Hedging Demand and Supply Risks in the Newsvendor Model

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Abstract

We consider a single-period inventory model where there are risks associated with the uncertainty in demand as well as supply. Furthermore, the randomness in demand and supply is correlated with the financial markets. Recent literature provides ample evidence on this issue. The inventory manager may then exploit this correlation and manage his risks by investing in a portfolio of financial instruments. The decision problem therefore includes not only the determination of the optimal ordering policy, but also the selection of the optimal portfolio at the same time. We analyze this problem in detail and provide a risk-sensitive approach to inventory management where one considers both the mean and the variance of the resulting cash flow. The analysis results in some interesting and explicit characterizations on the structure of the optimal policy.

Keywords. Newsvendor model, random supply, risk hedging, minimum-variance portfolio

1 Introduction

This work is motivated by Gaur and Seshadri [2005] who provides a convincing argument on hedging inventory risks through instruments in the financial markets. Their discussion is based on statistical evidence that an inventory index (Redbook) that represents average sales is very highly correlated with a financial index (S&P 500) that represents average asset prices. Therefore, this leads to the fact that risks associated with random demand may be hedged by investing in the financial markets. In another related research, Okyay et al. [2014] consider a model where there is additional uncertainty due to the randomness in supply. In this paper, we combine these 2 lines of research and take a risk-sensitive look at the standard single-period newsvendor model with random demand and supply. The inventory manager (IM) tries not only to maximize the expected profit or the cash flow at the end of the period as it is done in almost all of inventory management literature. But, more importantly, he also needs to consider decreasing the risk or the variance of the cash flow by investing in a portfolio of market instruments that are correlated with the random demand and supply.

The overwhelming majority of inventory literature relates to risk-neutral IMs who are concerned with the expected profit or cost criteria. Although this is a mathematically viable approach, it supposes that decision makers behave risk-neutrally which, in reality, is simply not necessarily true. That's why models with risk-neutrality assumptions have limited viability in practice. In recent years, the risk-sensitive behavior of the decision maker is typically addressed through mean-variance (MV) or semi-variance models, Value-at-Risk (VaR) or conditional VaR (CVaR) models, utility-based models, and models that aim to maximize some satisficing probability.

MV approach used in decision making under uncertainty originates from finance and it was introduced by Markowitz [1959] for the portfolio management problem. For a given value of mean return, the portfolio that minimizes the variance of the return is determined. In inventory management literature, pioneered by Lau [1980], the MV approach has also received significant attention. We refer the reader to Choi et al. [2008], Wu et al. [2009], Chen and Federgruen [2000], and Choi and Chiu [2012] for examples along the MV line of research. VaR and CVaR are also used in risk-sensitive decision making and inventory management. VaR and CVaR are widely used in financial mathematics and financial risk management as measure of the risk of loss on a specific portfolio of financial assets. Gan et al. [2004], Gotoh and Takano [2007], and Özler et al. [2009], among others, consider the VaR and CVaR criteria.

There are also other papers examining the control of risks with various other approaches. Ahmed et al. [2007] uses a coherent risk measure in a multi-period model while Wang and Webster [2009] focuses on loss aversion. Chen et al. [2007] propose a framework for incorporating risk aversion in multi-period inventory models as well as multi-period models that coordinate inventory and pricing strategies. Parlar and Weng [2003] provides an example using satisficing probability maximization, which refers to probability of achieving a certain level of profit. The utility approach is commonly used for modelling risk-sensitivity in inventory management where the objective function is the expected utility of the decision maker. Eeckhoudt et al. [1995] examine the effects of risk aversion in a single-period problem using a piecewise-linear payoff function and exponential utility function. In a recent paper, Sayın et al. [2014] consider a general utility-based model and provide several characterization on optimal ordering policies.

Our paper focuses on inventory management through financial hedging. Van Mieghem [2003] describes financial hedging as follows: “Risk-averse decision makers may be interested in mitigating risk in the capacity-investment decision. Mitigating risk, or hedging, involves taking counterbalancing actions so that, loosely speaking, the future value varies less over the possible states of nature. If these counterbalancing actions involve trading financial instruments, including short-selling, futures, options, and other financial derivatives, we call this financial hedging.” It should be noted that financial hedging is different than operational hedging which is widely studied in the inventory management literature through strategies like inventory pooling, component commonality, supplier diversification, etc. Within the literature on inventory models, few worked on controlling the inventory risk through borrowing and trading in financial markets. Anvari [1987] studies the capital asset pricing model to solve the single-period newsvendor model with no setup costs by investing some portion of capital in inventory and other on financial assets. The resulting optimal policy is characterized and compared with the classical expected utility maximization structure. Gaur and Seshadri [2005], another pioneer paper on this subject, use the S&P 500 index to construct static hedging strategies using both mean-variance and utility-maximization frameworks. An important aspect they pointed out is, the risk of inventory carrying can be replicated as a financial portfolio by using simple instruments like bonds, futures and options. Caldentey and Haugh [2006] use financial hedging methods for continuous-time models. Their paper views the non-financial operations of a corporation, as assets in the corporation's portfolio; thus, turning the problem into a financial hedging problem in incomplete markets. By dynamically hedging the profits of a corporation they propose a framework for modelling the operations of a non-financial corporation that also trades in the financial markets. Pınar et al. [2011] consider a quantity flexibility contract in a supply chain consisting of a buyer and a seller. It is assumed that random demand is perfectly correlated with a risky financial asset. They then investigate the joint ordering and portfolio optimization problem to evaluate the value of the contract. A recent paper by Chod et al. [2010] explores operational flexibility (an operational hedging strategy) and financial hedging together and finds that the two strategies may be substitutes or complements depending on the problem setting.

A model with supply uncertainty and risk hedging using futures is discussed in Kazaz and Webster [2011].

They consider a specific problem in agriculture involving a 2 stage decision process where the manager decides on the amount of land to rent in the first stage when the product yield of the land is random. In the second stage, given the realized yield of the first stage, the problem is to find the optimal selling price, amount of the final product to be produced from internally grown and externally purchased fruit, as well as the amount of fruit to be sold in the open market without converting to the final product. This is a specific model that applies to a problem faced in agriculture where yield is the only source of randomness. Our model is the well-known newsvendor model with random demand as well as random supply where supply randomness may be due to random yield, or capacity or both. The models, analysis, results and the corresponding cash flows are completely different. Regarding risk hedging, there is a resemblance to our model in Kazaz and Webster [2011] where the authors discuss the value of using fruit futures in mitigating the supply risk in their model. Assuming that there is a futures market for the fruit, they show that fruit futures do not have an impact on firm's profitability in the risk-neutral case due to the implicit assumption on no arbitrage in the futures market. They arrive at the same conclusion under yield independent trading costs. Finally, through a numerical illustration with the exponential utility function, they illustrate that using fruit futures has an impact on the optimal decisions. The ideas presented by the numerical illustration in Kazaz and Webster [2011] are very much related to our ideas since they also address the effect of risk hedging. However, as mentioned above, the fact that the cash flows are not related is a significant difference. More importantly, risk hedging is the central theme in our paper and this constitutes our main contribution to the available literature. We present a rather general model with demand and supply risks to be hedged. We do not suppose the presence of a futures market for the commodity in question. Our analysis is based on the assumption that there are a number of derivative securities present in the financial markets and the cash flow is hedged by investing in a portfolio of these derivatives. We do not only consider a numerical illustration to justify the impact of risk hedging, but we present a complete analysis on how to hedge optimally and discuss its impact on the optimal order quantity of the newsvendor model as well as the risk (or the variance of the cash flow). We present a computationally tractable procedure and demonstrate via a numerical illustration that it is possible to mitigate inventory risks through various instruments in the financial markets.

To position our paper in the literature in comparison to those discussed above, we want to mention that our model is one where the IM first identifies the optimal risk hedging financial portfolio that minimizes the variance of the cash flow for any given order quantity. Then, he chooses the optimal order quantity that maximizes the expected hedged cash flow. This approach is at the intersection of industrial and financial management related to inventory control. In this regard, our approach is similar to Gaur and Seshadri [2005] who investigate a newsvendor problem with a similar risk hedging perspective. We show that their approach can be generalized to supply uncertainty in addition to demand uncertainty and provide an explicit solution to the problem of finding the variance minimizing portfolio and the corresponding optimal inventory decisions. We think these are significant and non-trivial generalizations of the pioneering approach of Gaur and Seshadri [2005] and enhance the application scope of their framework considerably.

In addition to random demand D, there is additional uncertainty due to random supply such that the quantity that is received is $Q(y) = U \min\{K, y\}$ if the order quantity is y. Here, U represents the random proportion of nondefective items and K is the random capacity of the supplier or manufacturer. This is a rather general model that combines random yield U and capacity K. It is based on the assumption that the quantity received can be at most equal to the capacity K, and only a proportion U of nondefective items can enter the store. The reader is referred to Arifoğlu and Özekici [2010] for motivations and discussions on the literature related to random supply. A complete analysis and results on the classical newsvendor model with random supply can be found in Okyay et al. [2014].

