Risk Hedging in the Newsvendor Model

H. Kaan OKYAY, Fikri KARAESMEN and Süleyman ÖZEKICI [1]
Koç University, Department of Industrial Engineering, Istanbul, Turkey

Abstract. There is ample evidence that the demand for products held in an inventory system is often correlated with the returns of securities in financial markets. Therefore, the risks associated with the profit or cash flow in the inventory system can be hedged by investing in a portfolio of instruments in the financial system. In order to get insights, we take this idea to the extreme by supposing that random demand, as well as random supply, both depend "perfectly" on the price of a security in an almost arbitrary fashion. This allows one to represent the cash flow by a replicating portfolio of derivative securities and bonds. Thus, the value of the cash flow needs to be determined in terms of the prices of these financial instruments. The decisions of the inventory manager are therefore based on this pricing mechanism. In particular, in a complete market with some risk-neutral martingale measure that yields no arbitrage opportunities, the expected value of the cash flow should be determined using this measure. We discuss these issues in the context of a single period newsvendor model with random demand and supply.

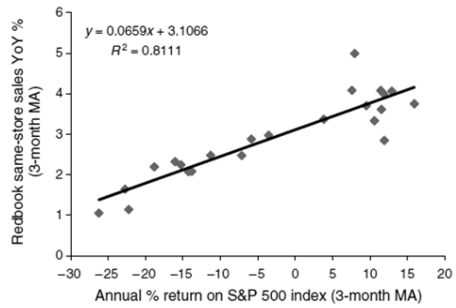
Keywords.
Newsvendor model, demand and supply risks, perfect hedging, replicating portfolio

Introduction

In a related article, Gaur and Seshadri [7] discuss the relationship between sales of inventory in stores and values of some financial index. As it is clearly illustrated by Figure 1 taken from Gaur and Seshadri [7], there is very strong statistical evidence that an inventory index (Redbook) that represents average sales is very highly correlated with a financial index (S&P 500) that represents average prices of stocks in the financial markets. They further discuss the case when the relationship is perfect so that the random demand in a period is a linear function of the price of a share of stock traded in the market. In the context of the newsvendor model, they obtain an explicit expression for the portfolio that replicates the periodic cash flow of the newsvendor model. The portfolio consists of a cash bond and derivatives (futures and European calls) of the stock. This naturally leads to the conclusion that the inventory manager (IM) should consider the value of the replicating portfolio at the beginning of the period in order to avoid arbitrage opportunities. In this paper, our primary aim is to extend this analysis to the case where this relationship is not necessary linear. Furthermore, we also take into account the fact that the risks in the inventory model is increased due to randomness in the supply in addition to the demand.

Although the main source of randomness is the demand in inventory models, there has been a lot of emphasis in recent years on models where the supply is random as well.

[1] Corresponding Author, E-mail: sozekici@ku.edu.tr.



Notes. The *Redbook* year-on-year same-store sales growth rate is computed as the growth rate of same-store sales during a month with respect to the same month in the previous year. We use time-series data for 25 months from November 1999 to November 2001. We compute three-month moving averages to reduce autocorrelation.

Figure 1. Redbook Same-Store Sales Growth Rate vs. Annual Return on the S&P 500 Index

Chopra and Sodhi [4] and Serel [18] discuss some of the issues related to randomness in supply and mention a number of real cases. For example, as reported in Norrman and Jansson [12], a fire at a supplier's plant disrupted the supply of radio-frequency chips to Ericsson in 2001 resulting in a loss of $400 million. Juttner [9] reports that in the same year, the continuity of production at Land Rover was threatened due to financial problems faced by the UK chassis manufacturer UPF Thompson. Kharif [10] states that Motorola failed to ship the phones promised to its major customers during the holiday season in 2003 due to component shortages.

Most of the literature on supply randomness considers the following models based on randomness in the capacity of the supplier and in the yield. An implicit assumption in almost all of the papers is the independence of demand and supply. Let y be the amount ordered and $Q(y)$ be the amount received.

