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The Value of Advance Demand Information in Production/Inventory Systems

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Abstract. Advance demand information, when used effectively, improves the performance of production/inventory systems. In this paper, we investigate the value of advance demand information in production/inventory systems. For a single-stage make-to-stock queue, we assess the value of using advance demand information under a variety of assumptions on the cost of obtaining advance demand information, and the delivery timing requirements. This analysis enables us to identify conditions under which advance demand information may bring significant benefits.

Keywords: production/inventory systems, make-to-stock queue, advance demand information, base stock policies

1. Introduction

Advance demand information, when used effectively, leads to a performance improvement in supply systems. Intuitively, better information on future demands leads to lower inventory levels for the same service level. The precise nature of this information–inventory interaction, however, is in general difficult to assess. This difficulty is even more pertinent for a production system than for an external replenishment system, because in the former case production capacity also enters the interaction in a non-trivial manner.

In this paper, we investigate the value of advance demand information on capacitated supply systems. Our model is that of a single-stage manufacturing system operating in a make-to-stock mode. The manufacturing capacity is modelled by a single-server queueing system that places finished items into a finished goods inventory after processing. The demand process generates the arrivals to this queueing system. In the absence of advance demand information, the model is a single-stage make-to-stock queue. As for the advance order mechanism, we assume that each customer, upon arrival, provides a due-date for the item requested. The demand and its due-date are assumed to comprise a firm order that can be neither cancelled nor modified. We attempt to respond to two basic questions in this setting: (1) how much advance demand information is sufficient, if information is obtained free-of-charge, and (2) what is the optimal amount of advance information that would minimize costs, if it has a price (e.g., a unit discount on the selling price) associated with it?

The above setting is particularly attractive for gaining insights into system performance as a function of demand lead times (the difference between the due-date of an order and the arrival time of the order). For an M/M/1 make-to-stock queue, system performance and the value of advance demand information is characterized exactly for two different models: (1) the case where the customer does not accept deliveries earlier than the due-date and (2) the case where the customer accepts deliveries earlier than the due-date. For the first model, the system performance is characterized by Buzacott and Shanthikumar (1989, 1994). We extend their results to explore in detail the value of advance demand information. We also present new results on the optimization of release times that lead to a more precise characterization than that in the above papers. The second model, to our knowledge, has not been investigated before. The conclusions in this case provide an interesting contrast to the findings from the first model.

The paper is structured as follows: a literature review is presented in section 2. In section 3 we present a model where the customers order in advance of their required delivery dates with the assumption that both earliness and lateness with respect to the required delivery date is penalized. Section 4 focuses on a model where only lateness with respect to the required delivery date is penalized. Section 5 comprises our conclusions.

2. Literature review

There are several papers that investigate the value of advance demand information and its interactions with inventories, using analytical models. One possible classification of this research can be based on the way the underlying supply system is modelled. Supply systems with exogenous lead times (pure inventory systems) behave significantly differently than supply systems with endogenously determined lead times (production/inventory systems).

For supply systems with exogenous lead times, Lambrecht, Muckstadt, and Luyten (1984) do not explicitly consider advance demand information but remark that in a standard multi-stage system, safety times are interchangeable with safety stocks. Milgrom and Roberts (1988) present a stylized analysis of advance demand information in a single-period newsvendor setting, where demand information can be obtained using market surveys at a cost. Hariharan and Zipkin (1995) model advance demand information through orders placed in advance and present a thorough study on the benefits of early information on demand for continuous-time, exogenous-replenishment inventory systems. Their analysis reveals that early information is a substitute for supply lead times and can reduce safety stock levels and costs significantly when used effectively. Bourland, Powel, and Pyke (1996) investigate a two-stage supply system where demand information from the downstream stage can be interpreted as advance demand information for the upstream stage (if transmitted in a timely manner). They show that timely demand information transmission can lead to significant supply chain savings. Gullu (1997) demonstrates that the value of forecast information can be significant in a two-echelon allocation problem consisting of a single depot and multiple retailers. DeCroix and Mookerjee (1997) analyze a periodic-review system where the supplier has the option to purchase advance demand information. They characterize the optimal information purchase policy and the value of dynamically purchasing advance demand information. Gallego and Ozer (2001) investigate optimal replenishment policies for a single stage periodic-review inventory system with advance demand information. Their numerical results show that under the optimal replenishment policy, advance demand information can lead to important cost reductions. The extension of the single-stage model to the multi-stage case is analyzed in Gallego and Ozer (2000). Chen (2001) models and investigates a market segmentation problem where customers get price discounts as a function of the advance demand information they provide. Finally, van Donselaar, Kopczak, and Wouters (2001) investigate the benefits of advance demand information in a project-based (i.e., a pure make-to-order) setting.

For capacitated supply systems which generate endogenous lead times due to congestion effects, Buzacott and Shanthikumar (1989, 1994) present a detailed analysis of a single-stage make-to-stock queue with advance demand information in the form of firm orders placed a fixed amount of time in advance of their due-dates. They then investigate how the optimal safety stock varies as a function of the lead time parameter which determines how advance demand information is utilized. Part of our work is based on the same basic model but extends the analysis to shed light onto the stock-information-capacity interactions and the value of advance demand information.

Karaesmen, Buzacott, and Dallery (2002) investigate the structure of optimal release timing and inventory control decisions, based on a discrete-time make-to-stock queue. Even though the exact optimal policy turns out to be complicated, there is a simple class of policies that are near-optimal. These policies, which are called BSADI (Base Stock policies with Advance Demand Information), require, in addition to the base-stock level, a parameter that sets the release lead time. The close-to-optimal performance of BSADI policies justifies their use as a benchmark to assess the value of advance demand information. Our paper presents this assessment for a continuous-time make-to-stock queue with advance demand information. In parallel to the current paper Karaesmen, Buzacott, and Dallery (2003) propose approximations for make-to-stock queues with general processing times. For a corresponding two-stage system, Liberopoulos and Koukoumialos (2003) present a simulation-based investigation of WIP-controlled BSADI policies for single-stage and two-stage make-to-stock systems. Liberopoulos and Tsikis (2003) present a modeling framework for multi-stage systems with advance demand information. Their framework also addresses lot sizing issues in this context. Benjaafar and Kim (2001) investigate advance demand information for a make-to-stock queue in the context of demand variability. Ozer and Wei (2001) analyze the structure of optimal policies for a capacitated periodic-review system. Finally, Wijngaard (2002) investigates the value of advance demand information for a production/inventory system where production capacity is modeled by a single resource with constant processing speed.

In other articles that investigate production/inventory systems from a slightly different perspective, Gullu (1996) and Toktay and Wein (2001) model the effects of forecast evolution on system performance for discrete-time make-to-stock queues. Specifically, Gullu (1996) investigates the structure of optimal policies and shows that using forecast information leads to inventory and cost reductions. Toktay and Wein (2001) extend and quantify these findings through an approximate heavy-traffic analysis. Our modelling framework is too simple to capture the subtleties of forecast evolution, but the simplicity of the basic model enables us to obtain explicit and intuitively appealing results on the performance-related effects of advance demand information.