The primary emphasis and contribution of this paper is on the impact of risk hedging in inventory models by investing in a portfolio of financial assets. Our approach supposes that there is correlation between the uncertainty in the inventory model and uncertainty in the financial markets. This correlation is exploited to provide the IM betters means of controlling the risk. The uncertainty in the inventory model includes both random demand and random supply, while the uncertainty in the financial market includes random asset prices. The analysis leads to a rather computationally tractable procedure to determine optimal portfolios that decrease the risk as measured by the variance of the cash flow. Our model contributes to the literature on risk-sensitive inventory management in 2 directions: (1) it includes random supply as another source of uncertainty, and (2) it uses risk hedging by investing in a financial portfolio. There are other means of risk minimization and hedging in inventory models. For example, Vaagen and Wallace [2008] discuss a mean-variance model in fashion supply chains where the risk associated with uncertain demand due to uncertain product popularity is hedged by increasing product variety through blending/fragmentation. Another example in the highly volatile fashion industry is provided by Choi [2013] where the IM is risk-sensitive and used the mean-variance approach with interest rate, budget, and profit target considerations. The approach in our work is quite different since we focus on risk hedging by investing in a financial portfolio of assets in the market.

In this paper, we consider the standard newsvendor model with random demand and supply which are correlated with the financial markets. Since both demand and supply are correlated with the market possibly due to similar economic cycles, they are also dependent. This corresponds to a number of interesting and practically relevant situations. During periods where demand surges, suppliers may have to ration production capacity between multiple customers leading to negative correlation between the supply quantity and the demand. We first review the standard newsvendor model with random supply briefly in Section 2. The same model with financial hedging is discussed later in Section 3 where characterizations on the structures of the optimal hedging portfolio and the optimal order quantity are provided. Special cases involving only random demand, random yield, and random capacity are discussed in Section 4. We illustrate how our results can be used by a number of examples and interpretations in Section 5. Concluding remarks and ideas for future work are presented in Section 6.

2 The Newsvendor Model with Random Supply

The newsvendor problem is a well-known single-item, single-period inventory problem in which the decision maker (or newsvendor) has to decide on how much to order. The replenishment decision is critical because if he orders too many, purchase cost will be unnecessarily high; on the contrary, there will be a missed opportunity for additional profit if he orders too few. In daily life, it is very common to encounter examples of newsvendor models, that's the foremost reason why these models are studied extensively. There is random demand D with a known distribution function that has a probability density function. Throughout this paper, we assume that all marginal, joint and conditional distributions corresponding to random demand D, random yield U and random capacity K have marginal, joint and conditional probability density functions. We suppose that the length of the period is T during which there is interest charged continuously with some rate r. Moreover, we suppose that there is a fixed sale price s, a fixed purchase cost c, a fixed shortage penalty $p \geq 0$, and a fixed salvage value v which satisfy $s > ce^{rT} > v > 0$ to avoid trivial situations. All cash flows occur at time T except for the cash payment made at time 0 to purchase inventory. This model is discussed earlier by Okyay et al. [2014] where there is no hedging and no interest so that $r = 0$. We now present their results adjusted accordingly to our setting with positive interest. Recall that the quantity that is received is $Q(y) = U \min\{K, y\}$ when there is random supply.

The aim of the newsvendor is to maximize the expected cash flow at the end of the period by choosing an ordering quantity y, or

$$\max_y E\left[CF\left(D, K, U, y\right)\right] \tag{1}$$

where $CF(D,K,U,y)$ is the random cash flow which can be written as

$$\begin{aligned} CF(D,K,U,y) &= -ce^{rT}U\min\{y,K\}+s\min\{D,U\min\{y,K\}\} \\ &\quad +v\max\{U\min\{y,K\}-D,0\}-p\max\{D-U\min\{y,K\},0\} \\ &= \left(v-ce^{rT}\right)U\min\{y,K\}+(s+p-v)\min\{D,U\min\{K,y\}\}-pD \end{aligned} \tag{2}$$

using the fact that $\max\{a-b,0\}=a-\min\{a,b\}$.

Theorem 1 *The optimal order quantity y^* satisfies*

$$\frac{E\left[U1_{\{K>y^*,D\leq Uy^*\}}\right]}{E\left[U1_{\{K>y^*\}}\right]}=\frac{s+p-ce^{rT}}{s+p-v}=\hat{p}. \tag{3}$$

Proof. Note that we can write

$$E[U\min\{K,y\}]=\int uf_U(u)du\left[\int_0^y xf_{K|U=u}(x)dx+y\int_y^{+\infty}f_{K|U=u}(x)dx\right]$$

where f_U is the probability density function of U and $f_{K|U=u}$ is the conditional probability density function of K given $U=u$. Leibnitz rule gives

$$\begin{aligned} \frac{dE[U\min\{K,y\}]}{dy} &= \int uf_U(u)du\left[yf_{K|U=u}(y)+\int_0^y f_{K|U=u}(x)dx-yf_{K|U=u}(y)\right] \\ &= \int uf_U(u)du\int_0^y f_{K|U=u}(x)dx \end{aligned}$$

or

$$\frac{dE[U\min\{K,y\}]}{dy}=E[U1_{\{K>y\}}]. \tag{4}$$

The argument for $E[\min\{D,UK,Uy\}]$ is somewhat similar and one can show that

$$\frac{dE[\min\{D,UK,Uy\}]}{dy}=E[U1_{\{K>y,D>Uy\}}]=E[U1_{\{K>y\}}]-E[U1_{\{K>y,D\leq Uy\}}] \tag{5}$$

so that the optimality condition can be written as

$$E[U1_{\{K>y\}}]\left(\left(s+p-ce^{rT}\right)-(s+p-v)\left(\frac{E[U1_{\{K>y,D\leq Uy,\}}]}{E[U1_{\{K>y\}}]}\right)\right)=0 \tag{6}$$

by taking the derivative of the expected cash flow in (1). This leads to the optimality condition (3). ■

The objective function is not necessarily concave. However, defining

$$g(y)=\frac{E\left[U1_{\{K>y,D\leq Uy\}}\right]}{E\left[U1_{\{K>y\}}\right]} \tag{7}$$

one obtains conditions for the existence and uniqueness of the solution. More precisely, if $g(y)$ is increasing in y, there exists $0\leq y^*\leq+\infty$ which satisfies the optimality condition $g(y^*)=\hat{p}$ provided that $g(0)\leq\hat{p}\leq g(+\infty)$. This solution is unique if $g(y)$ is strictly increasing. Moreover, the objective function is concave increasing on $[0,y^*)$ and decreasing on $[y^*,+\infty)$. Therefore, it is quasi-concave and the solution y^* is indeed the optimal solution that maximizes the objective function. Finally, $y^*=0$ if $g(0)\geq\hat{p}$ and $y^*=+\infty$ if $g(+\infty)\leq\hat{p}$.

It is clear that if $K=+\infty$ and $U=1$, we obtain the standard newsvendor model with no randomness in supply and $g(y)=P\{D\leq y\}$ is always increasing so that the optimality condition becomes

$$P\{D\leq y^*\}=\hat{p} \tag{8}$$

since $E\left[U1_{\{K>y^*,D\le Uy^*\}}\right]=E\left[1_{\{D\le y^*\}}\right]=P\{D\le y^*\}$ and $E\left[U1_{\{K>y^*\}}\right]=1.$

Similarly, in case $K=+\infty$, we have random yield only and $g(y)=E\left[U1_{\{D\le Uy\}}\right]/E[U]$ is also increasing and the optimality condition becomes

$$\frac{E\left[U1_{\{D\le Uy^*\}}\right]}{E[U]}=\hat{p} \tag{9}$$

since $E\left[U1_{\{K>y^*,D\le Uy^*\}}\right]=E\left[U1_{\{D\le Uy^*\}}\right]$ and $E\left[U1_{\{K>y^*\}}\right]=E[U]$.

Finally, if $U=1$, then the model involves random capacity only and $g(y)=P\{D\le y\mid K>y\}$ leads to the optimality condition

$$P\{D\le y^*\mid K>y^*\}=\hat{p} \tag{10}$$

since $E\left[U1_{\{K>y^*,D\le Uy^*\}}\right]=E\left[1_{\{K>y^*,D\le y^*\}}\right]=P\{K>y^*,D\le y^*\}$ and $E\left[U1_{\{K>y^*\}}\right]=E\left[1_{\{K>y^*\}}\right]=P\{K>y^*\}$.

Note that in all of the cases, the optimality condition is stated in terms of the same critical ratio $\hat{p}$. It is clear that $0\le\hat{p}\le1$ and this important parameter will appear in almost all of our results throughout this paper. Given the costs and revenues, (8) allows us to interpret $\hat{p}$ as the optimal probability of satisfying the periodic demand.

3 Newsvendor Model with Hedging

Risk hedging in inventory models, as well as other decision problems under uncertainty, is an important issue that should not be overlooked. After all, decision makers are not necessarily risk-neutral and the risk associated with a cash flow is one of the primary objectives. In the context of inventory models, Gaur and Seshadri [2005] discuss the relationship between sales of inventory in stores and values of some financial index. As it is clearly illustrated by Figure 1 taken from Gaur and Seshadri [2005], there is very strong statistical evidence that an inventory index (Redbook) that represents average sales is very highly correlated with a financial index (S&P 500) that represents average prices of stocks in the financial markets. They further discuss the case when the relationship is perfect so that the random demand in a period is a linear function of the price of a share of stock traded in the market. For the newsvendor model, they obtain an explicit expression for the portfolio that replicates the periodic cash flow. The portfolio consists of a cash bond and derivatives (futures and European calls) of the stock. This naturally leads to the conclusion that the IM should consider the value of the replicating portfolio at the beginning of the period in order to avoid arbitrage opportunities. Several cases of exactly replicating the cash flow by investing in such portfolios is also discussed in Okyay et al. [2011], but they only include unrealistic models where there are perfect deterministic relationships between demand/supply and financial variables. In this paper, our primary aim is to extend this line of research where these variables are only correlated.