- Random Yield: The amount ordered could be different from the amount received so that only a fraction enters the stockpile and

$$Q(y) = yU \tag{1}$$

  where U represents the proportion of nondefective items received. Henig and Gerchak [8] show that a non-order-up-to policy is optimal in this case. However, in a simplified version where U is either 0 or 1, also called the random availability model, Özekici and Parlar [15] establish the optimality of base-stock policies.
- Random Capacity: The supplier has some random replenishment capacity K so that

$$Q(y) = \min\{K, y\}. \tag{2}$$

  When an order is placed for y units, the suppliers will ship y if the total amount K of on hand inventory that they poses is greater than y. Or else, they will send all the inventory they poses, which is K. Erdem and Özekici [6] consider a periodically reviewed single-item inventory model in a random environment with random capacity and show that a base-stock policy is optimal.

- Random Yield and Capacity: This is another model that combines the previous two so that

$$Q(y) = U \min\{K, y\}. \tag{3}$$

  Once y units are ordered, the supplier can ship at most K and only a proportion U is received in good shape. In a recent article, Arifoğlu and Özekici [2] consider a model with random yield and fixed capacity operating in a hidden Markov environment. A state-dependent modified inflated base-stock policy is shown to be optimal.

In this paper, we consider the standard newsvendor model with random demand and supply which depend perfectly on the price of a share of stock. We first review the standard newsvendor model with random demand and supply briefly in Section 1. The same model with perfect hedging is discussed later in Section 2 where characterizations on the structures of the replicating portfolio and the optimal order quantity are provided. Special cases involving only random demand, random capacity, and random yield are discussed in Section 3. Concluding remarks and ideas for future work are presented in Section 4.

1. The Newsvendor Model with Random Supply

The newsvendor problem is a well-known single-item, single-period inventory problem in which the decision maker (or newsvendor) has to decide on how much to order. The replenishment decision is critical because if he orders too many, purchase cost will be unnecessarily high; on the contrary, there will be a missed opportunity for additional profit if he orders too few. In daily life, it is very common to encounter examples of newsvendor models, that's the foremost reason why these models are studied extensively. There is random demand D with a known distribution function that has a probability density function. Throughout this paper, we assume that all marginal, joint and conditional distributions corresponding to random demand D, random yield U and random capacity K have marginal, joint and conditional probability density functions. We suppose that the length of the period is T during which there is interest charged continuously with some rate r. Moreover, we suppose that there is a fixed sale price p, a fixed purchase cost c, a fixed shortage penalty h, and a fixed salvage value s which satisfy $p > ce^{rT} > s > 0$ and $p + h > ce^{rT}$ to avoid trivial situations. All cash flows occur at time T except for the cash payment made at time 0 to purchase inventory. This model is discussed earlier by Okyay et al. [13] where there is no hedging and no interest so that $r = 0$. We now present their results adjusted accordingly to our setting with positive interest.

The aim of the newsvendor is to maximize the expected cash flow at the end of the period by choosing an ordering quantity y, or

$$\max_y E\left[CF(D, K, U, y)\right] \tag{4}$$

where $CF(D, K, U, y)$ is the random cash flow which can be written as

$$CF(D,K,U,y) = \left(s - ce^{rT}\right) U \min\{y,K\} \\ + (p+h-s)\min\{D, U\min\{K,y\}\} - hD. \tag{5}$$

The optimal order quantity y^* satisfies

$$\frac{E\left[U1_{\{K>y^*, D\le Uy^*\}}\right]}{E\left[U1_{\{K>y^*\}}\right]} = \frac{p+h-ce^{rT}}{p+h-s} = \hat{p}. \tag{6}$$

The objective function is not necessarily concave. However, defining

$$h(y) = \frac{E\left[U1_{\{K>y, D\le Uy\}}\right]}{E\left[U1_{\{K>y\}}\right]} \tag{7}$$

one obtains conditions for the existence and uniqueness of the solution. More precisely, if $h(y)$ is increasing in y, there exists $0 \le y^* \le +\infty$ which satisfies the optimality condition $h(y^*) = \hat{p}$ provided that $h(0) \le \hat{p} \le h(+\infty)$. This solution is unique if $h(y)$ is strictly increasing. Moreover, the objective function is concave increasing on $0, y^*)$ and decreasing on $y^*, +\infty)$. Therefore, it is quasi-concave and y^* is indeed the optimal solution that maximizes the objective function. Finally, $y^* = 0$ if $h(0) \ge \hat{p}$ and $y^* = +\infty$ if $h(+\infty) \le \hat{p}$.

