Value of Advance Demand Information in Production and Inventory Systems with Shared Resources

Fikri Karaesmen

Abstract Advance Demand Information can improve the performance of production/inventory systems with random demand considerably. However, to exploit the potential of such information, replenishment policies must be adapted to the available information. We explore the impact and the benefits of Advance Demand Information under optimal replenishment policies for two different supply chain structures, one without inventory sharing and the other with complete inventory and capacity sharing. We perform this analysis for three different modeling frameworks with different degrees of sophistication in modeling the inventory and production dynamics. This enables a comparison of the factors that make advance demand information relatively more valuable.

1 Introduction

Investigating the benefits of Advance Demand Information (ADI) in production and inventory systems has been a significant research question in recent years. We view ADI as a general concept encompassing different types of future demand information: formal and subjective forecasts, early or advance orders and in general any signal providing information about future demand occurrences. Under this general definition, it is clear that ADI is has been around for a long time. It is therefore interesting that it was not modeled and investigated systematically for a long time in production/inventory control literature. It is likely that the recent increase in research effort was fueled by the information revolution which enabled more data and much easier analysis and exchange of such data. At the same time, models of production/inventory systems appear to have reached a maturity with well-established sophisticated tools for analysis. The combination of practical business needs and

Fikri Karaesmen
Dept. of Industrial Eng., Koç University, Sariyer, Istanbul, Turkey e-mail: fkaraesmen@ku.edu.tr

the existence of tools for analysis have rapidly generated a wide body of research in ADI applications in production/inventory systems.

This chapter has a double purpose. First, we would like to present some basic results on ADI in inventory and production systems from a common perspective. These results usually appear in a dispersed manner but studying them within the same perspective enables better comparisons and possibly an improved understanding. Second, there is a rich inventory control literature in modeling and understanding the benefits of resource sharing in terms of inventory pooling for uncapacitated inventory systems and capacity and inventory pooling for capacitated systems. Resource sharing is well-known to be of value but is not always feasible. In order to understand how the benefits from resource sharing interact with the benefits from ADI, we present an investigation of the value of ADI in situations with and without resource sharing.

We consider three different inventory models. The first model is a single-period newsvendor problem where demand uncertainty can be modeled but inventory dynamics are ignored. The second model is a dynamic model with Poisson demand arrivals and constant supply lead times, a standard model in continuous-time. This model captures inventory dynamics as well as demand uncertainty but does not take into account replenishment capacity. Finally, the third model is a make-to-stock queue: a dynamic model with Poisson demand arrivals and a limited production resource modeled by a queue server. This model is also relatively simple and well-established and has the virtue of capturing the effects of limited capacity. For all three models, we consider multi-location demand that can either be satisfied by dedicated resources (inventory or capacity) to each location or by shared resources. In this setting, we explore the benefits of ADI for the above three models with and without resource sharing.

In order to assess the value of information, we employ a common ADI investment problem. The firm decides to invest in ADI at each location at a cost per location. This is a basic linear cost investment problem where the ADI investment is traded-off against the gains from inventory related costs. The solution of the ADI investment problem is obtained for all three models which allows some structural comparisons about under what conditions to invest.

The chapter is structured as follows. Section 2 presents a short literature review. Section 3 describes the supply chain structures considered. Section 4-6 present the three models and their analysis. Finally, Section 7 summarizes the main findings in terms of the value of information and Section 8 presents the conclusions.

2 Literature Review

The literature review is divided into three subsection. We first address the general issue of modeling ADI. Next, we review the papers that consider uncapacitated models of inventory systems and finally we present a review of the literature using ADI for capacitated inventory systems.

2.1 Modeling ADI

ADI is a general concept that encompasses different types of future demand information. The models for ADI vary in the degree of complexity and sophistication. Static models tend to be simpler whereas dynamic models may capture the evolution of ADI and may be considerably more complicated.

Static models in the context of single-period random demand inventory systems usually view ADI as a modification of the demand distribution. In the simplest case, this may correspond to a standard deviation reduction as in Milgrom and Roberts [38]. In particular, Milgrom and Roberts [38] assume that ADI removes all of the uncertainty (standard deviation) when demand prior to ADI is normally distributed. Zhu and Thoneman [51] consider a similar ADI model with partial removal of standard deviation but they also incorporate the forecast update in their model. Several other papers make similar assumptions in different settings (see for example [9], [16]).

Dynamic production/inventory models allow richer demand information modeling. A simple but useful model assumes that all customers order a fixed time in advance of their due-dates. The time between the order instant and the due-date is called the demand lead time . Buzacott and Shanthikumar [4], [6] and Hariharan and Zipkin [21] propose and analyze such models with fixed demand lead times for capacitated and uncapacitated inventory systems respectively. Several papers use this type of advance order model (Chen [7], Karaesmen, Buzacott and Dallery [26], Wijngaard and Karaesmen [50], Kocaga and Sen [31], Marklund [37] for example). Other papers assume that demand lead times can be random and/or orders can be cancelled (Gayon, Benjaafar, and de Vericourt [15], Benjaafar, Cooper and Mardan [2], Kim, Ahn, and Righter [30] for example).