We want to mention that the financial portfolios used for hedging may include a variety of products in the markets. For example, Chen and Yano [2010] consider a supply chain where the demand is weather related and their emphasis is on hedging risks via weather-linked rebates. They report on cases involving, for example, Wal-Mart, Cadbury Schweppes, Coca-Cola, Unilever, Nestle, Weatherproof Garment Company who state that their sales are substantially affected by weather conditions. This directly implies that risks associated with uncertain demand can be hedged by using portfolios of weather derivatives in financial markets. We refer the reader to Chen and Yano [2010] for examples and motivations.

We now consider the newsvendor model where random demand and supply are correlated with the financial markets. The newsvendor has a more comprehensive task on hand. In addition to determining the order quantity y, he must also choose a portfolio of financial securities to hedge his risks. We suppose that the risk-sensitive newsvendor tries to minimize the variance of the hedged cash flow by choosing his portfolio for any possible order

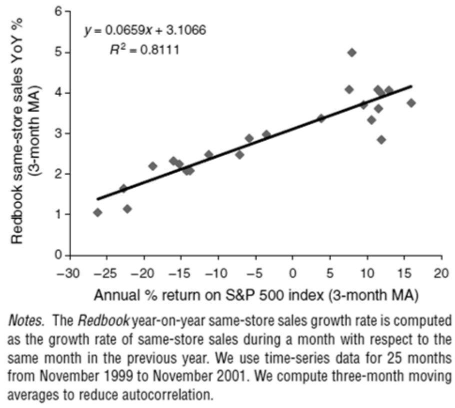

Figure 1: Redbook Same-Store Sales Growth Rate vs. Annual Return on the S&P 500 Index

quantity. Then, he chooses that order quantity that maximizes the expected value of the minimum-variance cash flow. Let $\mathbf{X} = (D, U, K)$ denote the vector of random variables corresponding to demand and supply uncertainties, and S denote the price of a primary asset in the market at the end of the period. The random vector $\mathbf{X}$ and the financial variable S are correlated. Suppose that there are $n \geq 1$ derivative securities in the market where $f_i(S)$ is the net cash flow of the ith derivative security of the primary asset at the end of the period. In other words, it is the payoff $\hat{f}_i(S)$ received at time T minus its investment cost f_i^T so that $f_i(S) = \hat{f}_i(S) - f_i^T$. For example, if the derivative is a call option with some strike price K, then $\hat{f}_i(S) = (S-K)^+ = \max\{0, S-K\}$ so that $f_i(S) = \max\{0, S-K\} - f_i^T$ where f_i^T is price of the call option compounded to time T. Will also let f_i^0 denote the price of the ith derivative security at the beginning of the period when it is purchased, and we then have $f_i^T = e^{rT} f_i^0$. If the market is complete with some risk-neutral probability measure Q, then it is well-known that $f_i^0 = e^{-rT} E_Q[\hat{f}_i(S)]$ and this will lead to $E_Q[f_i(S)] = E_Q\left[\hat{f}_i(S) - f_i^T\right] = 0$. We do not necessarily suppose that the market is complete. However, the consequences of such a market will be analyzed in our numerical illustrations in the last section.

Let α_i denote the amount of security i in the portfolio, and $CF(\mathbf{X}, y)$ denote the unhedged cash flow as a function of the random vector $\mathbf{X}$ and the decision variable y. Although S denotes the price of a single asset, our analysis actually holds as well when S is indeed a vector representing the price of a number of primary assets in the market. The total hedged cash flow is given by

$$CF_\alpha(\mathbf{X}, S, y) = CF(\mathbf{X}, y) + \sum_{i=1}^{n} \alpha_i f_i(S_i).$$

The problem now is to find the optimal portfolio $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_n)$ to minimize the variance of the total cash flow for a given order quantity y. The optimization problem is

$$\min_{\boldsymbol{\alpha}} \text{Var}\left(CF(\mathbf{X}, y) + \sum_{i=1}^{n} \alpha_i f_i(S)\right). \tag{11}$$

Once the optimal solution $\alpha^*(y)$ is determined for any order quantity y, the decision maker then chooses the optimal order quantity by solving

$$\max_{y \geq 0} E\left[CF(\mathbf{X}, y) + \sum_{i=1}^{n} \alpha_i^*(y) f_i(S)\right]. \tag{12}$$

Note that we do not impose nonnegativity restrictions on the portfolio $\boldsymbol{\alpha}$ so that shortselling is possible. The risk-sensitive newsvendor therefore makes sure that his risk (measured by the variance of the cash flow) is smallest for any order he gives. Our approach involves a two-step procedure where optimization problems (11) and (12) are solved one after each other. By solving (11), we decrease the risk measured by the variance of the cash flow for any order y that the newsvendor gives. This optimal minimum variance portfolio is denoted by $\alpha^*(y)$. Then, the newsvendor selects that order quantity y^* by solving (12). This gives the order quantity that maximizes the expected value of the hedged cash flow that includes the cash flow associated with the inventory management as well as financial cash flow obtained by investing in the financial portfolio. Note that if the market is risk-neutral, then $E\left[CF(\mathbf{X},y)+\sum_{i=1}^{n}\alpha_i^*(y)\,f_i(S)\right]=E\left[CF(\mathbf{X},y)\right]$ since $E\left[f_i(S)\right]=E\left[\hat{f}_i(S)-f_i^T\right]=0$ and the optimal order quantity y^* of (12) is equal to that of the risk-neutral case. However, by using the optimal portfolio $\alpha^*(y^*)$, the newsvendor decreases the variance of the cash flow. This is why our analysis, results, and illustrations mainly focus on the impact that financial hedging has on the newsvendor problem.

Since the main theme and purpose of the paper is to demonstrate how and how much risk reduction can be done through hedging, we consider the choice problem on the portfolio α to be more significant than the choice problem on the order quantity y. For this purpose we have not really formulated an optimization problem involving α and y at the same time. For any given y, the optimal portfolio which solves (11) is optimal in minimizing the variance of the cash flow. However, it follows that the two-step optimization procedure (11) and (12) actually solves the optimization problem

$$\max_{y\geq 0} E\left[CF(\mathbf{X},y)+\sum_{i=1}^{n}\alpha_i(y)\,f_i(S)\right] \tag{13}$$

such that

$$\operatorname{Var}\left(CF(\mathbf{X},y)+\sum_{i=1}^{n}\alpha_i(y)f_i(S)\right)\leq \operatorname{Var}\left(CF(\mathbf{X},y)+\sum_{i=1}^{n}\beta_i f_i(S)\right) \tag{14}$$

for all portfolios $\boldsymbol{\beta}=(\beta_1,\beta_2,\cdots,\beta_n)$ and $y\geq 0$. In this formulation, we require that for any order quantity y, only the minimum-variance portfolio should be used in hedging. The solution to this optimization problem is obtained by solving (11) and (12) sequentially.

Note that the optimal solution of (13) and (14), denoted by $(y^*,\alpha^*(y^*))$, gives the optimal order quantity and the optimal portfolio of the decision maker who wishes to maximize the expected return using minimum-variance financial portfolios. Decision makers do not all have the same perception towards risk. One can, of course, make other formulations that involve mean-variance, utility, downside risk, prospect, or other types of risk models. We believe that our formulation (13) and (14) is plausible while the two-step procedure (11) and (12) is tractable. Moreover, the two-step optimization approach can be exploited to numerically explore other interesting formulations. For instance, let us consider the mean-variance optimization problem. A theoretical study of this problem appears challenging and is beyond the scope of this paper. On the other hand, since we can characterize the variance minimizing portfolio for any given order quantity, we can numerically compute the expected returns and the corresponding minimum variances for an appropriate discretization of order quantities and generate an approximation of the efficient mean-variance frontier. This frontier represents the combinations of expected returns and variances that are available to a risk-averse decision maker. Clearly, our solution y^* is on this efficient frontier, but a decision maker may also consider other choices of y on the frontier depending on his attitude toward risk. For example, if $p=0$, a decision maker with no risk tolerance will choose $y=0$ and not y^* because it leads to no risk with, of course, no return.

When there is random yield and capacity so that $Q(y)=U\min\{K,y\}$, the cash flow is

$$CF(\mathbf{X},y)=\left(v-ce^{rT}\right)U\min\{K,y\}+(s+p-v)\min\{Uy,UK,D\}-pD$$

where $\mathbf{X}=(D,U,K)$ while the hedged cash flow becomes

$$CF_\alpha(\mathbf{X},S,y)=CF(\mathbf{X},y)+\sum_{i=1}^{n}\alpha_i f_i(S).$$

The objective function in (11) can be written as

$$\begin{aligned}\operatorname{Var}\left(CF_{\alpha}\left(\mathbf{X},S,y\right)\right) &= \operatorname{Var}\left(CF\left(\mathbf{X},y\right)+\sum_{i=1}^{n}\alpha_{i}f_{i}\left(S\right)\right) \\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}\alpha_{i}\alpha_{j}\operatorname{Cov}\left(f_{i}\left(S\right),f_{j}\left(S\right)\right) \\ &\quad +2\sum_{i=1}^{n}\alpha_{i}\operatorname{Cov}\left(f_{i}\left(S\right),CF\left(\mathbf{X},y\right)\right) \\ &\quad +\operatorname{Var}\left(CF\left(\mathbf{X},y\right)\right).\end{aligned}$$

We can rewrite the objective function in compact matrix notation as

$$\operatorname{Var}\left(CF_{\alpha}\left(\mathbf{X},S,y\right)\right)=\boldsymbol{\alpha}^{\mathbf{T}}\mathbf{C}\boldsymbol{\alpha}+2\boldsymbol{\alpha}^{\mathbf{T}}\boldsymbol{\mu}\left(y\right)+\operatorname{Var}\left(CF\left(\mathbf{X},y\right)\right) \tag{15}$$

where $\boldsymbol{\alpha}$ is a column vector, $\boldsymbol{\alpha}^{T}$ is the transpose of $\boldsymbol{\alpha}$, $\mathbf{C}$ is the covariance matrix of the securities with entries

$$C_{ij}=\operatorname{Cov}\left(f_{i}\left(S\right),f_{j}\left(S\right)\right) \tag{16}$$

and $\boldsymbol{\mu}(y)$ is a vector with entries

$$\mu_{i}\left(y\right)=\operatorname{Cov}\left(f_{i}\left(S\right),CF\left(\mathbf{X},y\right)\right) \tag{17}$$

that denotes the covariance between the financial securities and the cash flow.