It is clear that if $K = +\infty$ and $U = 1$, we obtain the standard newsvendor model with no randomness in supply and $h(y) = P\{D \le y\}$ is always increasing so that the optimality condition becomes

$$P\left\{D \le y^*\right\} = \frac{p+h-ce^{rT}}{p+h-s} = \hat{p}. \tag{8}$$

Similarly, in case $K = +\infty$, we have random yield only and

$$h(y) = E\left[U1_{\{D\le Uy\}}\right]/E[U] \tag{9}$$

is also increasing and the optimality condition becomes

$$\frac{E\left[U1_{\{D\le Uy^*\}}\right]}{E[U]} = \frac{p+h-ce^{rT}}{p+h-s} = \hat{p}. \tag{10}$$

Finally, if $U = 1$, then the model involves random capacity only and $h(y) = P\{D \le y \mid K > y\}$ leads to the optimality condition

$$P\left\{D \le y^* \mid K > y^*\right\} = \frac{p+h-ce^{rT}}{p+h-s} = \hat{p}. \tag{11}$$

Note that in all of the cases, the optimality condition is stated in terms of the same critical ratio $\hat{p}$.

2. Newsvendor Model with Perfect Hedging

We suppose that both demand and supply depend perfectly on the price of the stock. The newsvendor has to decide on the optimal order quantity y that maximizes the expected value of the cash flow or profit at time T when the period ends. Let $X = (D, U, K)$ denote the vector of random variables corresponding to demand and supply uncertainties, and $S = S_T$ denote the price of the stock at the end of the period. The random vector X is a deterministic function of the financial variable S so that

$$X=g(S) \tag{12}$$

for some well-behaving function g. This implies that $g(S) = (\mathcal{D}(S), \mathcal{K}(S), \mathcal{U}(S))$ where $D = \mathcal{D}(S)$, $K = \mathcal{K}(S)$, and $U = \mathcal{U}(S)$ for some functions $\mathcal{D}, \mathcal{K}$, and $\mathcal{U}$. We can also rewrite (5) as

$$\begin{aligned} CF\,(X, y) = CF\,(g(S), y) = \left(s - ce^{rT}\right)\mathcal{U}(S) \min\{y, \mathcal{K}(S)\} \\ + (p + h - s) \min\{\mathcal{D}(S), \mathcal{U}(S) \min\{\mathcal{K}(S), y\}\} - h\mathcal{D}(S). \end{aligned} \tag{13}$$

We further suppose that for any fixed y, the function

$$f(x) = CF(g(x), y) \tag{14}$$

is right-continuous on $0, +\infty)$ with a finite number of jumps in any finite interval and it can be written as a DC (difference of convex) function between the jumps. Let $\{x_n\}$ denote the discontinuities or jumps of f while $\{\nabla f_n\}$ denote the jump magnitudes. Figure 2 provides a typical graphical description of such functions.