More sophisticated advance demand information evolution models have been proposed for discrete time systems. Such models typically use a future demand information vector. In additive models, this vector includes all orders that have already arrived and that have due-dates in the future. Because orders are collected over time, if they are not subject to cancelations, the information vector is subject to additive updates. Gallego and Ozer [12], [13], Ozer and Wei [40], Ozer [39] and Dellaert and Melo [10] investigate such models. In contrast, Tan, Gullu and Erkip [42] propose a similar model but in their case arriving orders are subject to cancellations before they materialize. Van Donselaar, Kopzcak and Wouters [11] and Thonemann [45] consider supply chains consisting of several manufacturers that can produce similar products. In this case, the customer may provide information about which products will be ordered and which manufacturers may receive the order which specifies the ADI structure.

Finally, there is a stream of papers that incorporate the demand forecasting process in inventory management. Graves et al. [17] and Heath and Jackson [22] propose a coherent framework for the outputs of a demand forecasting process in a production/inventory system. This framework is called the Martingale Model of Forecast Evolution (MMFE) [22]. There are several papers that use the MMFE-based forecast information to analyze different poduction/inventory systems starting by [17] and [22]. Some other examples include Gullu [19], [20], Graves, Kletter and Hetzel [18], Toktay and Wein [46], Zhu and Thonemann [51] and Schoenmeyr and Graves [41]. Finally, Kaminsky and Swaminathan [24], [25] propose a simpler forecasting models where the forecasts consist of a forecast band comprising a pessimistic and optimistic forecast.

2.2 Uncapacitated Inventory Systems

In this subsection, we review the ADI literature that considers ordering policies for supply systems without capacity constraints. Some of these papers consider single-period models. Others consider dynamic models in discrete or continuous time with supply lead times that do not depend on the number of orders.

Milgrom and Roberts [38] consider a single-period random demand model where customers may reveal the exact demand at a cost. For this setting, they find the optimal level of investment of ADI. Zhu and Thonemann [51] consider a more sophisticated model within the same framework where demand information gets updated according to a MMFE type model.

The models in continuous time typically assume a Poisson order process with a constant demand lead time that is the customers order a fixed time in advance of their due-dates and late delivery penalties, if any, are incurred only after the due-date. This enables the supplier to initiate the ordering process before the required due-date. Hariharan and Zipkin [21] investigate the system with constant supply lead times and identify optimal ordering policies and the assess the benefits of ADI. In particular, they make the fundamental observation that demand lead times have an opposing effect to supply lead times and alleviate the need for inventories. Chen [7] explores a problem motivated by market segmentation issues where customers may be willing to provide different demand lead times depending on the financial incentives provided. Marklund [37] considers a single warehouse multiple retailer supply chain with advance order information and presents exact and approximate performance analysis considering different inventory allocation policies. Lu, Song and Yao [36] investigate assemble-to-order systems with ADI and establish that ADI improves fill-rate type service levels for such systems. Kocaga and Sen [31] study an inventory allocation problem with advance order information in the context of spare-parts inventories and show that ADI and efficient capacity allocation can lead to significant inventory cost savings.

A number of papers investigate the impacts of ADI on periodic-review inventory systems. Bourland, Powell and Pyke [3] study a two-stage supply chain and explore the effect of the retailer providing early information on its demand to the supplier. Güllü [20] considers a two-echelon allocation problem for a supply chain consisting of a single warehouse and multiple retailers under the MMFE model and shows that the value of forecast information can be significant. Graves et al. [18] consider the MMFE demand information model to investigate the trade-off between production smoothing and inventory optimization. DeCroix and Mookerjee [9] investigate a model where one-period ahead demand information can be obtained at a cost and establish optimal replenishment and ADI purchasing policies. Gilbert and Ballou [16] consider a make-to-order manufacturer whose customers may provide advance order information in return for a price discount. In this setting, they explore the optimal discount scheme for using ADI. Motivated by project-based supply chains, Van Donselaar et al. [11] consider supply systems with several products, several manufacturers and a single customer where the customer provides information about which manufacturers may get the upcoming order and which products are to be ordered. They show that this type of information is extremely valuable. Thonemann [45] extends this model to cover multiple types of information sharing and analyzes the inventory cost savings under different types of ADI sharing structures. Zhu and Thonemann [51] study a single-period problem with a single retailer and multiple customers under the possibility of an MMFE-based demand update for individual customers. They explore the problem of finding the optimal set of customers to share demand information given that such information is costly. Gallego and Ozer [12] establish the structure of optimal replenishment policies for a single-stage periodic-review inventory system with ADI using the additive demand information update framework. The analysis is extended to the multi-stage case in Gallego and Ozer [13] and to distribution systems in Ozer [39]. Dellaert and Melo [10] also consider the additive demand information update model to study a lot-sizing problem and propose lot-sizing heuristics that take into account ADI. Tan, Gullu and Erkip [42] investigate optimal ordering decisions for a single-stage inventory system under an imperfect ADI model where initial orders may be cancelled over time. Tan, Gullu and Erkip [44] consider a similar demand model for an inventory allocation problem in a two-demand class system. Tan [43] proposes a forecasting methodology for imperfect ADI. Wang and Toktay [48] investigate optimal ordering policies in a model with advance order information but where customers are willing to accept deliveries before the due-dates. Kunnumkal and Topaloglu [32] consider the problem of offering optimal price discounts to reduce the variability of demand which can be achieved through ADI. Schoenmeyr and Graves [41] study the safety stock optimization in a multi-stage inventory system under an MMFE-based demand information process. They show that the forecast evolution model can be incorporated into known safety stock optimization approaches.