It turns out that the optimal portfolio can be expressed in a compact formula as stated in the following proposition.

Proposition 2 *The minimum-variance financial portfolio is given by*

$$\boldsymbol{\alpha}^{*}\left(y\right)=-\mathbf{C}^{-1}\boldsymbol{\mu}\left(y\right). \tag{18}$$

Proof. By taking the gradient of (15) with respect to $\boldsymbol{\alpha}$ and setting it equal to zero, the first order optimality condition is obtained as

$$\mathbf{C}\boldsymbol{\alpha}+\boldsymbol{\mu}\left(y\right)=\mathbf{0}$$

where $\mathbf{0}=(0,0,\cdots,0)$ is the zero vector. The second order condition is also verified since the Hessian matrix of (15) is the covariance matrix $\mathbf{C}$ which is always positive definite. ■

By substituting $\boldsymbol{\alpha}^{*}(y)=-\mathbf{C}^{-1}\boldsymbol{\mu}(y)$ into the variance function (15), we can rewrite it as

$$\operatorname{Var}\left(CF_{\alpha^{*}(y)}\left(\mathbf{X},S,y\right)\right)=\operatorname{Var}\left(CF\left(\mathbf{X},y\right)\right)-\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\boldsymbol{\mu}(y). \tag{19}$$

Since a covariance matrix is C always positive definite, so is its inverse, and $\boldsymbol{\mu}(y)^{\mathbf{T}}\mathbf{C}^{-1}\boldsymbol{\mu}(y)\geq 0$ for any $y\geq 0$. Therefore, the hedged variance $\operatorname{Var}\left(CF_{\alpha^{*}(y)}\left(\mathbf{X},S,y\right)\right)$ is always less than or equal to that of the unhedged cash flow for any order quantity y. This show the impact of hedging; there is much to be gained by hedging in variance reduction. This, of course, depends on the correlation between the unhedged cash flow and payoffs of the derivative securities used for hedging. If there is no correlation and $\mu(y)=0$, then we have the same variance function and hedging has no effect.

Next we focus on the determination of the order quantity that maximizes the expected cash flow while using the minimum-variance portfolio $\boldsymbol{\alpha}^{*}\left(y\right)$. Then, the objective function is

$$\begin{aligned}E\left[CF_{\alpha^{*}(y)}\left(\mathbf{X},S,y\right)\right] &= \left(v-ce^{rT}\right)E\left[U\min\left\{K,y\right\}\right] \\ &\quad +\left(s+p-v\right)E\left[\min\left\{D,UK,Uy\right\}\right] \\ &\quad -pE\left[D\right]-\boldsymbol{\mu}\left(y\right)^{\mathbf{T}}\mathbf{C}^{-1}E\left[\mathbf{f}\left(S\right)\right]\end{aligned} \tag{20}$$

where $\mathbf{f}(S) = (f_1(S), f_2(S), \cdots, f_n(S))$ denotes the column vector of the payoffs of the derivative securities. Let $\boldsymbol{\mu}'(y)$ denote the gradient vector of $\boldsymbol{\mu}(y)$ obtained by the derivatives $\mu_i'(y) = d\mu_i(y)/dy$. The characterization of the optimal order quantity is provided in the following result.

Theorem 3 *The optimal order quantity that maximizes the expected cash flow while using the minimum-variance portfolio* $\boldsymbol{\alpha}^*(y)$ *satisfies*

$$\frac{E\left[U1_{\{D\leq Uy^*,K>y^*\}}\right]}{E\left[U1_{\{K>y^*\}}\right]} + \frac{\boldsymbol{\mu}'(y^*)^{\mathbf{T}}\,\mathbf{C}^{-1}E\left[\mathbf{f}(S)\right]}{(s+p-v)\,E\left[U1_{\{K>y^*\}}\right]} = \hat{p}. \tag{21}$$

Proof. By taking the derivative with respect to y of the expected cash flow (20), we obtain the optimality condition

$$\left(v - ce^{rT}\right)E\left[U1_{\{K>y\}}\right] + (s+p-v)\,E\left[U1_{\{D>Uy,K>y\}}\right] - \boldsymbol{\mu}'(y)^{\mathbf{T}}\,\mathbf{C}^{-1}E\left[\mathbf{f}(S)\right] = 0. \tag{22}$$

To get (22), we also use (4) and (5). The optimality condition can be rewritten as

$$\left(v - ce^{rT}\right)E\left[U1_{\{K>y\}}\right] + (s+p-v)\left(E\left[U1_{\{K>y\}}\right] - E\left[U1_{\{D\leq Uy,K>y\}}\right]\right) - \boldsymbol{\mu}'(y)^{\mathbf{T}}\,\mathbf{C}^{-1}E\left[\mathbf{f}(S)\right] = 0$$

or

$$\left(s+p - ce^{rT}\right)E\left[U1_{\{K>y\}}\right] - (s+p-v)\,E\left[U1_{\{D\leq Uy,K>y\}}\right] - \boldsymbol{\mu}'(y)^{\mathbf{T}}\,\mathbf{C}^{-1}E\left[\mathbf{f}(S)\right] = 0.$$

This leads to (21). ■

The characterization in Theorem 3 is similar to that of the standard random supply model (3) in the previous section. The only difference is that we now have an additional term

$$h(y) = \frac{\boldsymbol{\mu}'(y)^{\mathbf{T}}\,\mathbf{C}^{-1}E\left[\mathbf{f}(S)\right]}{(s+p-v)\,E\left[U1_{\{K>y\}}\right]}$$

and the optimal solution satisfies

$$g(y^*) + h(y^*) = \hat{p}. \tag{23}$$

Note that we need to impose additional restrictions on $g + h$ in for the existence and uniqueness of the optimal solution. In particular, if $g(y) + h(y)$ is increasing in y, there exists $0 \leq y^* \leq +\infty$ which satisfies the optimality condition $g(y^*) + h(y^*) = \hat{p}$ provided that $g(0) + h(0) \leq \hat{p} \leq g(+\infty) + h(+\infty)$. This solution is unique if $g(y) + h(y)$ is strictly increasing. Moreover, $y^* = 0$ if $g(0) + h(0) \geq \hat{p}$ and $y^* = +\infty$ if $g(+\infty) + h(+\infty) \leq \hat{p}$. If the new term $h(y)$ is positive and increasing, then the optimal order quantity will be smaller than the optimal order quantity without hedging, and vice versa. Moreover, if there is no correlation between demand, supply and the market, $h(y)$ will be equal to zero since $\boldsymbol{\mu}'(y) = \mathbf{0}$ and the model reverts to the random supply model in the previous section.

There is further simplification in our results when there is only $n = 1$ security. These results are summarized in the following corollary.

Corollary 4 *If only one security is used for financial hedging (i.e.,* $n = 1$*), then the minimum-variance portfolio consists of*

$$\alpha^*(y) = -\frac{Cov(f(S), CF(\mathbf{X}, y))}{Var(f(S))} = p\beta_D(+\infty) - \left(v - ce^{rT}\right)\beta_{K,U}(y) - (s+p-v)\,\beta_{D,K,U}(y) \tag{24}$$

where

$$\beta_D(y) = \frac{Cov(f(S), \min\{D, y\})}{Var(f(S))} \tag{25}$$

$$\beta_{K,U}(y) = \frac{Cov(f(S), U\min\{K, y\})}{Var(f(S))} \tag{26}$$

and

$$\beta_{D,K,U}(y) = \frac{Cov(f(S), \min\{Uy, UK, D\})}{Var(f(S))}. \tag{27}$$

Moreover, the optimal order quantity y^* that maximizes the expected cash flow for the minimum-variance portfolio satisfies

$$\frac{E\left[U1_{\{D\leq Uy^*, K>y^*\}}\right]}{E\left[U1_{\{K>y^*\}}\right]} + \frac{\beta'_{D,K,U}(y^*)\, E[f(S)]}{E\left[U1_{\{K>y^*\}}\right]} + \frac{\left(v - ce^{rT}\right)\beta'_{K,U}(y^*)\, E[f(S)]}{(s+p-v)\, E\left[U1_{\{K>y^*\}}\right]} = \hat{p} \tag{28}$$

where

$$\beta'_D(y) = \frac{Cov\left(f(S), 1_{\{D>y\}}\right)}{Var(f(S))} \tag{29}$$

$$\beta'_{K,U}(y) = \frac{Cov\left(f(S), U1_{\{K>y\}}\right)}{Var(f(S))} \tag{30}$$

and

$$\beta'_{D,K,U} = \frac{Cov\left(f(S), U1_{\{K>y, D>Uy\}}\right)}{Var(f(S))}. \tag{31}$$