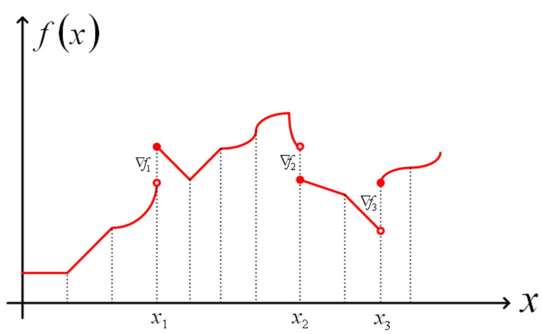


Figure 2. The Cash Flow

It follows that such functions can be represented as

$$f\,(x) = f\,(0) + f'_+\,(0)\,x + \sum_{x_n \le x} \nabla f_n + \int_0^{+\infty} (x - z)^+\,\mu_f\,(dz) \tag{15}$$

where f'_+ denotes the right-continuous version of the derivative of f and μ_f is a measure on $0, +\infty)$ with $\mu_f = f''$. For any cash flow $CF\,(g(S), y) = f(S)$, it is clear that this representation constitutes a replicating portfolio consisting of bonds (worth $f(0)$ at time T), $f'_+(0)$ futures of S, digital claims (with payoffs ∇f_n if $x_n \leq S$), and European call options (with payoffs $(S-z)^+$ where z is the strike price). Here, we define $a^+ = \max\{a, 0\}$ for any real number a.

Supposing that the financial market is complete with some martingale or risk-neutral measure Q that leads to arbitrage free pricing, the IM is faced with the decision problem

$$\max_y E_Q CF\,(g(S), y) \tag{16}$$

where each derivative security in (15) is priced using the risk-neutral measure Q.

The similarity between (5) and (13) also leads to the characterization

$$\frac{E_Q\left[\mathcal{U}(S)1_{\{\mathcal{K}(S)>y^*,\mathcal{D}(S)\leq\mathcal{U}(S)y^*\}}\right]}{E_Q\left[\mathcal{U}(S)1_{\{\mathcal{K}(S)>y^*\}}\right]} = \frac{p+h-ce^{rT}}{p+h-s} = \hat{p} \tag{17}$$

for the optimal order quantity y^*. The results in Section 1 can be extended naturally to obtain the optimality conditions

$$P_Q\left\{\mathcal{D}(S) \leq y^*\right\} = \hat{p} \tag{18}$$

in the standard newsvendor model with $K = +\infty$ and $U = 1$,and

$$\frac{E_Q\left[\mathcal{U}(S)1_{\{\mathcal{D}(S)\leq\mathcal{U}(S)y^*\}}\right]}{E_Q\left[\mathcal{U}(S)\right]} = \hat{p}. \tag{19}$$

in the random yield model with $K = +\infty$, and

$$P_Q\left\{\mathcal{D}(S) \leq y^* \mid \mathcal{K}(S) > y^*\right\} = \hat{p}. \tag{20}$$

in the random capacity model with $U = 1$. There is a clear relationship and similarity between the optimality condition (6) and (17). In the perfect hedging model, one should use the the risk-neutral measure Q instead of the ordinary probability measure in order to avoid arbitrage opportunities.

Note that our assumptions are quite restrictive since it is not in general possible to replicate the cash flow. In this case, the IM can not hedge the demand and supply risks perfectly. This leads to risk-sensitive decision making in inventory management. The overwhelming majority of inventory literature relates to risk-neutral IMs who are concerned with the expected profit or cost criteria. Although this is a mathematically viable approach, it supposes that decision makers behave risk-neutrally which, in reality, is simply not true. That's why models with risk-neutrality assumptions have limited viability in practice. In recent years, the risk-sensitive behavior of the decision maker is addressed implicitly through other criteria such as satisficing probability maximization, utility functions, Value-at-Risk (VaR) and other risk measures. For examples along this direction, the reader is referred to Li et al. [11], Eeckhoudt et al. [5], Agrawal and Seshadri [1], Parlar and Weng [17], Gaur and Seshadri [7], Caldantey and Hough [3], Wang et al. [19], Wu et al. [20], and Ozler et al. [16], among many others.