Finally, in other related work on production/inventory systems, Gilbert and Ballou (1999) investigate the capacity planning problem of a make-to-order supplier that can receive advance demand commitments through a pricing policy. Gavirneni, Kapuscinski, and Tayur (1999) consider a two-stage supply chain with a capacitated production system upstream. Using simulation, they provide a comparison of the case where the only information transmitted to the upstream stage is through downstream orders and the case where the upstream stage has access to end-client demand information. The simulation results confirm the benefits of early demand information.

3. Production/inventory systems with advance demand information and timely delivery requirements

In most models of production/inventory systems, demand is a random variable (or process), for which statistical information exists, and suppliers determine their inventory policies in order to deal with the uncertainty in the demand. Below, we consider a different situation where the medium/long term demand is still assumed to be random but there is more than statistical information about the timing of short term demand. The situation that we have mind is that of downstream customers who place orders with a future due-date. Because our objective is to gain basic insights on the value of advance demand information, we consider a simple model of advance demand information. Namely, all customers order exactly τ periods in advance of their required delivery date. This restrictive assumption is justified when the downstream customer plans his/her production according to an MRP-type system. In the auto industry, for example, manufacturers can transmit their orders to their suppliers several days in advance because their final assembly line schedule is determined in advance for optimization (line balancing) purposes.

The other complication in modelling the effects of advance demand information on inventories is determining the policy that will be employed. It is known that the exact optimal inventory policy can be complicated and may require very detailed information on the timing of future due-dates (see Karaesmen, Buzacott, and Dallery (2002)).

On the other hand, simple adaptations of base-stock policies usually have close-to-optimal performance. In this section, our analysis is based on such a policy, which we refer to as the base stock policy with advance demand information (BSADI). Recall that the simple base stock policy has a single parameter, S, the target inventory (or base stock) level. The BSADI policy has two parameters, S, the target inventory level, and L, the planned release lead time. As in the simple base stock system, a BSADI policy initially has S items in stock. The difference between the two systems comes from the production order release mechanism. In the simple case, a production order is released at each demand arrival. In other words, the release mechanism is triggered by an actual demand. In BSADI, a production order is released exactly L units before the due-date of an order. In this case, production orders are triggered by information signals rather than by actual demands. It is important to note that the release lead time L is a parameter of the policy which has to be optimized just like the base stock level. Finally, it has to be stressed that the release lead time is not unrestricted as it is constrained by the demand lead time. Namely, L has to be less than or equal to τ.

We are now ready to completely specify the modelling assumptions. The basic model is a make-to-stock queue where orders arrive according to a Poisson process with rate λ and the facility is modelled by an exponential server with processing rate μ. An order arriving at time t has a due-date equal to $t + \tau$. Inventory costs are incurred at rate h (per item per unit time) for items held in stock, and demands that are not satisfied at their due-dates are backlogged at cost rate b (per item per unit time). A BSADI policy is employed and the optimization problem is to determine the parameters S^* ($S^* \geqslant 0$) and L^* ($0 \leqslant L \leqslant \tau$).

Before moving on to the analysis of the make-to-stock queue, we will first look into a make-to-order system.

3.1. Analysis of make-to-order systems with timely delivery requirements

Consider the following make-to-order production system: All customers transmit their orders exactly τ periods in advance of the required delivery date. Following Hariharan and Zipkin (1995), we refer to τ as the *demand lead time*. The delivery date is firm and customers do not accept early deliveries. If the order is late with respect to its delivery date, a lateness (shortage) charge of b is applied (per order per unit time). Orders that are completed in advance of their due-dates incur a holding cost of h (per order per unit time). The supplier has random supply lead times W that are assumed to be unknown in advance. Let $F_W(\cdot)$ denote the probability distribution function of the random variable W. In particular, we assume that the supplier has (or uses) no information about the state of the replenishment process.

As far as the replenishment process is concerned, we follow the assumption that Haji and Newell (1971) make in a pioneering paper in which they address the issue of relating the distribution of the number of outstanding orders and the supply lead time in a queueing setting. This assumption is:

Assumption 1.

- All arriving orders enter to the supply system one at a time, remain in the system until they are fulfilled (there is no blocking, balking or reneging) and leave one at a time.
- Orders leave the system in the order of arrival (FIFO).
- New orders do not affect the supply lead time of previous orders (lack of anticipation).

The above assumption would typically hold for a capacitated system, but would not hold for an uncapacitated system (with random lead times).

Clearly, the supplier has to start the replenishment process earlier than the due-date. The question is, when exactly should the supplier start the replenishment process in order to minimize expected costs? Suppose that he starts it a fixed amount of time before the due-date. Specifically, let us define the parameter L, which determines the *release lead time*. Using this parameter, and assuming that an order arrives at time t for a delivery date of $(t+\tau)$, the replenishment process is started at time $(t+\max\{0, \tau - L\})$. The optimization question posed above can now be recast in the following form: what is the value of L that minimizes expected costs? Proposition 1 answers this question.

Proposition 1. Under assumption 1, the optimal release lead time for a make-to-order system with constant demand lead times τ is given by

$$L^* = \min\{L_\infty^*, \tau\}, \tag{1}$$

where L_∞^* is called the *optimal unconstrained release lead time* and is given by

$$F_W(L_\infty^*) = \frac{b}{h+b}. \tag{2}$$

Proof. Suppose that the supplier uses an unconstrained release lead time L_∞, where L_∞ is unconstrained in the sense that it does not take into account the constraint $L \leqslant \tau$, which ensures that a release cannot take place before the order that triggered it arrives. Let λ be the order arrival rate, then, the expected cost per unit time can be expressed as

$$C[L_\infty] = \lambda\left[h\int_0^{L_\infty}(L_\infty - w)\,\mathrm{d}F_W(w) + b\int_{L_\infty}^{\infty}(w - L_\infty)\,\mathrm{d}F_W(w)\right]. \tag{3}$$

The above formulation is similar to the formulation of the standard news-vendor problem, and its solution is obtained by the first order optimality condition that leads to expression (2). Note that the optimal unconstrained release lead time is set according to the well-known critical fractile rule (equation (2)).

Using the convexity of the cost function (3) and the constraint $L \leqslant \tau$, the optimal constrained value of L is then found to be the minimum of τ and the optimal unconstrained release lead time, which satisfies (2). □

Before moving on to capacitated make-to-stock systems, it is worth noting a particular application of proposition 1 in a queueing setting. Specifically, let us assume that the underlying supply system is modelled by the sole server of a queueing system. Then, an interesting question that arises is the determination of the release lead time, L. Using proposition 1, the optimal release lead time is given by a fractile of the supply lead time (waiting time in the queueing system). This leads to an explicit formula for the optimal release lead time if the underlying model of the make-to-order system is an M/M/1 queue.