Proof. The results follow directly from Proposition 2 and Theorem 3 by noting that if there is only one derivative security for hedging with payoff $f(S)$, then $C = \text{Cov}(f(S), f(S)) = \text{Var}(f(S))$ and $\mu(y) = \text{Cov}(f(S), CF(\mathbf{X}, y))$ where $CF(\mathbf{X}, y) = \left(v - ce^{rT}\right)U\min\{K, y\} + (s+p-v)\min\{Uy, UK, D\} - pD$. Note also that $\beta_D(+\infty) = \text{Cov}(f(S), \min\{D, +\infty\})/\text{Var}(f(S)) = \text{Cov}(f(S), D)/\text{Var}(f(S))$. To see (29), we first write

$$\text{Cov}(f(S), \min\{D, y\}) = E[f(S)\min\{D, y\}] - E[f(S)]\, E[\min\{D, y\}]$$

where

$$E[f(S)\min\{D, y\}] = \int f(s) g_S(s) ds \left[\int_0^y z f_{D|S=s}(z) dz + y\int_0^y f_{D|S=s}(z) dz\right].$$

Here, g_S is the probability density function of S and $f_{D|S=s}$ is the conditional probability density function of D given $S = s$. Using Leibnitz rule as in the proof of Theorem 1, we get

$$\frac{dE[f(S)\min\{D, y\}]}{dy} = E[f(S)1_{\{D>y\}}].$$

Now, if we simply take $f(S) = 1$ in this analysis as a special case, $dE[\min\{D, y\}]/dy = E[1_{\{D>y\}}]$ and

$$\begin{aligned}\frac{d\text{Cov}(f(S), \min\{D, y\})}{dy} &= E[f(S)1_{\{D>y\}}] - E[f(S)]\, E[1_{\{D>y\}}] \\ &= \text{Cov}\left(f(S), 1_{\{D>y\}}\right)\end{aligned}$$

which leads to (29). The arguments for (30) and (31) are somewhat similar and they are omitted for brevity. ∎

One may be able to use the formulas in Corollary 4 to obtain explicit analytical solutions. However, this may be a formidable task in general, but they can be estimated using simulation. We now need to take

$$h(y) = \frac{\beta'_{D,K,U}(y)\, E[f(S)]}{E\left[U1_{\{K>y\}}\right]} + \frac{\left(v - ce^{rT}\right)\beta'_{K,U}(y)\, E[f(S)]}{(s+p-v)\, E\left[U1_{\{K>y\}}\right]}$$

as the additional term in (23). Our results provide compact characterizations of minimum-variance portfolios and corresponding expected value maximizing order quantities. As a final remark on the solutions of (11) and (12), note that determination of the optimal portfolio (18) does not require any additional assumptions. This result provides the main contribution of this paper and it always holds. However, we need additional conditions in determining the optimal order quantity by solving (21). The conditions that we impose address the existence and uniqueness of the solution of the optimality condition. After all, the objective function is not necessarily concave when there is random supply as demonstrated by the fact that the derivative of the objective function in (6) is not necessarily decreasing. This is true even for the simpler case of the standard newsvendor problem without hedging. The remedy for this situation is obtained, for example, whenever $g(y)$ is increasing which leads to quasi-concavity of the objective function. In the model with hedging, this is not sufficient and we need additional conditions on $gy) + h(y)$. Recall that if the market is risk-neutral, then $E[\mathbf{f}(S)] = 0$ so that $h(y) = 0$ and we only need to impose restrictions on $g(y)$. Without such additional restrictions, the optimality condition (21) may have many solutions. They may give local minima, maxima or inflection points. In this case, additional computational effort is needed to identify all solutions and choose the one that maximizes the objective function. In the next section, we investigate some special cases of the results presented in this section in order to obtain insights for the optimal hedging portfolio and the optimal order quantity.

4 Special Models

In this section we consider a number of special cases that lead to simplified versions of our characterization results. They are obtained by setting the values of $K = +\infty$ and/or $U = 1$ in (24) - (31).

4.1 Random Demand Model

Suppose that the supply is certain with $K = +\infty, U = 1$ and the randomness of the profit comes only from demand. The hedged cash flow is given by

$$CF_\alpha(D, S, y) = \left(v - ce^{rT}\right) y + (s + p - v) \min\{D, y\} - pD + \sum_{i=1}^{n} \alpha_i f_i(S).$$

Let $\hat{\boldsymbol{\mu}}(y)$ denote a vector of functions where the ith entry is $\hat{\mu}_i(y) = \text{Cov}\big(f_i(S), 1_{\{D>y\}}\big)$. One can show that the optimality condition (21) simplifies to give the following optimality condition.

Corollary 5 *The optimal order quantity that maximizes the expected cash flow while using the minimum-variance portfolio* $\boldsymbol{\alpha}^*(y)$ *satisfies*

$$P\{D \le y^*\} + \hat{\boldsymbol{\mu}}(y^*)^{\mathbf{T}} \mathbf{C}^{-1} E[\mathbf{f}(S)] = \hat{p}. \tag{32}$$

Proof. This follows by noting that $E\left[U1_{\{K>y^*, D\le Uy^*\}}\right] = E\left[1_{\{D\le y^*\}}\right] = P\{D \le y^*\}, E\left[U1_{\{K>y^*\}}\right] = 1$, and

$$\mu_i(y) = (s + p - v)\,\text{Cov}(f_i(S), \min\{D, y\}) - p\text{Cov}(f_i(S), D).$$

This completes the proof since $d\text{Cov}(f_i(S), \min\{D, y\})/dy = \text{Cov}\big(f_i(S), 1_{\{D>y\}}\big) = \hat{\mu}_i(y)$ by (29) and $\mu_i'(y) = (s + p - v)\hat{\mu}_i(y)$. ■

The right-hand side of the optimality condition (32) is the same critical ratio $\hat{p}$. The left-hand side consists of a cumulative distribution function $g(y) = P\{D \le y\}$ that is always increasing, plus the additional term $h(y) = \hat{\boldsymbol{\mu}}(y)^{\mathbf{T}} \mathbf{C}^{-1} E[\mathbf{f}(S)]$. Note that this term's sign and behavior is crucial. If it is non-decreasing and positive, then the optimal ordering quantity will be smaller compared to unhedged optimal ordering quantity. If it is non-decreasing and negative, then the optimal ordering quantity will be larger compared to the unhedged optimal ordering quantity. Furthermore, if there is no correlation between S and D, $\boldsymbol{\mu}(y)$ will be zero; thus, the optimality condition in (32) reverts back to standard newsvendor model (8).

If there is a single hedging asset, then one can show that (24) reduces to

$$\alpha^*(y) = p\beta_D(+\infty) - (s + p - v)\beta_D(y) \tag{33}$$

and the optimal order quantity now satisfies

$$P\{D \le y^*\} + \beta'_D(y^*)\, E[f(S)] = \hat{p} \tag{34}$$

from (32).

Recall that the right-hand side of (34) $\hat{p}$ is the same critical ratio. The left-hand side consists of $g(y) = P\{D \le y\}$ which is increasing plus a covariance term $\beta'_D(y) = \mathrm{Cov}\big(f(S), 1_{\{D>y\}}\big)$ multiplied by the expected payoff of the derivative security $E[f(S)]$. Hence, the structure of the optimal solution depends on the covariance term's sign and shape. Assuming that the payoff $f(S)$ is non-negative,

- If $\mathrm{Cov}\big(f(S), 1_{\{D>y\}}\big) > 0$, the optimal order quantity with hedging will be smaller than the optimal order quantity without hedging.
- If $\mathrm{Cov}\big(f(S), 1_{\{D>y\}}\big) < 0$, the optimal order quantity with hedging will be larger than the optimal order quantity without hedging.
- If $\mathrm{Cov}\big(f(S), 1_{\{D>y\}}\big) = 0$, they will be equal.

To make further characterizations, we need some assumptions about the relationship between D and S. To this end, the concept of "positive association" between random variables is quite useful. Two random variables, such as D and S, are positively associated (PA) if $\mathrm{Cov}(g(D), h(S)) \ge 0$ is true for all pairs of non-decreasing functions g and h. Therefore, assuming D and S are PA variables implies that $\mathrm{Cov}\big(f(S), 1_{\{D>y\}}\big) \ge 0$ if the payoff $f(S)$ is an increasing function of S. Hence, $\beta_D(y)$ becomes an increasing function of y. Based on this, we observe that $\alpha^*(0) = p\beta_D(+\infty) > 0$, $\alpha^*(+\infty) = -(s+p-v)\,\beta_D(+\infty) < 0$ and as y increases $\alpha^*(y)$ decreases. This means that a lower amount of investment in the security is needed when the order quantity is higher.