Okyay et al. [14] consider the risk-sensitive version of our model where it is not possible to have the explicit representation in (13) and there is no replicating portfolio. They suppose that there are $n \geq 1$ derivative securities in the market where $f_i(S)$ is the payoff of the ith derivative security of the primary asset. They construct a model where the inventory cash flow $CF(X, y)$ in (5) is hedged by investing in a portfolio $\{\alpha_i\}$ of derivatives $\{f_i\}$. The total hedged cash flow is given by

$$CF_\alpha(X, S, y) = CF(X, y) + \sum_{i=1}^{n} \alpha_i f_i(S_i). \tag{21}$$

The first problem now is to find the optimal portfolio $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)$ to minimize the variance of the total cash flow for a given order quantity y. This optimization problem is

$$\min_\alpha \operatorname{Var}\left(CF(X, y) + \sum_{i=1}^{n} \alpha_i f_i(S) \right). \tag{22}$$

Once the optimal solution $\alpha^*(y)$ is determined for any order quantity y, the IM then chooses the optimal order quantity by solving

$$\max_y E\left[CF(X, y) + \sum_{i=1}^{n} \alpha_i^*(y) f_i(S) \right]. \tag{23}$$

One may of course use other approaches to model the this problem. One of them is the mean-variance model where the problem is to solve

$$\max_{\alpha, y} \left\{ E\left[CF_\alpha(X, S, y) \right] - \theta \operatorname{Var}\left(CF_\alpha(X, S, y) \right) \right\} \tag{24}$$

for different values of θ that represents the risk-sensitivity of the IM. It is also fairly common to represent the behaviour of investors by utility functions. This approach leads to the model

$$\max_{\alpha, y} u\left(CF_\alpha(X, S, y) \right) \tag{25}$$

where u is the utility function of the IM.

In this paper, we consider only models where perfect hedging is possible as described by (13). We now provide explicit characterizations on the structure of the replicating portfolios for a number of interesting special cases.

3. Special Models

In this section we consider a number of cases that lead to simplified versions of our characterization results. Note that the optimality conditions for the general model and its special cases are given by (17) - (20). These results simply state that, because the cash flow can be replicated by a portfolio of derivative securities, the pricing and calculations should be done by the risk-neutral probability measure to avoid arbitrage opportunities. In this section, we also provide the replicating portfolios for these special cases using the representations (13) and (15).

3.1. Hedging Demand Risks

In traditional newsvendor models the only uncertainty is generated by demand and supply is nonrandom. This implies that $K = +\infty$ and $U = 1$ so that the cash flow is

$$CF(S, y) = \left(s - ce^{rT}\right) y + (p + h - s) \min\{\mathcal{D}(S), y\} - h\mathcal{D}(S). \tag{26}$$

We now suppose that $\mathcal{D}$ is a twice differentiable and increasing function with an inverse. Since $S \geq 0$, $D = \mathcal{D}(S) \geq \mathcal{D}(0)$ and we can take the order quantity $y \geq \mathcal{D}(0)$ without loss of generality. This is based on the simple observation that the order quantity should be chosen so that it is over the minimum demand that can be realized during the period. We can now apply (15) to find a replicating portfolio and use it to hedge demand risks. It follows from (26) that for fixed y, taking $f(S) = CF(S, y)$, we get

$$\begin{aligned} f(0) &= \left(s - ce^{rT}\right) y + (p - s)\mathcal{D}(0) \\ f'_+(0) &= (p - s)\mathcal{D}'(0) \\ \mu_f(dz) &= (p + h - s)\mathcal{D}''(z) 1_{\{\mathcal{D}(z) < y\}} dz \\ &\quad - (p + h - s)\mathcal{D}'(z) 1_{\{z = \mathcal{D}^{-1}(y)\}} - h\mathcal{D}''(z)\, dz. \end{aligned}$$

The replicating portfolio can therefore be represented by

$$\begin{aligned} f(S) = &\left[\left(s - ce^{rT}\right) y + (p - s)\mathcal{D}(0)\right] \\ &+ \left[(p - s)\mathcal{D}'(0)\right] S \\ &- h\int_0^\infty (S - z)^+ \mathcal{D}''(z)\, dz \\ &+ (p + h - s)\int_0^{\mathcal{D}^{-1}(y)} (S - z)^+ \mathcal{D}''(z)\, dz \\ &- (p + h - s)\mathcal{D}'\left(\mathcal{D}^{-1}(y)\right)\left(S - \mathcal{D}^{-1}(y)\right)^+. \end{aligned}$$