2.3 Production/Inventory Systems

This subsection reviews the ADI literature that explicitly models production capacity constraints and the interaction between capacity and inventories. Such systems are prone to congestion and therefore their production lead times are endogenously determined. These endogenous lead times make the analysis challenging and presents interesting contrasts with respect to similar systems that have exogenous lead times. Some of the important issues for these systems are presented in detail in Buzacott and Shanthikumar [5] and Zipkin [52] for example.

Graves et al. [17] and Heath and Jackson [22] propose models that incorporate the forecast information captured by the MMFE framework in production and inventory planning. Gavirneni, Kapuscinski and Tayur [14] consider a two-stage supply chain with a capacitated production system upstream. Using simulation, they compare the cases where the upstream stage has access to end-customer demand information or not and show that there is significant value in this shared demand information. Gullu [19] uses the MMFE framework to model a single-stage production inventory system in discrete time and characterizes the structure of optimal policies and the value of forecast information system. A similar model is investigated by Toktay and Wein [46] who characterize optimal base stock levels under some approximations. Ozer and Wei [40] characterize the optimal production policy under the additive demand update framework. Kaminsky and Swaminathan [24], [25] investigate production policies and propose several heuristics under the forecast band model.

Another class of models explore advance order information for continuous time models of production/inventory systems represented by make-to-stock queues. Buzacott and Shanthikumar [4], [5], [6] study a single-stage M/M/1 make-to-stock queue with constant demand lead times and characterize the inventory-related performance measures as well as the demand-lead time inventory trade-off. Karaesmen, Buzacott and Dallery [26] investigate a version of the same system in discrete time and characterize optimal production policies. Karaesmen, Liberopoulos and Dallery [28], [27] explore the value of advance order information for M/G/1 and M/M/1 make-to-stock queues with constant demand lead times. Wijngaard [49] studies an M/D/1 make-to-stock system and characterizes the cost reduction due to ADI. Wijngaard and Karaesmen [50] further characterize the optimal policy structure for this system. Liberopoulos and Tsikis [34] propose a framework for describing production policies that incorporate advance order information for multi-stage production/inventory systems. Liberopoulos and Koukoumialos [33] explore the performance of single-stage and multi-stage policies that use such policies. Liberopoulos [35] investigates the inventory and demand lead time tradeoffs for M/D/1 and M/D/∞ make-to-stock systems. Claudio and Krishnamurthy [8] investigate multi-stage production/inventory systems that use ADI under kanban control using simulation. Iravani et al. [23] and Gayon, Benjaafar and de Véricourt [15] consider single-product multiple demand class systems. Iravani et al. [23] assume that the primary customers order at regular intervals and provide advance information but secondary customers request a single item at random times. The optimal production and stock allocation policy for this model is characterized. Gayon et al. consider a multi-class system with different lost sales costs for each demand class. The demand lead times are random and exponentially distributed. In addition, demand cancelations are allowed. The authors characterize the optimal production and inventory allocation policies for this system. Kim, Ahn and Righter [30] and Benjaafar, Cooper, and Mardan [2] also consider ADI models with demand cancelation. Both papers model the order evaluation over time through multiple stages with order cancelation probabilities at each stage and investigate optimal production policies.

3 Supply Chain Structures and Resource Sharing

Our objective is to analyze the value and the impacts of ADI in supply chains that are structurally similar but differ in terms of their inventory dynamics and supply capabilities. In particular, we analyze three different basic inventory models with random demand. We start with a single-period random demand also known as the newsvendor model. We next consider an uncapacitated supply model which receives a random demand process: this is the model of a supply system that has exogenous supply lead times. Finally, we investigate a supply model with production capacity and therefore is subject to congestion and experiences endogenous lead times. For each class of inventory system, ADI is modeled and incorporated in a different way that will be explained in the coming sections.

For the three basic inventory systems above, we consider a two-stage supply chain structure consisting of multiple customers that generate the demand and either multiple supply systems dedicated to each customer or a single supply system that servers all customers. The multiple dedicated supply system model represents the situations where inventories are planned individually for each customer and cannot be shared between customers due to product or customer specific restrictions. The single supply system, on the other hand, represents the centralization of resources and allows sharing inventories and capacity. Inventory centralization or pooling is a widely studied topic in inventory management and the structures studied here are standard in this body of work.

The two structures considered are depicted in Figure 1. The structure on the right satisfies demand using customer specific supply facilities and does not share inventories or capacity while the structure on the left pools inventories and capacity and allows complete sharing of resources. To maintain simplicity we assume that the customers are identical in terms of their backorder or lost sales costs which enables us to avoid to challenging inventory allocation problem under resource pooling that can arise under non-identical customer costs. We assess the value of ADI for both structures leading to a comparison of the structures as well as the three different inventory models.

4 A Static Model: Newsvendor Framework

In this section, we investigate a single-period random demand model under a simple model of ADI. This model ignores the inventory dynamics but still captures some of the important characteristics of the problem in terms of randomness and the effect of demand information. The analysis is inspired by Milgrom and Roberts [38], Zhu and Thonemann [51]. As for the effects of inventory sharing, we follow Uçkun, Karaesmen and Savaş [47] which investigates an inventory inaccuracy problem using similar models.

4.1 No Inventory Sharing

Let us assume that there are N customers with independent and identically distributed demands D_i $(i = 1,2,..N)$. Each D_i is normally distributed with mean μ and standard deviation σ. We assume that each customer is satisfied by its dedicated inventory and that inventory cannot be shared between different customers.