4.2 Random Yield Model

We now suppose that $K = +\infty$ so that $Q(y) = Uy$. Now, the cash flow becomes

$$CF_\alpha(D, U, S, y) = \big(v - ce^{rT}\big)\, Uy + (s+p-v)\min\{D, Uy\} - pD + \sum_{i=1}^{n} \alpha_i f_i(S). \tag{35}$$

Corollary 6 *The optimal order quantity that maximizes the expected cash flow while using the minimum-variance portfolio* $\boldsymbol{\alpha}^*(y)$ *satisfies*

$$\frac{E\left[U 1_{\{D \le Uy^*\}}\right]}{E[U]} + \frac{\boldsymbol{\mu}'(y^*)^{\mathbf{T}}\, \mathbf{C}^{-1} E[\mathbf{f}(S)]}{(s+p-v)\, E[U]} = \hat{p}. \tag{36}$$

Proof. This follows from the optimality condition in (21) by observing that we have $E\left[U 1_{\{K>y^*\}}\right] = E[U]$ and $E\left[U 1_{\{D \le Uy^*, K>y^*\}}\right] = E\left[U 1_{\{D \le Uy^*\}}\right]$ whenever $K = +\infty$. ■

Note that $g(y) = E\left[U 1_{\{D \le Uy\}}\right] / E[U]$ is increasing and one needs to impose additional restrictions on $h(y)$ as discussed before. If there is a single asset, then (24) becomes

$$\alpha(y^*) = p\beta_D(+\infty) - \big(v - ce^{rT}\big)\, y\beta_U - (s+p-v)\,\beta_{D,U}(y) \tag{37}$$

where

$$\beta_{D,U}(y) = \frac{\mathrm{Cov}\left(f(S), \min\{D, Uy\}\right)}{\mathrm{Var}\left(f(S)\right)}$$

and

$$\beta_U = \frac{\mathrm{Cov}\left(f(S), U\right)}{\mathrm{Var}\left(f(S)\right)}.$$

Moreover, the optimality condition (36) becomes

$$\frac{E\left[U1_{\{D\leq Uy^*\}}\right]}{E\left[U\right]}+\frac{\left(\left(s+p-v\right)\beta'_{D,U}\left(y^*\right)+\left(v-ce^{rT}\right)\beta_U\right)E\left[f\left(S\right)\right]}{\left(s+p-v\right)E\left[U\right]}=\hat{p} \tag{38}$$

where one can also show that

$$\beta'_{D,U}\left(y\right)=\frac{\text{Cov}\left(f\left(S\right),U1_{\{D>Uy\}}\right)}{\text{Var}\left(f\left(S\right)\right)}$$

using an analysis similar to the one in the proof of Corollary (4).

Recalling that $\mu_i\left(y\right)=\text{Cov}(f_i\left(S\right),CF\left(\mathbf{X},y\right))=\text{Cov}\big(f_i\left(S\right),\left(v-ce^{rT}\right)Uy+\left(s+p-v\right)\min\left\{D,Uy\right\}-pD\big)$,we can further take

$$\mu'_i\left(y\right)=\left(v-ce^{rT}\right)\text{Cov}\left(f_i\left(S\right),U\right)+\left(s+p-v\right)\text{Cov}\left(f_i\left(S\right),U1_{\{D>Uy\}}\right)$$

in (36) for the multiple asset case.

4.3 Random Capacity Model

As a final case, we consider the random capacity model where $U=1$ so that $Q\left(y\right)=\min\left\{y,K\right\}$ and the cash flow is

$$CF_\alpha\left(D,K,S,y\right)=\left(v-ce^{rT}\right)\min\left\{K,y\right\}+\left(s+p-v\right)\min\left\{D,K,y\right\}-pD+\sum_{i=1}^{n}\alpha_i f_i\left(S\right). \tag{39}$$

Corollary 7 *The optimal order quantity that maximizes the expected cash flow while using the minimum-variance portfolio* $\boldsymbol{\alpha}^*\left(y\right)$ *satisfies*

$$P\left\{D\leq y^*\mid K>y^*\right\}+\frac{\boldsymbol{\mu}'\left(y^*\right)^{\mathbf{T}}\mathbf{C}^{-1}E\left[\mathbf{f}\left(S\right)\right]}{\left(s+p-v\right)P\left\{K>y^*\right\}}=\hat{p}. \tag{40}$$

Proof. This follows from the optimality condition in (21) by noting that $E\left[U1_{\{K>y^*\}}\right]=P\left\{K>y^*\right\}$ and $E\left[U1_{\{D\leq Uy^*,K>y^*\}}\right]=P\left\{D\leq y^*,K>y^*\right\}$ whenever $U=1$. ■

Furthermore, if there is only 1 asset, then (24) yields

$$\alpha^*\left(y\right)=p\beta_D\left(+\infty\right)-\left(v-ce^{rT}\right)\beta_K\left(y\right)-\left(s+p-v\right)\beta_{D,K}\left(y\right) \tag{41}$$

where

$$\beta_K\left(y\right)=\frac{\text{Cov}\left(f\left(S\right),\min\left\{K,y\right\}\right)}{\text{Var}\left(f\left(S\right)\right)}$$

and

$$\beta_{D,K}\left(y\right)=\frac{\text{Cov}\left(f\left(S\right),\min\left\{D,K,y\right\}\right)}{\text{Var}\left(f\left(S\right)\right)}.$$

Moreover, the optimality condition (40) further reduces to

$$P\left\{D\leq y^*\mid K>y^*\right\}+\frac{\left(\left(v-ce^{rT}\right)\beta'_K\left(y^*\right)+\left(s+p-v\right)\beta'_{D,K}\left(y^*\right)\right)E\left[f\left(S\right)\right]}{\left(s+p-v\right)P\left\{K>y^*\right\}}=\hat{p} \tag{42}$$

where

$$\beta'_K\left(y\right)=\frac{\text{Cov}\left(f\left(S\right),1_{\{K>y\}}\right)}{\text{Var}\left(f\left(S\right)\right)}$$

and

$$\beta'_{D,K}\left(y\right)=\frac{\text{Cov}\left(f\left(S\right),1_{\{D>y,K>y\}}\right)}{\text{Var}\left(f\left(S\right)\right)}$$

using an analysis similar to the one in the proof of Corollary (4).

Finally, $\mu_i(y) = \text{Cov}(f_i(S), CF(\mathbf{X}, y)) = \text{Cov}\big(f_i(S), \big(v - ce^{rT}\big)\min\{K, y\} + (s+p-v)\min\{D, K, y\} - pD\big)$ leads to

$$\mu_i'(y) = \big(v - ce^{rT}\big)\,\text{Cov}\left(f_i(S), 1_{\{K>y\}}\right) + (s+p-v)\,\text{Cov}\left(f_i(S), 1_{\{D>y, K>y\}}\right)$$

in (40) for the multiple asset case.

The results on these important cases lead to a number of managerial insights. The first observation is that the optimal order quantity y^* is not affected by hedging if $E[\mathbf{f}(S)] = 0$,or if the expected cash flow from the financial portfolio is zero. In other words, if the prices of the financial assets is the portfolio do not lead to arbitrage opportunities, there is no gain expected in the cash flow. The formulas (32), (36), and (40) trivially reduce to (8), (9), and (10) respectively. If this is not true and $E[\mathbf{f}(S)] \neq 0$, then the optimal order quantity is affected by hedging as it is implied by the second term on the right-hand side of (32), (36), and (40). Whether this will lead to increased or decreased order quantities depends on a number of factors. For example, if $\boldsymbol{\mu}'(y^*)^{\mathbf{T}}\,\mathbf{C}^{-1}E[\mathbf{f}(S)] > 0$, then the optimal order quantity will be higher provided that the usual assumptions on $g(y) + h(y)$ holds. The opposite is true when $\boldsymbol{\mu}'(y^*)^{\mathbf{T}}\,\mathbf{C}^{-1}E[\mathbf{f}(S)] < 0$. There is no quick and easy way to check this condition, it depends on correlations among the random variables involved. This condition can be further simplified in case there is only one asset used for hedging. They are presented by (34), (38), and (42). The discussion at the end Section 4.1 sheds some light on this issue. The most important point that needs to be emphasized is that, no matter what the order quantity is, one can always determine the hedging portfolio and this will surely have a positive impact on the risk or variance of the cash flow. The formulas (18), (24), (33), (37), and (41) explain explicitly how this can be done. The numerical analysis requires the computation of a number of β values. If the computation can not be done analytically, one can always use simulation for estimation. As a matter of fact, we will use this approach in the section on numerical illustrations next.

5 Numerical Illustrations

In this section, we present numerical examples to quantify the effects of employing financial hedging to compensate for demand and supply risks. We consider three different models: the random demand case and two generalizations, first with random capacity and second with random yield. We want to point out that all of our numerical calculations are done using Monte Carlo simulations throughout the remainder of this section. This is because explicit analytical expressions for variances and covariances can not be found. We use Matlab as the simulation tool. Cash flows are generated by using the simulated values of S, D, K, and U whenever needed. We generate $20,000$ instances to calculate the stock prices, demand, capacity, yield, and payoff of the derivative securities. Then, we use the formulas (33), (37), and (41) obtained in Section 4 to calculate the optimal portfolios using estimates of the covariances in these formulas. Finally, we generate another $20,000$ instances to obtain the corresponding cash flows. For each illustration, we calculate the means and variances of the cash flows in order to emphasize the impact of financial hedging. Since our primary aim is to discuss hedging, we will not consider secondary problems involving the choice of the order quantity obtained from (34), (38), and (42). As a matter of fact, we will construct a risk-neutral model with $E[f(S)] = 0$ and the optimal order quantity is not affected by hedging. This also implies that the expected values of the unhedged and hedged cash flows are equal so that we can make a fair comparison among various hedging portfolios.