Corollary 1. Consider a make-to-order system where the customer orders arrive according to a Poisson process with rate λ and the supply system is a single-server with exponential processing times with rate μ. If the demand lead time is τ, then

$$L^* = \min\{L^*_\infty, \tau\},$$

where

$$L^*_\infty = -\frac{\log(h/(h+b))}{\mu(1-\rho)}. \tag{4}$$

Proof. The result is a direct application of proposition 1, after noting that the customer waiting time distribution of an M/M/1 queue is given by $F_W(w) = 1 - e^{-\mu(1-\rho)w}$. □

Corollary 1 characterizes the optimal unconstrained release lead time for an M/M/1 queue. It suggests a rolling-horizon-based part release policy, where all orders are released to the manufacturing stage L^*_∞ time units before their due-dates. This leads to an alternative interpretation of L^*_∞. Namely, L^*_∞ can be viewed as the optimal planning horizon for this class of part release policies.

It is important to note that no claims are made for the overall optimality of a release policy that plans releases according to corollary 1. The main advantage of a fixed release time policy is its simplicity. On the other hand, one may envision a more general class of policies where the release timing depends on the actual load of the system (so that the parameter L is updated dynamically as a function of the actual system load). This class of policies is considerably more difficult to analyze and is beyond the scope of this paper. A recent paper by Denton and Gupta (2000) presents an overview of resource scheduling problems with due-dates and investigates a special case of this latter class of problems.

3.2. Analysis of an M/M/1 make-to-stock queue with timely delivery requirements

This section presents the analysis of the BSADI policy for an M/M/1 make-to-stock queue and is based on the model of Buzacott and Shanthikumar (1994). The optimization of the parameters parallels the analysis in Karaesmen, Buzacott, and Dallery (2002). The base-stock level S is restricted to be a non-negative integer variable. It will be seen, however, that it is easier to operate in a continuous state space for optimization purposes. To this end, we denote by $\hat{S}$ the unrestricted base stock level, a non-negative continuous variable.

Similarly to the analysis of the make-to-order system, it is helpful, at first, to reintroduce the unconstrained release lead time L_∞. L_∞ is unconstrained in the sense that it does not take into account the constraint $L \leqslant \tau$. For fixed S and L_∞, the total expected average inventory and backorder related cost can be expressed as

$$C(S, L_\infty) = h\left(S + \lambda L_\infty - \frac{\rho}{1-\rho}\right) + (h+b)\frac{\rho^{S+1}}{1-\rho}\mathrm{e}^{-\mu L_\infty(1-\rho)}.$$

One can then optimize the expected average cost with respect to the parameters S and L_∞ to gain insights into the behaviour of the system under the optimal selection of parameters.

It can be seen that for a fixed value of L_∞, the optimal base stock level is given by

$$S^*(L_\infty) = \begin{cases} \lfloor \hat{S}^*(L_\infty) \rfloor, & \text{if } L_\infty \leqslant L_\infty^*, \\ 0, & \text{if } L_\infty \geqslant L_\infty^*, \end{cases}$$

where

$$\hat{S}^*(L_\infty) = \frac{\log(h/(h+b))}{\log\rho} + \frac{\mu(1-\rho)}{\log\rho}L_\infty,$$

and

$$L_\infty^* = -\frac{\log(h/(h+b))}{\mu(1-\rho)}. \tag{5}$$

It is important to note that the optimal base stock level behaves differently depending on whether L_∞ is greater or smaller than the critical value L_∞^*. When $L_\infty \leqslant L_\infty^*$, $S^*(L_\infty)$ is linearly decreasing in L_∞. At $L_\infty = L_\infty^*$, $S^*(L_\infty) = 0$, and any further increase in L_∞ beyond L_∞^* is useless in terms of base stock level reduction.

Setting the release lead time to L_∞^* enables then a zero base-stock level, which means that the system becomes equivalent to the make-to-order system described in the previous section. Moreover, the above piece-wise definition for the optimal base stock level reflects into the cost function as well. It turns out that for a fixed unconstrained release lead time L_∞, the optimal cost (obtained by optimizing the base stock level) is given by

$$C\big(\hat{S}^*(L_\infty), L_\infty\big) = \begin{cases} h\left[\dfrac{\log(h/(h+b))}{\log\rho} + \left(\dfrac{\mu(1-\rho)}{\log\rho} + \lambda\right)L_\infty\right], & \text{if } L_\infty \leqslant L_\infty^*, \\ h\left[\lambda L_\infty - \dfrac{\rho}{1-\rho}\right] + (h+b)\dfrac{\rho}{1-\rho}\mathrm{e}^{-\mu L_\infty(1-\rho)}, & \text{if } L_\infty \geqslant L_\infty^*. \end{cases}$$

It can easily be checked that $C(\hat{S}^*(L_\infty), L_\infty)$ is decreasing in L_∞ when $L_\infty \leqslant L_\infty^*$, and is increasing in L_∞ when $L_\infty \geqslant L_\infty^*$. Hence, $C(\hat{S}^*(L_\infty), L_\infty)$ is minimized at $L_\infty = L_\infty^*$. The optimal (S, L_∞) values are then $(0, L_\infty^*)$. At these values, the system operates in a make-to-order mode and all production orders are released exactly L_∞^* periods before their due-dates.

We will hence refer to L_∞^* as the optimal unconstrained release lead time. We would ideally like to have enough forward visibility (i.e., sufficient demand lead time) so that all orders can be released according to this parameter. In reality, the offered demand lead time τ may not be sufficient to reach this ideal situation. In this case, similarly to proposition 1, the optimal policy parameters are simply $(S^*(L^*), L^*)$ where $L^* = \min\{L_\infty^*, \tau\}$. At the other extreme, more demand lead time than what is required may be offered. In this case, the production order can be released exactly L_∞^* time units before its due-date. This requires delaying the production release order by $\tau - L_\infty^*$ from the time of arrival of a customer order (see (Karaesmen, Buzacott, and Dallery, 2002)).

In summary, the optimal parameters and the overall system behavior can be expressed in terms of the optimal unconstrained release lead time as follows:

1. When $\tau \leqslant L_\infty^*$, $L^* = \tau$ and $S^* = S^*(\tau)$. In this region, the optimal cost is linearly decreasing in τ.

2. When $\tau \geqslant L_\infty^*$, $S^* = 0$ and $L^* = L_\infty^*$. In this region, increasing the demand lead time τ does not have any effect on the optimal cost (if the ideal optimal unconstrained release lead time L_∞^* is employed).

