio (α)
S1	2352.0	155,739	0.1678	–
S2	2352.0	34,967	0.0795	−3.5191
S3	2351.4	101,383	0.1354	−1.8514
S4	2352.1	34,912	0.0794	−3.6528, 0.2511

without the utility maximization objective and 66% for the models with the utility maximization objective.

7. Concluding remarks

In this paper, we investigate the single-period, single-item inventory problem when the decision-maker (newsvendor) is risk-averse. We use the expected utility theory framework where the risks result not only from random demand, but also from random supply. In our initial model, there is no risk hedging and we obtained optimality conditions for the order quantity. Although the objective function is concave for the random yield case, this is not necessarily true when capacity is random. However, we are able to establish quasi-concavity under reasonable conditions and this allows us to obtain the optimality condition. We also presented various results on the effect of the model parameters on the optimal order quantity.

The second part of the paper focuses on models where the randomness in demand and supply is correlated with financial markets. We consider the opportunities of financial hedging to mitigate inventory risks. In our context, the decision-maker needs to choose the financial portfolio and the order quantity at the same time. We provide a two step solution approach to this problem. In the first step, we find the optimal portfolio that minimizes the variance of the cash flow for any order quantity. Then, in the second step, we find the optimal order quantity that maximizes the expected utility of the cash flow by using the characterization for the optimal portfolio. Although the minimization of the variance is a convex optimization problem, we do not necessarily have concavity in the maximization of the expected utility of the cash flow. However, under some conditions, one can establish quasi-concavity and find explicit characterizations for the optimal order quantities. Finally, some numerical illustrative numerical examples on these models are presented. The effects of risk-tolerance and some other parameters on the optimal order quantities are examined. Moreover, we also analyze the effect of risk-sensitivity and financial hedging on the variance of the problem. We conclude that as risk-tolerance increases, the optimal order quantity also increases. We further observe that financial hedging reduces the variance of the problem significantly.

This line of research can be extended in several directions by future research. The model can involve multi-period, infinite-period, or multi-product models. Furthermore, Bayesian models, models where demand and supply are modulated by a random environment, and hidden Markov models are other suitable areas for extensions. Another line of research is to combine risk hedging using financial instruments with other risk mitigation methods like the ability to set prices.

Acknowledgement

This research is supported by the Turkish Scientific and Technological Research Council through Grant 110M620. F. Karaesmen's research is partially supported by the TÜBA-GEBİP programme.

Appendix A

In this Appendix we provide the proofs of the main results presented in the paper.

A.1. Proof of Theorem 1

Proof. Note that

$$E[u(CF(D,K,W,y))] = \int_0^\infty g_W(w)\,dw \int_0^\infty g_{K|w}(z)\,dz \times \left(\int_0^{w(z\wedge y)} u(CF_-(x,w(z\wedge y)))g_{D|K=z,W=w}(x)\,dx + \int_{w(z\wedge y)}^\infty u(CF_+(x,w(z\wedge y)))g_{D|K=z,W=w}(x)\,dx \right)$$

where we let $a \wedge b = \min\{a,b\}$ for any real a and b. We can also show that

$$\frac{d}{dy}E[u(CF(D,K,W,y))] = -(c-v)E[Wu'(CF(D,K,W,y))1_{\{D\le Wy,K>y\}}] + (s+p-c)E[Wu'(CF(D,K,W,y))1_{\{D>Wy,K>y\}}] \quad (42)$$

and the optimality condition becomes

$$g(y) = E[Wu'(CF(D,K,W,y))1_{\{K>y\}}](-(s+p-v)h(y)+(s+p-c)) = 0 \quad (43)$$

where

$$h(y) = \frac{E[Wu'(CF(D,K,W,y))1_{\{D\le Wy,K>y\}}]}{E[Wu'(CF(D,K,W,y))1_{\{K>y\}}]}. \quad (44)$$

Noting that $P\{K>y\}>0$ for all y and $E[W]>0$ by our assumption, $E[Wu'(CF(D,K,W,y))1_{\{K>y\}}]>0$ since $u'>0$, and (43) can also be written as $-(s+p-v)h(y)+(s+p-c)=0$. This clearly leads to the optimality condition (12). □

A.2. Proof of Proposition 2

Proof. By taking the gradient of the objective function (24) and setting it equal to zero, the first order condition is obtained as

$$\frac{\partial}{\partial \boldsymbol{\alpha}}(Var(CF_{\boldsymbol{\alpha}}(\mathbf{X},S,y))) = 2\mathbf{C}\boldsymbol{\alpha} + 2\boldsymbol{\mu}(y) = 0$$

and the Hessian is

$$\frac{\partial^2}{\partial \boldsymbol{\alpha}^2}(Var(CF_{\boldsymbol{\alpha}}(\mathbf{X},S,y))) = 2\mathbf{C} \succcurlyeq 0$$

as the covariance matrix $\mathbf{C}$ is always positive definite. So, the second order condition is satisfied and the first order condition gives the optimality condition. □