Using standard assumptions, we assume that each customer location chooses an order quantity before observing the demand. Unsatisfied demand is lost and unsold inventory is salvaged. Let r be unit sale price per item, w be the purchasing cost per item and s be the salvage value per item. It is useful to define the critical fractile α expressed as a ratio of the financial parameters: $\alpha = (r-w)/(r-s)$. We also denote by $\Pi(Q)$ the profit obtained for some order quantity Q.

Let Q_i be the order quantity at location i. The expected profit at location i, $E[\Pi(Q)]$, is given by:

$$E[\Pi(Q_i)] = (r-w)\mu - (r-w)E[(D-Q)^+] - (w-s)E[(Q-D)^+] \tag{1}$$

where $(x)^+$ denotes $\max(x,0)$.

Let Q_i^* be the optimal order quantity which maximizes the expected profit given in (1), it is well-known that Q_i^* is characterized by the critical fractile rule: $F_{D_i}(Q_i^*) = \alpha$ where $F_{D_i}(x)$ is the cumulative distribution function of the random variable D_i and α depends on the financial parameters as defined above.

To further exploit the critical fractile rule in the case of normally distributed demand, let $\phi(z)$ and $\Phi(z)$ denote the probability density function and the cumulative density function of a standard normal random variable Z and let z_R be the solution of $\Phi(z_\alpha) = \alpha$. The optimal order quantity can then be expressed as:

$$Q_i^* = \mu + z_\alpha \sigma. \tag{2}$$

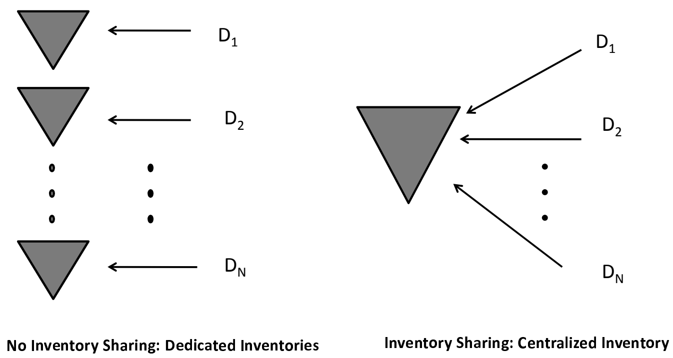

Fig. 1 The Two Supply Chain Structures

Further, Q_i^* can then be plugged back in (1) and using the properties of normally distributed random variables, the following expression for the optimal expected profit at location i can be obtained:

$$E[\Pi(Q_i^*)] = (r-w)\mu - (r-s)\phi(z_\alpha)\sigma.$$

Next, we focus on the total profit of the supply chain over N customers. To facilitate the forthcoming analysis, let us define by Π_n the total supply chain profit where n customers are providing ADI. Therefore Π_0 denotes the total profit over all customers without ADI. Its expected value is given by:

$$E[\Pi_0] = N(r-w)\mu - N(r-s)\phi(z_\alpha)\sigma.$$

Let us now investigate the case where Advance Demand Information is available possibly at a cost. We first assume the following form of ADI: at any location demand can be completely revealed in advance of the ordering decision at a unit cost of k per location. This may be possible by an early commitment contract along with improved information sharing and delayed ordering.

Let location i be one of the locations where ADI is available. For any realization of demand d, the order quantity $Q_i = d$. Obviously, $E[Q_i] = \mu$ since $E[D_i] = \mu$. The expected profit for this location is:

$$E[\Pi] = (r-w)\mu.$$

Let us now express the total profit if the firm chooses to obtain ADI at n ($n = 0, 1, 2, ...N$) locations and uses the optimal order quantity given in (2) for the remaining $N-n$ locations

$$E[\Pi_n] = N(r-w)\mu - (N-n)(r-s)\phi(z_\alpha)\sigma - nk.$$

We can now focus on the problem of optimal ADI investment. Let us find the optimal number of locations to obtain ADI in order to maximize the expected profit. To this end, treating n momentarily as a continuous variable, we note that:

$$\frac{dE[\Pi_n]}{dn} = (r-s)\phi(z_\alpha)\sigma - k. \quad (3)$$

Because $E[\Pi_n]$ is linear in n (from (3)), the optimal value of n is either 0 or N. Then $n^* = 0$ or $n^* = N$ depending on the value of the unit ADI investment cost k. It is clear that there is a threshold level $\bar{k}_{NIS}$ for this cost where it is optimal to invest in ADI at all N customers if $k < \bar{k}_{NIS}$ (where the index NIS denotes No Inventory Sharing) and not to invest at any customer otherwise. Using (3), this threshold is given by:

$$\bar{k}_{NIS} = (r-s)\phi(z_\alpha)\sigma. \quad (4)$$

The investment threshold $\bar{k}_{NIS}$ is clearly increasing in σ. Increasing demand variability justifies higher levels of ADI investments. The effect of financial parameters is less clear because they influence $\phi(z_\alpha)$ in addition to the term $(r-s)$ but it is safe to say that increasing r also justifies higher investments except in very special cases.