The above properties implicitly comprise the answers to the question how do optimal information requirements change as a function of capacity? Clearly, if advance demand information were free and available, the length of the horizon would be determined by L_∞^*, since the cost function is strictly decreasing in τ for values of τ less than L_∞^*, and is constant for values of τ greater than L_∞^*. Because L_∞^* is an increasing function of ρ, more advance demand information is required when the average system load is high.

It is interesting to note that the make-to-stock queueing system has the identical optimal unconstrained release lead time and consequently the identical release lead time as the corresponding make-to-order queueing system (see corollary 1). Although a formal proof is lacking, we conjecture that in general (under assumption 1), the optimal unconstrained release time of any make-to-stock queueing system is identical to the optimal unconstrained release time of the corresponding make-to-order system, which is given by proposition 1.

We can then interpret L_∞^*, similarly to the make-to-order case, as the optimal planning horizon for BSADI policies. By corollary 1, this planning horizon is determined by the critical fractile of the supply lead time. This interpretation is different than the one in Buzacott and Shanthikumar (1994), where the optimal planning horizon is expressed as a safety factor times the mean supply lead time. Incidentally, for the M/M/1 make-to-stock queue the two interpretations lead to the same result. It should be noted, however, that the critical fractile interpretation holds for the discrete-time make-to-stock queue in Karaesmen, Liberopoulos, and Dallery (2002), whereas the other interpretation fails.

In exploring the value of advance demand information, it is helpful to express the optimal cost as a function of the offered demand lead time τ. Returning to the the cost

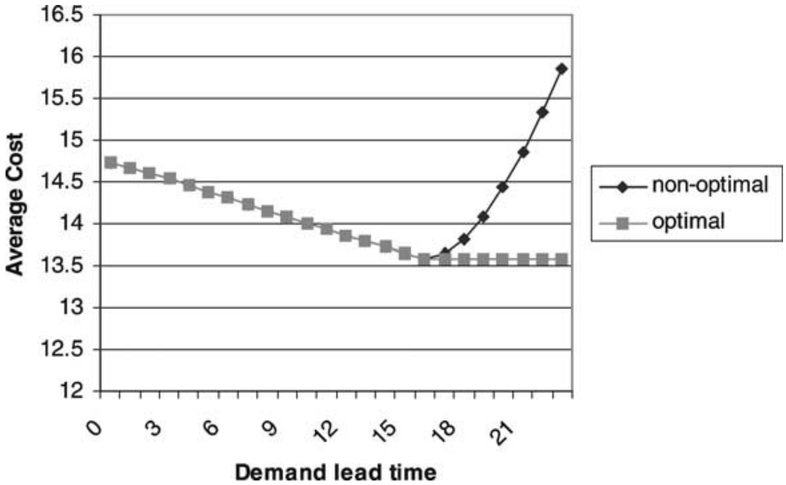

Figure 1. The optimal average cost with immediate (non-optimal) and optimal release for a BSADI policy with parameter values: $\lambda = 0.85$, $\mu = 1$, $h = 1$, $b = 10$.

function, based on the above principles, the optimal (using the optimal pair (S^*, L^*)) expected average cost can be expressed, in terms of τ, as follows:

$$C^*(\tau) = \begin{cases} h\left[\dfrac{\log(h/(h+b))}{\log\rho} + \left(\dfrac{\mu(1-\rho)}{\log\rho} + \lambda\right)\tau\right], & \text{if } \tau \leqslant L^*_\infty, \\ h\log\left(\dfrac{h+b}{h}\right)\dfrac{\rho}{1-\rho}, & \text{if } \tau \geqslant L^*_\infty. \end{cases}$$

Figure 1 depicts the average inventory related cost as a function of the demand lead time τ for the case where the release is determined in an optimal way (i.e., $L = \min\{\tau, L^*_\infty\}$) and for the case where an immediate release takes place (i.e., $L = \tau$). The figure shows the importance of optimizing the release lead time, especially if the demand lead time τ is large.

Figure 2 depicts the evolution of the optimal base stock level and the optimal cost as a function of the demand lead time τ. It is important to remark the linear decrease in both the optimal base stock level and the corresponding optimal cost. In terms of the release policy, we observe two different regimes: to the left of the dotted vertical line, immediate release is optimal (i.e., $L^* = \tau$), whereas to the right of this threshold releases are delayed with respect to order arrivals (i.e., we set $L^* = L^*_\infty$).

From the above expression, we can see that if τ is not constrained (for example, $\tau \to \infty$), $C_{\min}$, the minimum cost that can be attained is given by

$$C_{\min} = h\log\left(\frac{h+b}{h}\right)\frac{\rho}{1-\rho}.$$

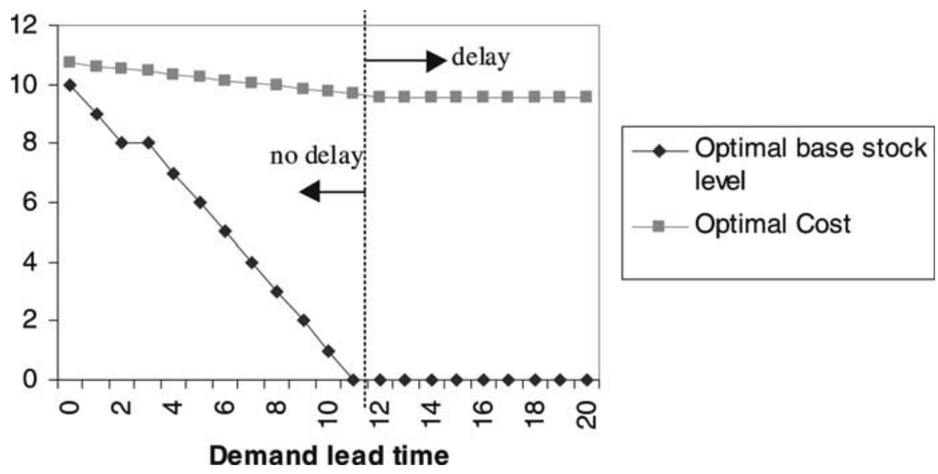

Figure 2. Optimal base stock level and optimal cost as a function of the demand lead time.

3.3. Value of advance demand information: fixed cost of advance demand information

In the previous section, it was shown that having longer demand lead times decreases (at least in a non-strict sense) inventory-related costs. It was also shown that extending demand lead times too much may be unnecessary and that there is an optimal horizon beyond which we would not be interested in further information on future demand. If we make the rather simplifying assumption that advance demand information on demands can be obtained free-of-charge, we would be interested in selecting the optimal length of the horizon, or in other words the optimal advance demand information requirements. An interesting question which arises and which has to do with the impact of production capacity on advance demand information requirements is: when do we need longer horizons of advance demand information? When the average system load is relatively large or small? How does the value of information change as a function of capacity? In order to characterize the value of advance demand information, let us define the difference $\Delta C \equiv C(\hat{S}^*, 0) - C_{\min}$. ΔC can be interpreted as the absolute value of advance demand information (going from zero demand lead time to the optimal demand lead time).