Since $f(S) = CF(S, y)$ for any y, we can replicate the cash flow $CF(S, y)$ by a portfolio consisting of

$$\left(s - ce^{rT}\right) y + (p - s)\mathcal{D}(0)$$

cash bonds,

$$(p - s)\mathcal{D}'(0)$$

futures, and a possibly infinite mixture

$$(p+h-s)\left[\int_0^{\mathcal{D}^{-1}(y)}(S-z)^+\,\mathcal{D}''(z)\,dz-\mathcal{D}'\left(\mathcal{D}^{-1}(y)\right)\left(S-\mathcal{D}^{-1}(y)\right)^+\right]$$
$$-h\int_0^{\infty}(S-z)^+\,\mathcal{D}''(z)\,dz$$

of European call options.

The optimality condition (18) can be rewritten as

$$P_Q\left\{\mathcal{D}(S)\le y^*\right\}=P_Q\left\{S\le\mathcal{D}^{-1}(y^*)\right\}=F_S(\mathcal{D}^{-1}(y^*)=\hat{p} \tag{27}$$

so that

$$y^*=\mathcal{D}\left(F_S^{-1}(\hat{p})\right) \tag{28}$$

where F_S is the distribution of S under the risk-neutral measure Q.

If there is a linear relationship $\mathcal{D}(S)=a+bS$ as in Gaur and Seshadri [7], we have $\mathcal{D}(0)=a$, $\mathcal{D}'(z)=b$,and $\mathcal{D}''(z)=0$ so that the portfolio consists of $\left(s-ce^{rT}\right)y+(p-s)\,a$ cash bonds, $(p-s)\,b$ futures, and $-(p+h-s)b$ European call options with strike price $(y-a)/b$ for $y\ge\mathcal{D}(0)=a$. Moreover, the optimal order quantity satisfies the optimality condition

$$P_Q\left\{S\le\frac{y^*-a}{b}\right\}=\hat{p}. \tag{29}$$

3.2. Hedging Joint Demand and Supply Risks

We now investigate a couple of cases when supply is also random by analyzing random capacity and random yield models separately.

3.2.1. Random Capacity Model

Let $U=1$, $D=\mathcal{D}(S)$ and $K=\mathcal{K}(S)$ where both $\mathcal{D}(S)$ and $\mathcal{K}(S)$ are twice differentiable and increasing functions with an inverse. Then

$$\begin{aligned}CF(S,y)=\left(s-ce^{rT}\right)&\min\{\mathcal{K}(S),y\}\\&+(p+h-s)\min\{\mathcal{D}(S),\mathcal{K}(S),y\}-h\mathcal{D}(S).\end{aligned} \tag{30}$$

The relationship between $\mathcal{D}$ and $\mathcal{K}$ determines the structure of the replicating portfolio. We now examine the following three cases which do not necessarily cover all possibilities. As before, we take $y\ge\mathcal{D}(0)$.

- Case 1: $\mathcal{K}(x) > \mathcal{D}(x)$ for all $x \geq 0$. One can show that

$$f(0) = \left(s - ce^{rT}\right) \min\{\mathcal{K}(0), y\} + (p - s)\mathcal{D}(0)$$

$$f'_+(0) = \left(s - ce^{rT}\right) \mathcal{K}'(0) 1_{\{\mathcal{K}(0)<y\}} + (p - s)\mathcal{D}'(0)$$

$$\begin{aligned} \mu_f(dz) = {} & \left(s - ce^{rT}\right) \mathcal{K}''(z) 1_{\{\mathcal{K}(z)<y\}} dz - \left(s - ce^{rT}\right) \mathcal{K}'(z) 1_{\{z=\mathcal{K}^{-1}(y)\}} \\ & + (p + h - s)\mathcal{D}''(z) 1_{\{\mathcal{D}(z)<y\}} dz \\ & - (p + h - s)\mathcal{D}'(z) 1_{\{z=\mathcal{D}^{-1}(y)\}} - h\mathcal{D}''(z)\, dz. \end{aligned}$$