Imperfect ADI

Let us now focus on the case where ADI is not perfect and assume that ADI does not enable the removal of all demand uncertainty but only a proportion. As a simple model, we assume that a fraction t $(0 < t < 1)$ of all demand uncertainty is removed by ADI investment at a location at a cost of k_t. This implies that after ADI, the demand at a location is normally distributed with mean μ and variance $(1-t)\sigma^2$. For a location j with ADI investment, the optimal order quantity is then given by:

$$Q_j^* = \mu + z_\alpha \sqrt{(1-t)}\sigma.$$

Let us now consider the case where the ADI investment takes place at n locations. The total expected profit is given by:

$$E[\Pi_n] = N(r-w)\mu - ((N-n) + n\sqrt{1-t})(r-s)\phi(z_\alpha)\sigma - nk_t.$$

$E[\Pi_n]$ in the above expression is linear in n. The investment threshold is given by:

$$\bar{k}_{t,NIS} = (1-\sqrt{1-t})(r-s)\phi(z_\alpha)\sigma.$$

We see that the optimal investment threshold is increasing in σ as before but it is lower than the threshold in the perfect ADI case when $t < 1$. Naturally, the threshold is also increasing in t. As the quality of ADI improves, ADI investments become more attractive.

4.1.1 Inventory Sharing

We consider a similar supply chain structure with N locations as in the previous section but now we assume that a central inventory can be shared among different locations after demand is realized. the objective is to compare the benefits of ADI with respect to the structure of the supply chain.

The central inventory is planned to maximize the total supply chain profit taking into account the total demand. Because demand at each location is normally distributed, the total demand also has a normal distribution with mean $N\mu$ and variance $N\sigma^2$. It then follows the optimal order quantity for the supply chain without ADI is given by:

$$Q^* = N\mu + \sqrt{N} z_\alpha \sigma.$$

The total supply chain profit then follows as:

$$E[\Pi_{0,IS}] = N(r-w)\mu - \sqrt{N}(r-s)\phi(z_\alpha)\sigma.$$

Let us first consider the case of perfect ADI at n locations. Once again, this is assumed to remove the uncertainty completely at these locations at a cost of k per location. This makes the total supply chain demand normally distributed with mean $N\mu$ and variance $(N-n)\sigma^2$. The optimal order quantity is given by:

$$Q_n^* = N\mu + \sqrt{(N-n)}z_\alpha\sigma$$

and the corresponding expected optimal profit is:

$$E[\Pi_{n,IS}] = (r-w)N\mu - \sqrt{(N-n)}(r-s)\phi(z_\alpha)\sigma - nk.$$

This expression is not linear in n as before. Differentiating with respect to n we obtain:

$$\frac{dE[\Pi_{n,IS}]}{dn} = -\frac{((r-s)\phi(z_\alpha)\sigma}{2\sqrt{(N-n)}} - k.$$

It can be verified that the function $E[\Pi_n]$ is convex in n. Therefore the optimal value of n is again an extreme value: $n^* = 0$ or $n^* = N$.

The optimal investment threshold is then given by:

$$\bar{k}_{IS} = \frac{(r-s)\phi(z_\alpha)\sigma}{\sqrt{N}}. \tag{5}$$

It is interesting to compare the threshold for case without inventory sharing given in (4) with the threshold in (5). For $N \geq 2$, the investment threshold is lower under no inventory sharing. This is intuitive because uncertainty has a greater impact in this setting. Alternatively, under inventory sharing, there may be little reason to invest in ADI if N is large.

Imperfect Information

The above analysis can be extended to imperfect ADI following the approach and the notation in Section 4.1. The expected optimal profit under imperfect ADI when n locations are invested in is:

$$E[\Pi_{n,IS}] = N(r-w)\mu - (r-s)\phi(z_\alpha)\sqrt{(N-n)\sigma^2 + n(1-t)\sigma^2} - k_t n.$$

Once again, $E[\Pi_{n,IS}]$ can be verified to be convex in n and $n^* = 0$ or $n^* = N$.

The investment threshold is given by:

$$\bar{k}_{t,IS} = \frac{(1-\sqrt{(1-t)})(r-s)\phi(z_\alpha)\sigma}{\sqrt{N}}.$$

As before, the above is lower than the corresponding threshold under no inventory sharing and is lower than the threshold under perfect ADI.

Correlated Demand

As a final case, let us investigate the effects of demand correlation on ADI investment decisions. First, clearly such correlation does not have any impact on the results under no inventory sharing since locations are managed independently. However, the correlation structure makes a difference when inventory is shared.

Let us assume that demand at different locations has a multi-variate normal distribution. The marginal distributions at each location are normal with mean μ and variance σ^2 as before. In addition, the demands of any two locations are pairwise correlated with correlation coefficient β (where $-1/N-1 < \beta \leq 1$). It turns out then that the total demand variance $\beta^2 = (N + N(N-1)\beta)\sigma^2$.

The optimal order quantity is given by:

$$Q^* = N\mu + z_R\beta$$

and the corresponding profit is:

$$E[\Pi_{0,IS}] = N(r-w)\mu - (r-s)\phi(z_R)\beta.$$

It is more difficult to model partial ADI investment in this case but we can investigate the case of full investment. Assume that ADI is implemented at all locations, the corresponding expected profit is then:

$$E[\Pi_{N,IS}] = N(r-w)\mu - Nk.$$

Therefore, the all or nothing investment threshold is given by:

$$\bar{k}_{IS} = \frac{\sqrt{1-\beta+\alpha N}(r-s)\phi(z_R)\sigma}{\sqrt{N}}.$$

It is seen that the investment threshold is increasing in the correlation coefficient β for $-1/N-1 < \beta \leq 1$. Consider now the case of extreme positive correlation where $\beta = 1$. This makes the threshold equal to: $(r-s)\phi(z_R)\sigma$ just like in the NIS case. Higher costs of ADI investment are justified as the demand correlation between the locations increases.