To assess the value of advance demand information we will compare the optimal inventory related costs between a system that does not use advance demand information and a system that uses the optimal amount of advance demand information. Some straightforward algebra leads to the following expression for the value of advance demand information:

$$\Delta C = h \log\left(\frac{h+b}{h}\right)\left[\frac{1}{\log(1/\rho)} - \frac{\rho}{1-\rho}\right].$$

The above expression implies that the value of advance demand information is increasing in ρ. The lower the average system load, the less we would pay to purchase the information. Further analysis reveals that the value of advance demand information diminishes to zero as ρ tends to zero, i.e.,

$$\lim_{\rho \to 0} \Delta C = 0.$$

More interestingly, the value of this information approches a constant as ρ approaches 1 (from below), i.e.,

$$\lim_{\rho \to 1} \Delta C = \frac{h}{2} \log\left(\frac{h+b}{h}\right).$$

From the above results, it seems, at first, that advance demand information is highly beneficial when the average load is high (production capacity close to demand rate) but not very useful otherwise. However, before concluding hastily, it is interesting to verify the value of advance demand information relative to the initial costs. To this end, let us define the relative difference $\Delta_{\mathrm{r}} C \equiv \Delta C / C^*(S^*, 0)$. In this case, some manipulation of the cost function leads to

$$\Delta_{\mathrm{r}} C = 1 - \frac{\rho \log(1/\rho)}{1-\rho}. \tag{6}$$

The relative value of information is then decreasing in ρ. In particular

$$\lim_{\rho \to 0} \Delta_{\mathrm{r}} C = 1 \quad \text{and} \quad \lim_{\rho \to 1} \Delta_{\mathrm{r}} C = 0.$$

Remark. Expression (6) also appears in a different context in an unpublished technical report by Buzacott and Shanthikumar (1989).

In summary, although the absolute value of advance demand information increases as the average load increases, the relative benefits may be insignificant when the average load is high. A highly simplified conclusion seems difficult to obtain. Nevertheless for moderate values of ρ (say $\rho < 0.8$), relative benefits are over 10% of the initial cost and absolute benefits may be very significant depending on the ratio of b to h. It should be remarked, however, that optimal demand lead time requirements also increase in the ratio b/h.

3.4. Value of advance demand information: horizon dependent cost of advance demand information

In the previous section, we used the idealized setting of free-of-charge advance demand information (with unrestricted demand lead times) to characterize the maximum value that can be obtained from this information. In this section, we analyze the more realistic case where advance demand information is obtained at a cost that depends on the demand lead time provided. The cost of information is of course a proxy for contractual terms such as unit price discounts for ordering earlier. We will investigate two cases in detail. In order to understand the basic interactions, we first study the case of a fixed demand rate and production capacity. We then focus on the case where the demand rate may vary.

Let $\gamma(\tau)$ denote an increasing and differentiable cost function characterizing the cost (the unit discount offered) of having the customer provide a demand lead time of τ. The total expected average cost function can now be expressed as

$$C(S,\tau) = h\left(S + \lambda\tau - \frac{\rho}{1-\rho}\right) + (h+b)\frac{\rho^{S+1}}{1-\rho}\mathrm{e}^{-\mu\tau(1-\rho)} + \gamma(\tau).$$

Proposition 2. If the cost of advance demand information is linear (corresponding to a linear unit price discount schedule), i.e., if $\gamma(\tau) = c\tau$, then the optimal demand lead time required is:

(i) $\tau^* = 0$, if $c > -h(\mu(1-\rho)/\log\rho + \lambda)$;

(ii) $\tau^* = L^*_\infty$, if $c < -h(\mu(1-\rho)/\log\rho + \lambda)$;

(iii) $\tau^* = \tau$, τ: $0 \leqslant \tau \leqslant L^*_\infty$, if $c = -h(\mu(1-\rho)/\log\rho + \lambda)$.

Proof. First note that

$$\frac{\partial C(\hat{S}^*,\tau)}{\partial\tau} = \begin{cases} h\left(\dfrac{\mu(1-\rho)}{\log\rho} + \lambda\right) + \gamma'(\tau), & \text{if } \tau \leqslant L^*_\infty, \\ \gamma'(\tau), & \text{if } \tau \geqslant L^*_\infty. \end{cases}$$

By the above expression, when advance demand information is free-of-charge ($\gamma(\tau) = 0$), the optimal cost decreases linearly in τ in the region where $\tau < L^*_\infty$. The result follows by comparing the rate of decrease in inventory related costs with the rate of increase in advance demand information costs $\mathrm{d}(\gamma(\tau))/\mathrm{d}\tau = c$. □

Proposition 2 has interesting managerial implications. Assume that advance demand information can be obtained through a contractual agreement with a customer through a linear unit price discount schedule. Proposition 2 states that the optimal demand lead time under this assumption is either zero or equal to the optimal unconstrained release lead time L^*_∞ determined by equation (5). In other words, the firm should negotiate with its customers in order to push the demand lead time as far ahead as L^*_∞; otherwise, the advance demand information is too costly to be of any use.

With linear price discounts, it is seen that advance demand information is interesting only when the cost rate of the discount is below a threshold level that depends on the holding costs, the average demand and the average capacity levels. Are there other price-discount schedules which may modify these results? In particular, is it interesting in certain cases to obtain some advance demand information regardless of its cost? The next proposition investigates this question.

Proposition 3. If the cost of advance demand information is $\gamma(\tau) = c\tau^p$ (where $1 < p < \infty$ and $0 < c < \infty$), then the optimal demand lead time is:

$$\tau^* = \min\left\{L^*_\infty, \left[\frac{-h}{cp}\left(\frac{\mu(1-\rho)}{\log \rho} + \lambda\right)\right]^{1/(p-1)}\right\}. \tag{7}$$

Proof. As in proposition 2, we have to compare the rate of increase in the cost of advance demand information with the rate of decrease in inventory related costs. The (candidate) optimal demand lead time τ is then determined as the solution of a simple equation which gives the second term in (7). At the same time, it is known that inventory related costs are decreasing in τ in the region where $\tau < L^*_\infty$, thereby eliminating the need to go beyond L^*_∞. □

Proposition 3 complements the earlier insights in an intuitive way. Under certain (non-linear) unit price discount schedules, which are increasing and convex in the demand lead time, the proposition states that it is always valuable to obtain some amount of advance demand information.

3.5. Marginal value of advance demand information and the effects of capacity

In this section, our goal is to investigate the effects of capacitated production on optimal advance demand information requirements. Assume that the cost of advance demand information (the unit price discount) does not depend on the average system load. Under what conditions should a manufacturing firm *purchase* advance demand information: when its average load is low or high?