This provides the structure of the replicating portfolio consisting of the cash bond, futures and European calls through (15). Moreover, the optimality condition can be written as

$$P_Q\left\{\mathcal{D}(S) \leq y^* \mid \mathcal{K}(S) > y^*\right\} = P_Q\left\{S \leq \mathcal{D}^{-1}(y^*) \mid S > \mathcal{K}^{-1}(y^*)\right\} = \hat{p}. \tag{31}$$

- Case 2: $\mathcal{D}(x) > \mathcal{K}(x)$ for all $x \geq 0$. In this case,

$$f(0) = \left(p + h - ce^{rT}\right) \mathcal{K}(0) - h\mathcal{D}(0)$$

$$f'_+(0) = \left(p + h - ce^{rT}\right) \mathcal{K}'(0) - h\mathcal{D}'(0)$$

$$\begin{aligned} \mu_f(dz) = {} & \left(s - ce^{rT}\right) \mathcal{K}''(z) 1_{\{\mathcal{K}(z)<y\}} dz - \left(s - ce^{rT}\right) \mathcal{K}'(z) 1_{\{z=\mathcal{K}^{-1}(y)\}} \\ & + (p + h - s)\mathcal{K}''(z) 1_{\{\mathcal{K}(z)<y\}} dz \\ & - (p + h - s)\mathcal{K}'(z) 1_{\{z=\mathcal{K}^{-1}(y)\}} - h\mathcal{D}''(z)\, dz. \end{aligned}$$

Once again, the structure of the replicating portfolio is found through (15). Of course, the optimality condition is the same as (31).

- Case 3: $\mathcal{K}(x) > \mathcal{D}(x)$ for all $0 \leq x < \bar{y}$ and $\mathcal{K}(x) < \mathcal{D}(x)$ for all $x > \bar{y}$ when there is a unique $\bar{y}$ for which $\mathcal{K}(\bar{y}) = \mathcal{D}(\bar{y})$. In Case 1 and 2, the functions $\mathcal{K}$ and $\mathcal{D}$ do not intersect. Now, we take a look at the case when they are equal at some unique $\bar{y}$. Following the same steps as in previous examples, we now obtain

$$f(0) = \left(s - ce^{rT}\right) \min\{\mathcal{K}(0), y\} + (p - s)\mathcal{D}(0)$$

$$f'_+(0) = \left(s - ce^{rT}\right) \mathcal{K}'(0) 1_{\{\mathcal{K}(0)<y\}} + (p - s)\mathcal{D}'(0)$$

$$\begin{aligned} \mu_f(dz) = {} & \left(s - ce^{rT}\right) \mathcal{K}''(z) 1_{\{\mathcal{K}(z)<y\}} dz - \left(s - ce^{rT}\right) \mathcal{K}'(z) 1_{\{\mathcal{K}(z)=y\}} \\ & + (p + h - s)\mathcal{D}''(z) 1_{\{z\leq\bar{y}, \mathcal{D}(z)\leq y\}} dz \\ & + (p + h - s)\mathcal{K}''(z) 1_{\{z>\bar{y}, \mathcal{K}(z)\leq y\}} dz \\ & - (p + h - s)\mathcal{D}'(z) 1_{\{z=\bar{y}\}} + (p + h - s)\mathcal{K}'(z) 1_{\{z=\bar{y}\}} \\ & - (p + h - s)\mathcal{D}'(z) 1_{\{\mathcal{D}(z)=y\}} dz \\ & - (p + h - s)\mathcal{K}'(z) 1_{\{\mathcal{K}(z)=y\}} dz - h\mathcal{D}''(z)\, dz. \end{aligned}$$

The replicating portfolio consists of the cash bond, futures and European calls by (15) and the the optimality condition is the same as (31).