5 Inventory Systems with Exogenous Lead Times

In this section, we consider a continuous review inventory system that receives Poisson demand processes from multiple locations. The replenishment system has ample capacity but there are processing lead times. In Zipkin's terminology [52], this is the case of a supply system with exogenous lead times. Similarly to the previous section, we model ADI and explore the ADI investment structure under two different scenarios: without any inventory sharing between locations and with inventory sharing between locations. The inventory model is considerably more sophisticated with respect to Section 4 but we use a similar framework to enable some comparisons. The basic model that is employed in this section follows Hariharan and Zipkin [21]. The model with random replenishment lead times is summarized in Karaesmen [29].

5.1 No Inventory Sharing

The firm receives demand from N different locations and maintains a dedicated inventory for each location. It is assumed that inventory cannot be shared between locations.

The demand at each location is an independent Poisson Process with rate λ. Whenever there is inventory available, the demand is satisfied from inventory and it is backordered otherwise. As in Section 4, r denotes the unit sale price and w denotes the unit purchasing costs. In addition, holding costs of h (per item per time) and backorder costs of b (per item per time) are incurred for inventory and backorders respectively. It is assumed that there are no fixed ordering costs.

Each order takes a supply lead time of L time units to replenish. We assume first that L is constant and investigate the case of random supply lead times later.

Let us use the notation of Zipkin [52] for inventory related quantities. Let $I(t)$ and $B(t)$ denote the inventory on hand and the number of backordered items at time t respectively ($I(t),\ B(t) \geq 0$). $IN(t) = I(t) - B(t)$ is called the net inventory and $IN(t)$ can be positive or negative. Let $IO(t)$ denote the number of items on order. $IO(t)$ corresponds to the items that are already in the supply system but have not yet reached the inventory. Finally, $IP(t) = IN(t) + IO(t)$ is the inventory position.

It is well-known that the optimal replenishment policy (in the absence of fixed ordering costs) is a base stock policy. This policy has a single parameter S which is the base stock level. It then stipulates to order whenever the inventory position $IP(t)$ falls behind the base stock level S.

Let IO_i be stationary random variable corresponding to the outstanding orders at location i. The long-run average profit for location i can be expressed as:

$$E[\Pi] = (r-w)\lambda - hE[(S-IO_i)^+] - bE[(IO_i - S)^+]. \tag{6}$$

The above problem has the newsvendor structure similar to (1) of Section 4 where the random demand in (1) has been replaced by the random number of (steady-state) outstanding orders. It therefore follows that the profit maximizing base stock level is again given by the critical fractile formula. Because IO is a discrete random variable:

$$S^* = min_S \left\{ F_{IO_i}(S) \geq \frac{b}{h+b} \right\}.$$

When demand is a Poisson process with rate λ, IO_i is a Poisson random variable with mean λL. Unfortunately, this does not lead to an explicit expression for the base stock level. We therefore approximate IO by a normal random variable with mean λL and standard deviation $\sqrt{\lambda L}$. This approximation is known to be accurate if λ is not too small.

Let the critical ratio $\alpha' = b/(h+b)$ and let $z_{\alpha'}$ be the solution of $\Phi(z'_R) = \alpha'$. Under the normal approximation,the optimal base stock level is obtained as:

$$S^* = \lambda L + z_{\alpha'}\sqrt{\lambda L}$$

and the expected profit per unit time at location i can be expressed as:

$$E[\Pi] = (r-w)\lambda - (h+b)\phi(z_{\alpha'})\sqrt{\lambda L}.$$

It is seen from the above that the optimal expected profit is decreasing in the supply lead time L at a square root rate.

If we now consider the supply chain consisting of N locations, the total optimal expected profit per unit time simply becomes:

$$E[\Pi_N] = N(r-w)\lambda - N(h+b)\phi(z_{\alpha'})\sqrt{\lambda L}.$$

Let us now focus on the case where a location provides ADI by ordering earlier than its due-date for a cost of k (per location per unit time). The time between the order instance and the due-date is known as the *demand lead time*. Let us assume that the demand lead times of all locations are l (if the ADI investment cost is paid at the location) and assume first that $l > L$. When demand lead-times are longer than supply lead times, the replenishment system is able to function in a make-to-order mode if each order is released exactly L time units before its due-date. In this case, no inventory is needed and all orders are satisfied right on time. The corresponding expected profit per unit time at such a location is $(r-w)\lambda$.

Let us now consider that the firm invests in ADI at n locations and functions as before for the remaining $N-n$ locations. The optimal expected profit for such a system is given by:

$$E[\Pi_n] = N(r-w)\lambda L - \sqrt{(N-n)}(h+b)\sqrt{\lambda L}\phi(z_{\alpha'}) - nk.$$

As before, the above expression can be verified to be convex in n. This again leads to $n^* = 0$ or $n^* = N$. The investment threshold is then:

$$\bar{k}_{NIS} = (h+b)\phi(z'_R)\sqrt{\lambda L}.$$

Short Demand Lead Times

The case of ample demand lead time ($l > L$) is very similar to the case of perfect demand information in Section 4. Let us now consider the case where $0 < l < L$. If ADI investment is made at location i, all orders at that location will be released exactly l units before their due-dates. Hariharan and Zipkin [21] show that the inventory dynamics of such a system are equivalent to those of a system with zero demand lead times and supply lead times of $L-l$. In other words, IO_i is a Poisson random variable with mean $\lambda(L-l)$. The optimal base stock level under the normal approximation then becomes:

$$S^* = \lambda(L-l) + z_{\alpha'}\sqrt{\lambda(L-l)}.$$

Assuming that the ADI investment takes place at n locations, the optimal expected profit is:

$$E[\Pi_n] = N(r-w)\lambda - (h+b)\phi(z_{\alpha'})(\sqrt{(N-n)\lambda L} + \sqrt{n\lambda(L-l)}) - nk_l.$$