5.1 Random Demand Case

As the base scenario, we take the setting of the example in Gaur and Seshadri [2005] where the demand risk is hedged by a stock in the financial market. Let the initial stock price S_0 be \$660 and the interest rate be

$r = 10\%$ per year. Assume that $T = 6$ months and that the return S_T/S_0 has a lognormal distribution with $\mu = 10\%$ per year and $\sigma = 20\%$ per year. We assume that the demand $D = b\,S_T + \epsilon$ where $b = 10$ and ϵ has a normal distribution with mean zero and standard deviation σ_ϵ. The financial parameters are as follows: $s = \$1$, $c = \$0.6, p = 0$, and $v = \$0.1$. Finally, we let the order quantity be $y = 7000$ in almost all of our illustrations. This is a round figure that is close to the order quantity $\overline{y} \cong 6719.30$ that satisfies

$$P\{D = 10S_T \leq 6719.30\} = \hat{p} = (s + p - ce^{rT})/(s + p - v) \cong 0.41$$

for the lognormal distribution in the random demand case. Our primary objective is to illustrate the effect of hedging by a financial portfolio in order to decrease the variance while keeping the mean cash flow intact.

It is important to note that Gaur and Seshadri [2005] have shown that when $\sigma_\epsilon = 0$ there is a risk-free portfolio that consists of stock futures and call options with strike price of $\$y/b = \$700$ (and settlement date T). Let us denote by $\hat{f}_1(S) = S$ the future and by $\hat{f}_2(S) = (S - 700)^+$ the call option. It turns out that if the optimal investment amounts in these instruments are chosen to be $\alpha_1 = -(s - v)b$ and $\alpha_2 = (s - v)b$, the variance of the hedged profit is zero. This shows that a portfolio consisting of stock futures and appropriately selected call options on the stock is sufficient to remove all variability under this special case. The conclusion follows from the observation that

$$CF_\alpha\left(D, S, y\right) = \left(v - ce^{rT}\right) y + (s - v) \min\left\{D, y\right\} + \alpha_1 \left(S - f_1^T\right) + \alpha_2((S - (y/b))^+ - f_2^T)$$

which can be rewritten as

$$\begin{aligned} CF_\alpha\left(D, S, y\right) &= \left(v - ce^{rT}\right) y + (s - v)\, bS - (s - v)b((S - (y/b))^+ \\ &\quad +\alpha_1 \left(S - f_1^T\right) + \alpha_2((S - (y/b))^+ - f_2^T) \\ &= \left(v - ce^{rT}\right) y - \alpha_1 f_1^T - \alpha_2 f_2^T. \end{aligned} \tag{43}$$

after noting that $\min\left\{D, y\right\} = D - (D - y)^+$ and substituting $D = bS$. The hedged cash flow (43) is made deterministic by choosing the portfolio $\alpha_1 = -(s - v)b = -9$ and $\alpha_2 = (s - v)b = 9$. Moreover, it is equal to

$$CF_\alpha\left(D, S, y\right) = \left(v - ce^{rT}\right) y + (s - v)be^{rT}\left(f_1^0 - f_2^0\right)$$

independent of D or S where f_1^0 and f_2^0 are the purchase prices of the two derivatives at the beginning of the period. In our analysis, we will focus on the effect of the portfolios on the variance of the cash flow. We can make fair comparisons on risk reduction when the expected values of the cash flows are the same with or without financial hedging. That is why we suppose that $E\left[S - f_1^T\right] = E\left[(S - (y/b))^+ - f_2^T\right] = 0$ and the expected gain from the financial portfolio is 0. This leads to the expected cash flow

$$E\left[CF_\alpha\left(D, S, y\right)\right] = \left(v - ce^{rT}\right) y + (s - v)\, bE\left[S\right] - (s - v)bE\left[((S - (y/b))^+\right].$$

Although perfect hedging is possible when there is a perfect linear relation $D = bS$, this is not realistic at all. It can not be achieved when $D = b\,S_T + \epsilon$ with $\sigma_\epsilon > 0$ and minimum-variance portfolios should be determined by using our results. For hedging demand variability, we consider three types of financial portfolios. The first portfolio consists of the future on the stock only and has the structure $\alpha_1 \approx \alpha_{1,1} f_1(S)$, the second portfolio consists of the call option on the stock with strike price y/b only and has the structure $\alpha_2 \approx \alpha_{2,2} f_2(S)$. Finally, the third portfolio uses both instruments jointly and has the structure $\alpha_3 \approx \alpha_{3,1} f_1(S) + \alpha_{3,2} f_2(S)$. The optimal portfolios in the three cases are determined using Proposition 2 and Corollary 4 where the intermediate covariance calculations are done using the estimators from Monte Carlo simulations.

σ_ϵ	$E[CF]$	Var(CF)	Var(CF_{α_1})	Var(CF_{α_2})	Var(CF_{α_3})	$\alpha_{1,1}$	$\alpha_{2,2}$	$\alpha_{3,1}$	$\alpha_{3,2}$
0	2095.54	220185.58	64459.51	172989.59	0.00	-4.46	-4.03	-9.00	9.00
100	2094.06	222868.73	67335.25	175316.46	3924.57	-4.46	-4.05	-8.96	8.93
500	2055.20	287406.26	133664.83	232866.11	86700.63	-4.43	-4.34	-8.27	7.67
900	1976.77	434629.42	281201.20	369730.28	251124.68	-4.43	-4.73	-7.45	6.13

Table 1: The variances of the cash flows and the optimal investment amounts for different hedging portfolios ($y = 7000$)

Table 1 shows the variance reductions in the cash flows that are made possible by financial hedging. It is seen that a deterministic profit is possible using the financial portfolio $(-9f_1(S) + 9f_2(S))$ when there is perfect correlation between the stock price and demand ($\sigma_\epsilon = 0$). In addition, significant variance reductions are achieved when the standard deviation of the demand error is small ($\sigma_\epsilon = 100$) corresponding to the case of a strong correlation. The reductions decrease when the correlation decreases but even when $\sigma_\epsilon = 900$ the variance of the cash flow (434629.42) can be lowered by more than 40% (251124.68) using the appropriate financial portfolio $(-7.45f_1(S)+6.13f_2(S))$. Figure 2 depicts similar results graphically for a larger set of demand standard deviation values.

As for the optimal portfolio structure, it is always optimal to sell the future since demand and stock price are taken to be positively correlated. On the other hand, in the optimal portfolio, the call option is bought when used as the second instrument along with the future but is sold when it is used as the sole instrument. It is also interesting to note that using a portfolio consisting only of the future on the stock is very effective and achieves most of the variance reduction benefits. On the other hand, the call option serves to fine tune the portfolio along with the investment in the stock but is not as effective when used alone.

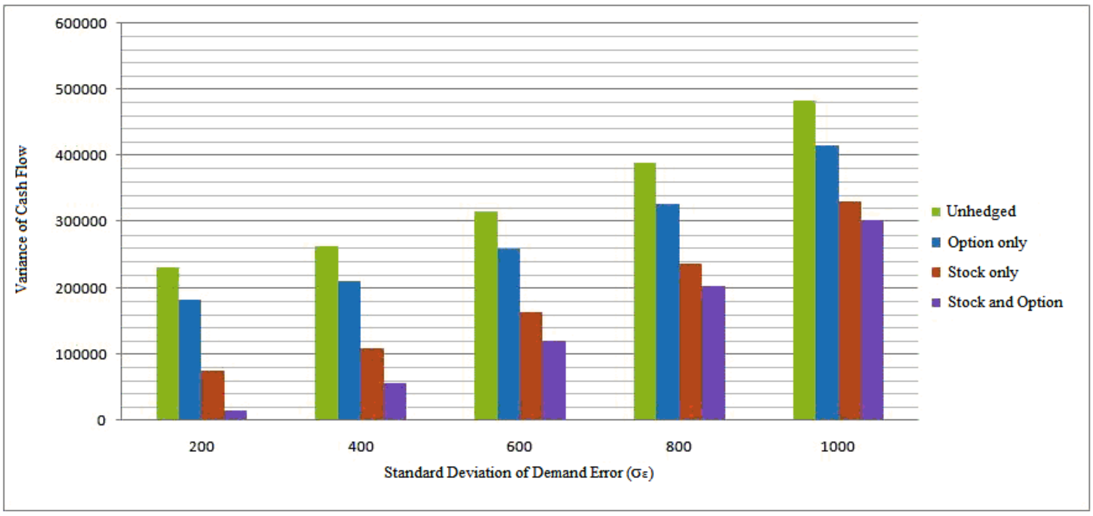

Figure 2: Effects of different hedging portfolios on the variance of the cash flow

5.2 Random Capacity Model

For the base examples, we use the same assumptions as in Section 5.1. In addition, we assume the following relationship between the stock price and the capacity $K = k\,S_T + \eta$ where $k = 10$ and η has a normal distribution with mean zero and standard deviation σ_η.