3.2.2. Random Yield Model

In this model, we let $K = +\infty$, $D = \mathcal{D}(S)$ and $U = \mathcal{U}(S)$ where both $\mathcal{D}(S)$ and $\mathcal{U}(S)$ are twice differentiable and increasing functions with an inverse. Then,

$$CF(S, y) = \left(s - ce^{rT}\right) \mathcal{U}(S)\, y + (p - s + h) \min\{\mathcal{U}(S)\, y, \mathcal{D}(S)\}. \tag{32}$$

For any y, suppose that there is a unique $\bar{y}(y)$ where $\mathcal{U}(x)\, y$ and $\mathcal{D}(x)$ intersect each other so that $\mathcal{U}(\bar{y}(y))\, y = \mathcal{D}(\bar{y}(y))$. Then,

$$\begin{aligned}
f(0) &= \left(s - ce^{rT}\right) \mathcal{U}(0)\, y + (p - s + h) \min\{\mathcal{U}(0)\, y, \mathcal{D}(0)\} - h\mathcal{D}(0) \\
f'_{+}(0) &= \left(s - ce^{rT}\right) y\mathcal{U}'(0) + (p - s + h)\, y\mathcal{U}'(0)\, 1_{\{\mathcal{U}(0)y < \mathcal{D}(0)\}} \\
&\quad + (p - s + h)\, \mathcal{D}'(0)\, 1_{\{\mathcal{U}(0)y > \mathcal{D}(0)\}} - h\mathcal{D}'(0) \\
\mu_f(dz) &= \left(s - ce^{rT}\right) y\mathcal{U}''(z)\, dz + (p - s + h)\, y\mathcal{U}''(z)\, 1_{\{\mathcal{U}(z)y < \mathcal{D}(z)\}} dz \\
&\quad + (p - s + h)\, \mathcal{D}''(z)\, 1_{\{\mathcal{U}(z)y > \mathcal{D}(z)\}} dz \\
&\qquad + (p - s + h) \left(\mathcal{D}'(\bar{y}(y)) - y\mathcal{U}'(\bar{y}(y))\right) - h\mathcal{D}''(z)\, dz.
\end{aligned}$$

This provides the replicating portfolio through (15) and optimality condition becomes

$$\frac{E_Q\left[\mathcal{U}(S)\, 1_{\{\mathcal{D}(S) \le \mathcal{U}(S)y^*\}}\right]}{E_Q[\mathcal{U}(S)]} = \hat{p}. \tag{33}$$

We want to point out that we have imposed some restrictions on $\mathcal{D}, \mathcal{K}$ and $\mathcal{U}$ in order to make explicit representations on the replicating portfolios. However, one can determine these portfolios in more general cases provided that the cash flow function $f(x) = CF(g(x), y)$ in (13) has the structure in (15).

4. Concluding Remarks

In this paper, we considered a number of inventory models where random demand and supply depend on the price of a primary asset. Moreover, the dependence leads to a representation of the cash flow by a replicating portfolio that consists of a mixture of derivative securities of the primary asset. As a consequence, the cash flow is valued using the market prices of the securities and the optimal order quantity is determined by using these prices. In the presence of a risk-neutral probability measure, this implies that the objective function should be the expected value of the cash flow where the expectation is computed using the risk-neutral measure to avoid arbitrage opportunities. We also provide explicit representations of the replicating portfolio in a number of cases involving random demand and supply.

One can extend this line of research in several directions by combining risk management with inventory management. This will eventually require models where the risk sensitivity of the IM is somehow built into the model. For example, one can construct utility-based models where the objective is to maximize the expected utility of the cash flow which may be hedged by some portfolio of financial instruments. Another line of future research involves multi-period and infinite-period models where the optimality of base-stock policies or its variations should be investigated. Another promising approach is to use binomial models to represent demand and supply risks in inventory model, and proceed with the usual risk management tools of financial models. Moreover, it is well-worth studying the possibility of continuous hedging through financial instruments in continuous extensions.

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