The above is expression is convex in n and the all-or-nothing investment structure is maintained. The investment threshold is:

$$\bar{k}_{l,NIS} = (h+b)\phi(z_{\alpha'})(\sqrt{\lambda L} - \sqrt{\lambda(L-l)}).$$

Random Replenishment Times Let us briefly consider the case of random replenishment times. The replenishment times L are now assumed to be independent and identically distributed random variables and order crossing is allowed. This is the model of a supply system with several parallel supply channels. The complication is that the demand lead time l is not always greater than or less than the supply lead time.

This system is analyzed in Karaesmen [29]. The analysis uses the following two quantities:

$$\gamma_1 = \lambda\int_0^l (l-x)dF_L(x) \quad (7)$$

$$\gamma_2 = \lambda\int_l^\infty (x-l)dF_L(x) \quad (8)$$

where F_L is the cumulative distribution function of the replenishment lead time.

Following [29], the number of outstanding orders can be written as the difference of two quantities: $IO = IO_1 - IO_2$ where IO_1 and IO_2 are independent Poisson random variables with means γ_1 and γ_2 respectively.

To employ a similar approximation as before, we approximate IO by normal random variable with mean $\gamma_1 - \gamma_2$ and standard deviation $\sqrt{\gamma_1^2+\gamma_2^2}$.

We can then write:

$$E[\Pi_n] = (r-w)N\lambda - (h+b)\phi(z_{\alpha'})\left((N-n)\sqrt{\lambda E[L]} + n\sqrt{\gamma_1^2+\gamma_2^2}\right) - kn.$$

The investment threshold is then given by:

$$\bar{k}_{l,NIS} = \max\{0, (h+b)\phi(z_{\alpha'})(\sqrt{\lambda L} - \sqrt{\gamma_1^2 + \gamma_2^2})\}.$$

This threshold is lower than the corresponding one with constant lead times that are equal to $E[L]$.

5.2 Inventory Sharing

Let us now review the model of Section 5.1 under the assumptions of inventory sharing: a central inventory is held and the pooled demand from all locations can be satisfied from this inventory.

Let us first consider the system without ADI, the total demand is a Poisson process with rate $N\lambda$. The centralized base stock level under the normal approximation is then:

$$S^* = N\lambda L + \sqrt{N} z_{\alpha'} \sqrt{\lambda L}.$$

The optimal expected profit is given by:

$$E[\Pi_{0,IS}] = N(r-w)\lambda - \sqrt{N}(h+b)\phi(z_{\alpha'})\sqrt{\lambda L}.$$

Let us assume now that ADI is implemented at the first n locations and that demand lead times are greater than supply lead times ($l > L$). The demand from the first n locations can then be met at zero cost in a make-to-order mode and the total steady-state outstanding orders in this system are given by:

$$IO = \sum_{i=n+1}^{N} IO_i$$

because IO_i are independent Poisson random variables, IO is also a Poisson random variable with mean $(N-n)\lambda$. The optimal base stock level using the normal approximation is then obtained as:

$$S^* = (N-n)\lambda L + \sqrt{(N-n)} z_{\alpha'} \sqrt{\lambda L}$$

and the total expected profit per unit time as:

$$E[\Pi_{n,IS}] = (r-w)\lambda - \sqrt{N-n}(h+b)\phi(z'_R)\sqrt{\lambda L} - nk.$$

Once again, the convexity of $E[\Pi_{n,IS}]$ in n can be established. The optimal threshold for full investment is then:

$$\bar{k}_{IS} = \frac{(h+b)\phi(z_{\alpha'})\sqrt{\lambda L}}{\sqrt{N}}.$$

As in Section 4, this threshold is lower than the one without inventory sharing. Once again, inventory sharing results in lower variability and the relative benefit of ADI is lower in this environment.

Short Demand Lead Times

Let us now assume that supply lead times are longer than lead times. As for the case of $l < L$, a similar analysis can be performed to that of Section 5.1. This leads to:

$$E[\Pi_{n,IS}] = (r-w)\lambda - (h+b)\phi(z_{\alpha'})\sqrt{n\lambda(L-l)+(N-n)\lambda L} - nk$$

and to an optimal investment threshold of:

$$\bar{k}_{l,IS} = \frac{(h+b)\phi(z_{\alpha'})(\sqrt{\lambda(L)} - \sqrt{\lambda(L-l)})}{\sqrt{N}}.$$

We observe that this threshold lower than the threshold with long lead times and is in fact increasing in l. Earlier orders by customers make the ADI investment more attractive.

6 Capacitated Systems

The previous sections investigated the effects of ADI and optimal investment levels while ignoring capacity limitations. In this section, we focus on production/inventory systems where the production capacity endogenously generates lead times through the congestion effect. Once again, we try to maintain the parallels to the previous sections to explore similarities and contrasts. In particular, we investigate the effects of supply chain structure in terms of resource sharing. However, in contrast with the previous sections, we consider production capacity in addition to the inventory as the shared resource. The basic model without ADI is described in Buzacott and Shanthikumar [5] or Zipkin [52]. The model with ADI is based on Buzacott and Shanthikumar [6]. The differences between the short versus long demand lead time cases are discussed in Karaesmen et al. [27] and the approximations are based on Karaesmen et al. [28].