A.3. Proof of Theorem 3

Proof. The derivative of (30) is

$$\begin{aligned}\frac{d}{dy}E[u(CF_{\boldsymbol{\alpha}^*(y)}(x,S,y))] &= -(ce^{rT}-v)\int_0^y E_x[u'(CF_-(x,S,y))]g_D(x)\,dx\\ &+(s+p-ce^{rT})\int_y^\infty E_x[u'(CF_+(x,S,y))]g_D(x)\,dx\\ &-\boldsymbol{\mu}'(y)^{\mathbf{T}}\mathbf{C}^{-1}\int_0^\infty E_x[\mathbf{f}(S)u'(CF_-(x,S,y))]g_D(x)\,dx\\ &= -(ce^{rT}-v)E[u'(CF_{\boldsymbol{\alpha}^*(y)}(D,S,y))1_{D\le y}]\\ &+(s+p-ce^{rT})E[u'(CF_{\boldsymbol{\alpha}^*(y)}(D,S,y))1_{D>y}]\\ &-\boldsymbol{\mu}'(y)^{\mathbf{T}}\mathbf{C}^{-1}\mathbf{f}(S)E[u'(CF_{\boldsymbol{\alpha}^*(y)}(D,S,y))] \qquad (45)\end{aligned}$$

where the derivative of $\boldsymbol{\mu}(y)$ equals

$$\boldsymbol{\mu}'(y) = \frac{d}{dy}Cov(\mathbf{f}(S), CF(D,y)) = (s+p-v)Cov(\mathbf{f}(S), 1_{\{D>y\}}). \qquad (46)$$

Therefore, using (45) and (46), the first order optimality condition can be written as

$$g(y) = E[u'(CF_{\boldsymbol{\alpha}^*(y)}(D,S,y))]((s+p-ce^{rT})-(s+p-v)h(y)) = 0 \qquad (47)$$

where

$$h(y) = \frac{E[u'(CF_{\boldsymbol{\alpha}^*(y)}(D,S,y))1_{\{D\le y\}}]+Cov(\mathbf{f}^{\mathbf{T}}(S),1_{\{D>y\}})\mathbf{C}^{-1}E[\mathbf{f}(S)u'(CF_{\boldsymbol{\alpha}^*(y)}(D,S,y))]}{E[u'(CF_{\boldsymbol{\alpha}^*(y)}(D,S,y))]}.$$

Note that by our assumption $u'>0$, then the first order condition in (47) can be rewritten as $g(y)=0$ or $h(y)=\widehat{p}$ which gives (31). □

A.4. Proof of Theorem 4

Proof. We can write the objective function as

$$\begin{aligned}&E[u(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))]\\ &= \int_0^\infty g_W(w)\,dw\left(\int_0^y g_{K|W=w}(z)\,dz\right.\\ &\times\left(\int_0^{wz} E_{x,z,w}[u(CF_-(x,S,w(z\wedge y)))]g_{D|K=z,W=w}(x)\,dx\right.\\ &\left.\left.+\int_{wz}^\infty E_{x,z,w}[u(CF_+(x,S,w(z\wedge y)))]g_{D|K=z,W=w}(x)\,dx\right)\right)\\ &+\int_0^\infty g_W(w)\,dw\left(\int_y^\infty g_{K|W=w}(z)\,dz\right.\\ &\times\left(\int_0^{wy} E_{x,z,w}[u(CF_-(x,S,w(z\wedge y)))]g_{D|K=z,W=w}(x)\,dx\right.\\ &\left.\left.+\int_{wy}^\infty E_{x,z,w}[u(CF_+(x,S,w(z\wedge y)))]g_{D|K=z,W=w}(x)\,dx\right)\right) \qquad (48)\end{aligned}$$

where $E_{x,z,w}$ denotes the conditional expectation given $D=x, K=z$, and $W=w$.

The derivative of the objective function (48) is

$$\begin{aligned}&\frac{d}{dy}E[u(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))]\\ &= \int_0^\infty g_W(w)\,dw\left(\int_0^y g_{K|W=w}(z)\,dz\right.\\ &\left(\int_0^{wz}\mathbf{E}^1 g_{D|K=z,W=w}(x)\,dx + \int_{wz}^\infty \mathbf{E}^2 g_{D|K=z,W=w}(x)\,dx\right)\\ &+\int_y^\infty g_{K|W=w}(z)\,dz\left(\int_0^{wy}\mathbf{E}^3 g_{D|K=z,W=w}(x)\,dx+\right.\\ &\left.\left.\int_{wy}^\infty \mathbf{E}^4 g_{D|K=z,W=w}(x)\,dx\right)\right)\\ &= -\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}E[\mathbf{f}(S)u'(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))]\\ &-(ce^{rT}-v)E[Wu'(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))1_{\{D\le Wy, K>y\}}]\\ &+(s+p-ce^{rT})E[Wu'(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))1_{\{D>Wy, K>y\}}]\end{aligned}$$

where

$$\mathbf{E}^1 = E_{x,z,w}[(-\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\mathbf{f}(S))u'(CF_-(x,S,w(z\wedge y)))]$$
$$\mathbf{E}^2 = E_{x,z,w}[(-\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\mathbf{f}(S))u'(CF_+(x,S,w(z\wedge y)))]$$
$$\mathbf{E}^3 = E_{x,z,w}[(-(ce^{rT}-v)w-\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\mathbf{f}(S))u'(CF_-(x,S,w(z\wedge y)))]$$

and

$$\mathbf{E}^4 = E_{x,z,w}[((s+p-ce^{rT})w-\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\mathbf{f}(S))u'(CF_+(x,S,w(z\wedge y)))].$$

Then, the first order condition can be written as

$$g(y) = (s+p-v)E[Wu'(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))1_{\{K>y\}}]\left(\left(\frac{s+p-ce^{rT}}{s+p-v}\right)-h(y)\right) = 0 \qquad (49)$$

where

$$h(y) = \frac{E[Wu'(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))1_{\{D\le Wy,K>y\}}]}{E[Wu'(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))1_{\{K>y\}}]} + \left(\frac{\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}}{(s+p-v)}\right)\frac{E[\mathbf{f}(S)u'(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))]}{E[Wu'(CF_{\boldsymbol{\alpha}^*(y)}(\mathbf{X},S,y))1_{\{K>y\}}]}.$$

Note that $u'>0$ and $P\{K>y\}>0$ by our assumption, and the optimality condition $g(y)=0$ leads to $h(y)=\widehat{p}$ which is identical to (33). □

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