We first assume that there is ample demand in the market with respect to the capacity available (i.e., $P\{D > K\} = 1$) and that capacity is perfectly correlated with the stock ($\sigma_\eta = 0$). To gain an understanding of the variance reductions achievable, we use the same three types of portfolios from the previous subsection (future alone, call option alone and future and call option together). It can be shown that if the strike price of the call option is taken as $y/k = 700$ and the optimal investment amounts in the instruments are chosen to be $\alpha_{3,1} = -(s - c\,e^{rT})k = -3.69$ and $\alpha_{3,2} = (s - c\,e^{rT})k = 3.69$, the variance of the hedged profit is zero.

y	$E[CF]$	$\text{Var}(CF)$	$\text{Var}(CF_{\alpha_1})$	$\text{Var}(CF_{\alpha_2})$	$\text{Var}(CF_{\alpha_3})$	$\alpha_{1,1}$	$\alpha_{2,2}$	$\alpha_{3,1}$	$\alpha_{3,2}$
6000	2082.60	5410.10	3721.80	4611.30	0.00	-0.44	-0.34	-3.69	3.69
8000	2405.60	83725.00	6276.60	69809.00	0.00	-2.98	-4.21	-3.69	3.69
10000	2435.20	117340.00	194.14	115380.00	0.00	-3.67	-11.57	-3.69	3.69
12000	2435.80	118780.00	0.08	118620.00	0.00	-3.69	-159.46	-3.69	3.69

Table 2: The variances of the cash flows and the optimal hedging portfolios when capacity is perfectly correlated with the stock ($\sigma_\eta = 0$) and demand is ample ($D > K$)

Table 2 shows that financial hedging in the random capacity model has similar properties as in the random demand case. In particular, a portfolio consisting only of the future is extremely effective and achieves most of the variance reduction. Of course, adding the call option as a second instrument removes all of the rest of the remaining variance as before. Note that there is a significant benefit to holding a financial portfolio in this situation because the optimal expected cash flow is increasing in the order quantity. But increasing the order quantity also increases the variance without financial hedging. The financial hedge removes the variance and enables the newsvendor to order higher quantities while tolerating reasonable variances on the cash flow. Note that it is not realistic to have $\sigma_\eta = 0$ so that there is perfect linear relationship between stock price and capacity. In reality, this will not be true but there may be some level of correlation between them and this can be exploited to reduce the variance of the cash flow.

Next, in order to explore the effect of imperfect correlations with the market, we return to the base example and take $D = 10\,S_T + \epsilon$. Further, it is assumed that $K = 9\,S_T + \eta$. Note that as σ_ϵ and σ_η increase, the correlations between the demand and the market, and the capacity and the market weaken. At the same time, the correlation between the demand and the capacity also weakens. Table 3 reports the optimal portfolios and the resulting variances as σ_ϵ and σ_η are varied together. It can be observed that, once again, significant reductions in variance can be achieved by hedging. The reductions are naturally most important when the market correlation is strong. For instance, the case $\sigma_\epsilon = \sigma_\eta = 400$ corresponds to correlation coefficients of 0.9 with the market and the variance can be reduced by a factor of more than 5. Even in the case when the correlations with the market are relatively low ($\sigma_\epsilon = \sigma_\eta = 1000$ corresponds to correlation coefficients lower than 0.7), the variance reduction is considerable.

Last, we fix the demand-market correlation by letting $\sigma_\epsilon = 600$ and vary the capacity-market correlation by varying σ_η. The results are summarized in Table 4. The variance reductions are comparable to those in Table 3. It appears that in this case a strong correlation between the market and the capacity is sufficient to achieve significant variance reductions even if the demand-market correlation is relatively lower.

5.3 Random Yield Model

In this subsection, we briefly present an example for the random yield model. The functional form relating the stock price to the yield can take many forms and we take the following plausible example where $U =$

$\sigma_\epsilon = \sigma_\eta$	$E[CF]$	Var(CF)	Var(CF_{α_1})	Var(CF_{α_2})	Var(CF_{α_3})	$\alpha_{1,1}$	$\alpha_{2,2}$	$\alpha_{3,1}$	$\alpha_{3,2}$
200	2150.80	63174.00	9829.10	51930.00	3782.90	-2.47	-3.15	-3.29	3.24
400	2124.10	78679.00	20981.00	65698.00	15243.00	-2.57	-3.39	-3.37	3.15
600	2073.10	112240.00	45949.00	95739.00	40719.00	-2.76	-3.82	-3.52	3.01
800	2010.90	162890.00	87211.00	142060.00	82674.00	-2.95	-4.29	-3.65	2.80
1000	1943.40	228810.00	144350.00	203510.00	140520.00	-3.11	-4.73	-3.76	2.58

Table 3: The variances of the cash flows and the optimal hedging portfolios in the case of imperfect demand - market and demand - capacity correlation ($y = 7000$)

σ_η	$E[CF]$	Var(CF)	Var(CF_{α_1})	Var(CF_{α_2})	Var(CF_{α_3})	$\alpha_{1,1}$	$\alpha_{2,2}$	$\alpha_{3,1}$	$\alpha_{3,2}$
0	2122.90	79958.00	19439.00	66631.00	13147.00	-2.64	-3.43	-3.47	3.30
200	2116.10	84115.00	23056.00	70294.00	17061.00	-2.65	-3.50	-3.46	3.22
400	2098.20	95573.00	32573.00	80570.00	27045.00	-2.69	-3.64	-3.47	3.10
600	2073.10	112240.00	45949.00	95739.00	40719.00	-2.76	-3.82	-3.52	3.01
800	2043.90	132470.00	61976.00	114390.00	56819.00	-2.84	-4.00	-3.60	2.99
1000	2013.20	154830.00	79871.00	135270.00	74632.00	-2.93	-4.16	-3.69	3.01

Table 4: The variances of the cash flows and the optimal hedging portfolios ($y = 7000$)

$1 - e^{-(1/S_0)(\gamma+S_T)}$. As in the previous examples, we take γ to be normally distributed with mean zero and standard deviation σ_γ.

We take the base scenario of Section 5.1 and use the identical portfolio consisting of the stock itself and a call option. We assume that $\sigma_\gamma = 100$ leading to a market-yield correlation of 0.7 and vary the market-demand correlation by changing σ_ϵ. The results are summarized in Figure 3. As before, it can be seen that significant variance reductions are achievable.

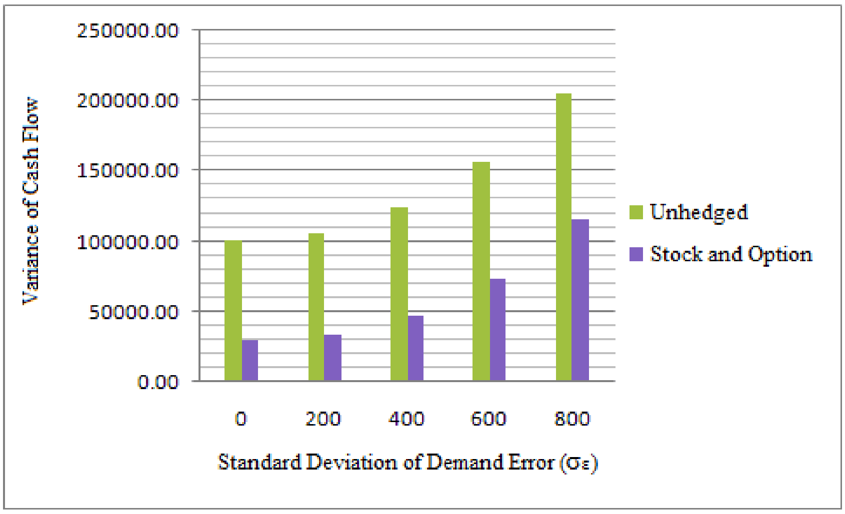

Figure 3: Effect of financial hedging on variance of the cash flow in the case of random yield

5.4 Summary of Numerical Observations

Sections 5.1 – 5.3 present numerical examples for three different models. In Section 5.1, the only uncertainty is due to random demand and capacity and yield are certain. In Section 5.2, both capacity and demand are random and in Section 5.3 both demand and yield are random. The common threads to the analysis can be summarized as follows:

- For plausible order quantities, there is considerable variability in the cash flow in all cases. This should be a cause for concern.
- The variability in cash flow can be effectively mitigated by financial hedging when the uncertainty in the inventory operation is correlated to the uncertainty in a financial instrument. The effectiveness of financial hedging depends on the degree of correlation but significant improvements in variability are possible even for moderate levels of correlation
- The most effective financial portfolio involves a combination of futures and call options on the correlated instrument. However, a portfolio consisting only of the future on the instrument is also extremely effective. On the other hand, a portfolio consisting only of a call option has limited effectiveness.

6 Concluding Remarks

In this paper, we take a risk-sensitive approach in managing a single-period, single-item inventory (newsvendor) model. The risks or uncertainties in our model are generated by random demand as well as random supply due to capacity and yield. The combined randomness of demand and supply enhances the level of uncertainty, thus leading to an increased risk for the newsvendor. Furthermore, based on statistical evidence, we suppose that the randomness in the inventory model correlated with the randomness in the financial markets. We consider the opportunities of financial hedging to mitigate inventory risks when the demand and/or supply are correlated with the price of a financial asset. The correlation is not necessarily perfect and this prevents us from constructing a replicating portfolio; thus, limiting our ability to remove the variability of the cash flow completely. Moreover, the inexistence of such a portfolio implies that the cash flow will remain random, making the analysis more challenging. Since the cash flow can not be replicated, we choose to minimize the variance of the hedged cash flow by holding a portfolio of financial securities. Therefore, in our context, the newsvendor selects the order quantity and the financial portfolio simultaneously. Once the minimum-variance portfolio is determined for each order quantity, the newsvendor then chooses that order quantity that maximized the expected cash flow. The structure of the optimal portfolio and the optimal order quantity clearly depends on the correlation between the random variables of the inventory model and the random financial variables.

Our analysis shows that financial hedging is a useful tool to further improve management of inventory. In our analysis, we illustrated how it can be done for a specific well-known inventory model. There are many suitable areas for extensions, such as using utility functions to represent the risk-sensitivity of the decision maker. Another line of future research involves multi-period and infinite-period models where the optimality of base-stock policies or its variations should be investigated. In continuous-time models, it is well-worth studying the possibility of continuous hedging through financial instruments.

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