In order to answer the previous question, we will focus on measuring the improvement in cost obtained by a small increase in τ. More precisely, we will investigate the partial derivative of the function $C(\hat{S}^*, \tau)$ as a function of the system load ρ. In order reduce the number of parameters involved, we will fix the production rate μ at 1 (in other words, time is measured in units respective to the average processing time) and vary ρ by varying the demand rate λ.

In the previous subsection, we saw that a determining factor for the optimal amount of advance demand information is the rate of decrease in inventory related costs. Let us denote by $\phi(\rho)$ the rate of decrease in the optimal cost function. This function manifests an interesting non-monotonic behavior as summarized in the next proposition.

Proposition 4. Let $\phi(\rho)$ be the function that describes the rate of decrease in inventory related costs for a fixed value of τ (and for $\mu = 1$), where $\tau < L^*_\infty$. Then,

(i) $\lim_{\rho\to 0} \phi(\rho) = 0$;

(ii) $\lim_{\rho\to 1} \phi(\rho) = 0$;

(iii) $\phi(\rho)$ is a uni-modal function of ρ in the interval $(0, 1)$ and is maximized when $\rho = 0.166413$. The maximum value of $\phi(\rho)$ is $0.298426h$.

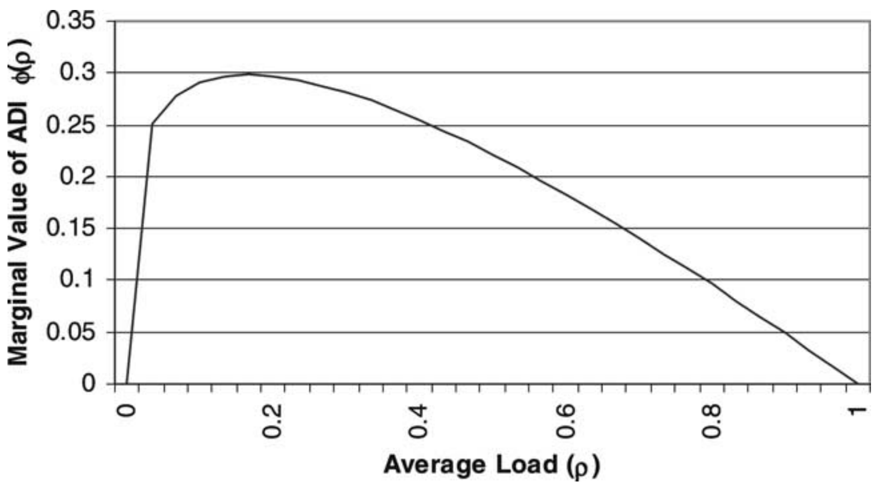

Figure 3. The rate of decrease in the optimal cost function as a function of the system load.

Proof. When $\mu = 1$, we can express $\phi(\rho)$ as

$$\phi(\rho) = -\frac{\partial C(\hat{S}^*, \tau)}{\partial \tau} = h\frac{(1-\rho)}{\log \rho} + \rho.$$

The properties (i)–(iii) then follow in a straightforward manner by analyzing the right-hand side of the above expression. □

The function $\phi(\rho)$ is displayed in figure 3 for $h = 1$. The rate of cost decrease is insignificant at extreme loads. Clearly, with the "marginal cost decrease" interpretation, for advance demand information to be of value, the average load has to be somewhat significant (for instance $\rho > 0.1$) but not extremely high (i.e., not higher than $\rho > 0.9$).

3.6. Some effects of processing time variability

Our analytical investigation is based on the M/M/1 make-to-stock queue. This is a restrictive setting but is often sufficient in order to develop an understanding of the important qualitative properties due to congestion (i.e., limited production capacity). Below, we present some simulation-based results that explore the effects of less variable (than the exponential) processing times.

For the simulation study, we consider two processing time distributions: a deterministic distribution and a two-stage Erlang distribution with balanced means. In particular, we compare three production/inventory systems that are identical in all parameters (including the mean processing time) except for the distribution of the processing time. We then compute through a discrete-event simulation the optimal average cost (using the optimal base stock level and optimal release lead time) as a function the demand lead time for the two systems and use the analytical results for exponential processing times. For the experiments, the average processing time is equal to 1. The other parameters

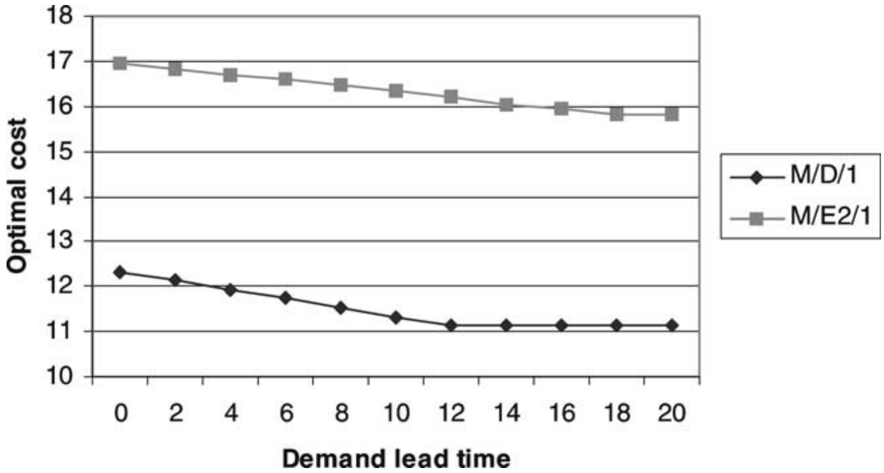

Figure 4. Effects of variability: the optimal cost as a function of the demand lead time.

are as follows: $\lambda = 0.9$, $h = 1$, $b = 10$. Below, is a summary of some of the key observations.

Effects of variability on the average cost for a given lead time. As usual, variability has a degrading effect on performance. For a given demand lead time, the system with the lowest processing time variability achieves the lowest cost, and the cost difference can be significant. Figure 4 depicts the optimal cost for two systems with deterministic and Erlang-2 processing times respectively (the exponential processing time case which generates significantly higher optimal costs is not depicted). The system with deterministic processing times generates significantly smaller costs regardless of the demand lead time. The same holds when comparing the Erlang-2 and exponential processing times (where the Erlang-2 system's costs are significantly smaller).

Linear decrease in optimal cost as function of the demand lead time. The linear decrease in optimal costs (and optimal base stock levels) that was seen in the M/M/1 system is also observed in figure 4 (and in other simulation experiments not reported here). This linear decrease (until a certain demand lead time) seems to be a general qualitative property of make-to-stock queues.

Effects of variability on planning horizons. In figure 4, the M/D/1 system reaches its minimum cost at a demand lead time of 12 while the M/E2/1 system requires a demand lead time of 18. Note that, the M/M/1 system achieves its minimum cost at a demand lead time of 24. As other simulation results also confirm, processing time variability leads to an increase in the planning horizon.

Effects of variability on the relative value of advance demand information. For the example in figure 4, the relative cost decrease, $\Delta_r C$, is 9.6%, 6.9%, and 5.1% respectively, for deterministic, Erlang-2 and exponential processing times. As expected, processing time variability diminishes the relative value of advance demand information. It is also observed that the "less variable" system reduces costs at a higher rate with additional demand lead time than a corresponding "more variable" system.

4. Production/inventory systems with advance demand information and early deliveries

In the previous section we investigated a model where each customer announces a future due-date and requires timely deliveries. In this case, lateness is penalized through the backorder cost but earliness with respect to the due-date is also implicitly penalized since a customer does not accept an order earlier than its due-date. The underlying assumption behind the model is that the "customer" himself/herself is not the end-client but a downstream member of the supply chain serving the end-customer. In a typical example, the downstream member assembles components which are delivered directly to the assembly line by the suppliers. Late deliveries have high penalties since they disrupt the assembly schedule, but early deliveries are not accepted either since they generate excess stock.

Let us now take an alternative point of view and assume that the customer who submits the due-date information accepts early (with respect to the due-date) deliveries. This is the case where the customer is the end-client of the supply chain. A typical situation is on-line retail where customers are proposed a particular delivery due-date. Lateness is still highly undesirable, but most customers would now be content if they received the ordered articles before the announced due-date. The concept of timeliness is now asymmetric and earliness does not have a negative effect.

Let us consider the following model. Customer orders arrive according to a Poisson process λ. Each customer requires a delivery exactly τ periods after the order. The processing times are exponentially distributed with rate μ. Holding costs are incurred at rate h and backorder costs at rate b for orders that not fulfilled before their due-dates.

Let us now describe a highly plausible control policy for the manufacturer. If there is an outstanding order and an item in stock, the item should be delivered immediately regardless of the due-date of the order, because early deliveries are not costly. This has the positive effect of reducing holding costs for the manufacturer. The manufacturer releases a new part to the manufacturing stage with each arriving order, since faster replenishment is more critical than timely replenishment. The above characteristics suggest an order-based base-stock policy (with an initial base stock level S) where each arriving order generates the release of a new part to the manufacturing stage. As for deliveries, each order is immediately fulfilled from stock when possible or fulfilled as rapidly as possible (in the order of their arrival). Bollon (2001) numerically investigates the structure of the optimal policy for a two-stage system with exponential processing times at each stage and early deliveries and exponentially distributed demand lead times. For this case, he observes that the optimal policy is of the above type.

4.1. Analysis of an M/M/1 make-to-stock queue with early deliveries

We start by an equivalence result that facilitates the analysis of the model. A direct sample-path comparison yields the following property.

Property. From the point of view of the manufacturer, the above system (advance demand information with early deliveries/order base-stock policy) is equivalent to a standard base-stock system (with arrival rate λ, processing rate μ, holding cost h and no advance demand information), where the backorder cost starts to incur after a delay of τ.

We can then analyze the system with advance demand information as an equivalent standard base-stock system with a modified backorder cost function. We compute below the average backorder cost for this modified cost function.

Let W be the waiting time of a customer in a standard base-stock system. Let us denote the cumulative waiting time distribution of an order by F_W ($F_W(t) = P\{W \leqslant \tau\}$). The probability that an order waits less than or equal to τ time units (as a function of the base-stock level S) is given by (see (Buzacott and Shanthikumar, 1993))

$$F_W(\tau) = 1 - \rho^S \mathrm{e}^{-\mu(1-\rho)\tau}.$$

We can then obtain the average backorder time of an arriving order as

$$E[\text{backorder time of a customer}] = \int_\tau^\infty (t-\tau)\,\mathrm{d}F_W(t) = \rho^S \frac{\mathrm{e}^{-\mu(1-\rho)\tau}}{\mu(1-\rho)}.$$

Finally, using Little's law, we can obtain the expected number of backorders in the system

$$E_\tau[B] = \rho^S \mathrm{e}^{-\mu(1-\rho)\tau} \frac{\rho}{1-\rho}.$$

Recall that the expected number of backorders in the standard base stock system (with base stock level S) without advance demand information is

$$E_{\tau=0}[B] = \rho^S \frac{\rho}{1-\rho}.$$

In other words, $E_\tau[B]$ is related to $E_{\tau=0}[B]$ in a simple way:

$$E_\tau[B] = \mathrm{e}^{-\mu(1-\rho)\tau} E_{\tau=0}[B].$$

Let us define a modified unit backorder cost

$$b_\tau = b\mathrm{e}^{-\mu(1-\rho)\tau}.$$

Then the system with advance demand information and early deliveries allowed is equivalent to a standard system with no advance demand information and unit backorder

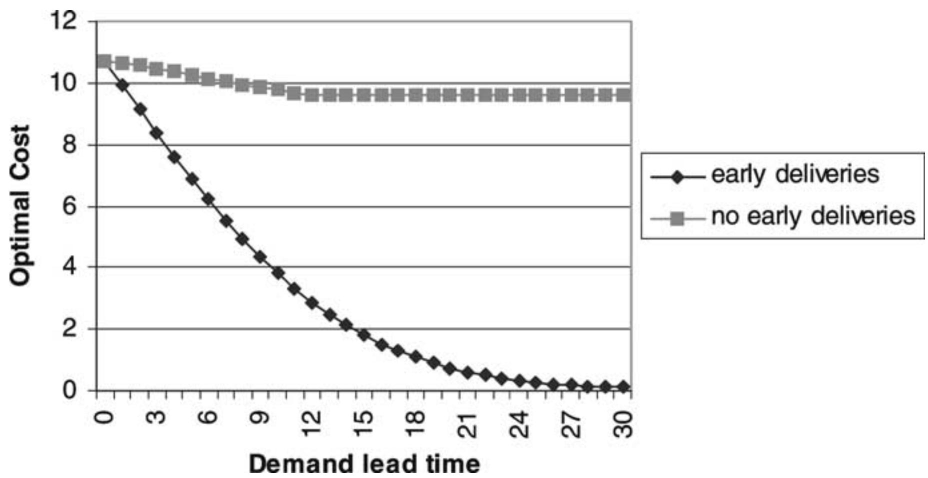

Figure 5. The difference in the optimal cost for the two delivery modes as a function of the demand lead time.

cost b_τ. The optimal base stock level for such a system is given by (see (Buzacott and Shanthikumar, 1993))

$$\hat{S}^*_\tau = \frac{\log(h/(h+b_\tau))}{\log\rho}. \tag{8}$$

The optimal cost then follows as

$$C\big(\hat{S}^*_\tau\big) = h\hat{S}^*_\tau = h\frac{\log(h/(h+b_\tau))}{\log\rho}. \tag{9}$$

Expressions (8) and (9) reveal the effects of advance demand information when early deliveries are accepted. Increasing the demand lead time has the same effect as decreasing the backorder cost in a standard base-stock system. This reflects immediately onto a base-stock level reduction and a cost reduction. It should also be noted that for the same base-stock level, the average backorder cost with early deliveries is identical to the average backorder cost in the corresponding system with timely deliveries. Obviously, for the same base-stock level, the system with timely deliveries holds more average inventory and is therefore more costly.

Figure 5 underlines the significant difference in optimal inventory cost that is attained in the two delivery modes. When timely delivery requirements are required (i.e., early deliveries are not accepted), advance demand information brings somewhat modest benefits. On the other hand, when early deliveries are allowed, the cost reduction is important. The parameters used in this example are: $\lambda = 0.8$, $\mu = 1$, $h = 1$, and $b = 10$.

4.2. *The value of advance demand information: fixed cost of advance demand information*

In order to compare the system with early deliveries with the system with timely deliveries, let us first note that as the demand lead time increases, the early-delivery system operates in a make-to-order mode and rarely misses a due-date. It can then be shown that

$$C_{\min} = \lim_{\tau \to \infty} C(S_\tau^*) = 0,$$

which implies that the maximum potential cost decrease is given by

$$\Delta C = C(S_0^*) \cong h \frac{\log(h/(h+b))}{\log \rho}.$$

It should be remarked that, in this case advance demand information can lead to a zero cost system if demand lead times are sufficiently long. This implies that the maximum relative difference, $\Delta_{\mathrm{r}} C$ is equal to 1, regardless of the system load ρ.

4.3. *The value of advance demand information: horizon dependent cost of advance demand information*

As in section 3.4, in this section we investigate the case where a unit price discount is given to customers as a function of the demand lead time. This leads to the following proposition (which is a counterpart of proposition 2).

Proposition 5. If the cost of obtaining advance demand information is linear in the demand lead time (i.e., $\gamma(\tau) = c\tau$), then the optimal demand lead time is given by τ^*, where τ^* is the solution of

$$\frac{\partial C^{(e)}(\hat{S}^*, \tau)}{\partial \tau} - c = 0,$$

where

$$\frac{\partial C^{(e)}(\hat{S}^*, \tau)}{\partial \tau} = h \frac{b\mathrm{e}^{-\mu(1-\rho)\tau} \mu(1-\rho)}{(b\mathrm{e}^{-\mu(1-\rho)\tau} + h) \log \rho}.$$

Proof. The proof follows along the same lines as the proof of proposition 2 by checking the optimality conditions. □

Figure 6 depicts τ^* as a function of c, the cost (per unit time) of advance demand information. The parameters taken for this example are: $\mu = 1$, $h = 1$ and $b = 10$. We observe that for the same level of price discount, a manufacturer with lower average load should negotiate for longer demand lead times. Indeed, from figure 6, we observe that the optimal demand lead time becomes less sensitive to price discounts as the load increases. At $\rho = 0.9$, the optimal demand lead time varies between 16 and 0 as a function of the price discount, whereas at $\rho = 0.7$ the variation is between 50 and 0.

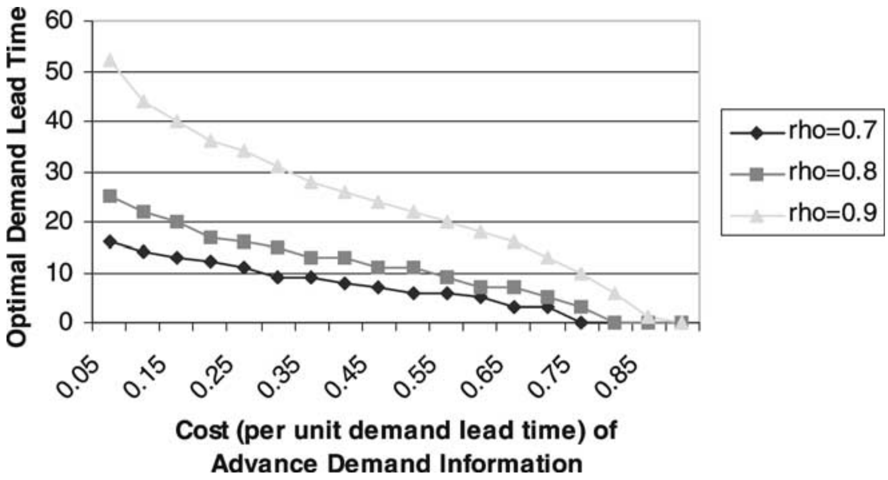

Figure 6. The optimal demand lead time τ^*, when $c(\tau) = c_p\tau$.

4.4. Marginal value of advance demand information and the effects of capacity

Proposition 4 stated an interesting non-monotonic effect of system load on the cost reduction that can be expected for a small increase in the demand lead time. The proposition below investigates the corresponding issue for the model with early deliveries. As before, we set $\mu = 1$ so that the variation in ρ implies a variation in the demand rate λ.

Proposition 6. Let $\phi(\rho)$ be the function that describes the rate of decrease in inventory related costs for a fixed value of τ ($\tau > 0$). Then,

(i) $\lim_{\rho \to 0} \phi(\rho) = 0$;

(ii) $\lim_{\rho \to 1} \phi(\rho) = hb/(b + h)$;

(iii) $\phi(\rho)$ is increasing in ρ.

Proof. The proof follows by a direct computation using equation (9). □

Proposition 6 implies that with early deliveries, the marginal value of advance demand information is increasing in the system load and is maximized as the system load goes to 1. Interestingly, from case (ii) of proposition 6, the fraction $hb/(h+b)$ is an upper bound on the cost reduction that can be attained by a unit demand lead time increase.

5. Conclusion

There is no doubt that advance demand information enhances the performance of production/inventory systems. In this paper, in order to refine this intuition, we investigated the factors that have an impact on the extent of the cost reduction that can be achieved through advance demand information.

The first important remark relates to capacitated production. The average system load is a determining factor for the value of advance demand information. The relative benefits of advance demand information disappear in extremely high system loads. Moreover, in heavy load conditions, the cost reduction per additional unit demand lead time is extremely small and the optimal planning horizon (demand lead time) is extremely long. The consolation is that the absolute value of advance demand information can be significant even at high loads given that demand lead times are sufficiently long.

The second important point has to do to with obtaining advance demand information through price discounts offered to customers. It was found that if price discounts are proportional to demand lead times, the optimal demand lead time is either zero or the optimal unconstrained release time (which depends on the supply lead time), depending on the price schedule. This implies that if customers have the power to set prices aggressively, the manufacturer may not be willing to operate using advance demand information. This clearly creates a supply-chain inefficiency since there are potential savings to be made by the use of advance demand information.

Finally, the nature of the delivery requirements have a significant impact on the value of advance demand information. If customers order in advance and accept deliveries earlier than the due-date, the manufacturer decreases costs in a significantly. This is in contrast with a timely delivery arrangement where early deliveries are not accepted, in which case the value of advance demand information is relatively modest.

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