6.1 No Inventory and Capacity Sharing

We consider a production inventory system receiving customer demand from N locations. Locations generate demands according to independent Poisson processes at rate λ. Each location has a dedicated processing resource. This dedicated resource processes items one by one with exponential processing times with rate μ (where

$\mu > \lambda$). Demand is satisfied from inventory whenever possible and is backordered otherwise. We assume the same profit/cost structure as in Section 5.2.

It is known that base stock policies are optimal for replenishing the above production/inventory system. The processor should be processing whenever the net inventory $IN(t)$ is below the base stock level S and stops processing when $IN(t)$ reaches S. Equivalently, a production order is released whenever the inventory position $IP(t)$ falls below S as in Section 5.2. In fact, the optimal profit as a function of S can be represented as (6) of Section 5.2.

Let us define $\rho = \lambda/\mu$ which is a measure of the average load. For performance analysis and optimization, the probability distribution of the number of outstanding orders is critical. Let us consider location i which constitutes an M/M/1 make-to-stock queue. It is well-known that, for this system, the stationary random variable IO_i (corresponding to the number of outstanding orders at location i) has a geometric distribution:

$$P(IO_i = j) = (1-\rho)\rho^j \text{ for } j = 0,1,2,...$$

Let us consider the optimization problem to maximize the expected profit per unit time by choosing the optimal base stock level. Using the critical fractile formula, there is an explicit expression for the optimal value of S:

$$S^* = \left\lfloor \frac{\log(1-\alpha')}{\log(\rho)} \right\rfloor \tag{9}$$

where $\lfloor x \rfloor$ denotes the largest integer that is greater than or equal to x and $\alpha' = b/(h+b)$.

The exact optimal profit per unit time at location i can also be written explicitly but for the rest of the analysis, we employ an approximation that is known to be very accurate:

$$E[\Pi] \cong (r-w)\lambda - h\left(\frac{\log(1-\alpha')}{\log(\rho)}\right). \tag{10}$$

The total expected optimal profit per unit time is then:

$$E[\Pi_N] = N(r-w)\lambda - Nh\left(\frac{\log(1-\alpha')}{\log(\rho)}\right).$$

Let us now assume that demand locations order l units of time in advance of their desired due-dates at a unit cost of k. From known results, there are two different release policies. If l is shorter than a critical lead time τ^c, it is optimal to release all advance orders when they arrive (i.e. l units of time in advance). Otherwise, advance orders should be delayed by $l - \tau^c$ time units and should be released τ^c units in advance of their due-dates. Let us consider the case where $l < \tau^c$. The stationary distribution of the outstanding number of orders can be obtained explicitly when IO is positive.

$$P(IO_i = j) = e^{-\mu(1-\rho)l}(1-\rho)\rho^j \text{ for } j = 0,1,2,...$$

Unfortunately, because supply lead times are random, IO is not always non-negative. Some orders are released in advance and experience shorter supply lead times than planned and may reach the inventory before their-due dates. This complicates the analysis but the optimal base stock level for a given location can still be obtained by the critical fractile, yielding:

$$S^* = \begin{cases} \left\lfloor \frac{\log(1-\alpha')}{\log(\rho)} + \frac{\mu(1-\rho)l}{\log(\rho)} \right\rfloor & \text{if } l < \tau^c \\ 0 & \text{otherwise.} \end{cases}$$

In addition, the critical demand lead time τ_c is given by ([27]):

$$\tau^c_{NIS} = \frac{-\log(1-\alpha')}{\mu(1-\rho)}. \tag{11}$$

To advance the analysis, let us first assume that the demand lead time l is longer than the critical value τ^c. Using the results from [28], for the expected optimal profit per unit time of a given location we can write:

$$E[\Pi] = (r-w)\lambda + h\log\left(1-\alpha'\right)\frac{\rho}{1-\rho} - k.$$

If investment is made at n locations, the total expected profit per unit time is:

$$E[\Pi_n] = N(r-w)\lambda + nh\log\left(1-\alpha'\right)\frac{\rho}{1-\rho} - (N-n)\frac{\log(1-\alpha')}{\log(\rho)} - k. \tag{12}$$

The expected optimal profit is linear in n and the optimal investment threshold is:

$$\bar{k}_{NIS} = -h\log\left(1-\alpha'\right)\left(\frac{\rho}{1-\rho} + \frac{1}{\log(\rho)}\right). \tag{13}$$

It can be verified that the above term is always non-negative for $0 < \rho < 1$.

Next, we explore the expected profits in the case of short demand lead times ($l < \tau^c$). Again using the results from [28], we have:

$$E[\Pi] = (r-w)\lambda - h\left(\frac{\log(1-\alpha')}{\log(\rho)} + \left(\frac{\mu(1-\rho)}{\log(\rho)} + \lambda\right)l\right) - k.$$

If ADI investment is made at n locations and the remaining locations use the previous ordering policies, the total profit of the system becomes:

$$E[\Pi_n(S^*)] = N(r-w)\lambda - Nh\left(\frac{\log(1-\alpha')}{\log(\rho)}\right) + hn\left(\left(\frac{\mu(1-\rho)}{\log(\rho)} + \lambda\right)l\right) - nk.$$

The above expression is linear in n and all-or-nothing investment is again optimal. The threshold for making the full investment is obtained to be:

$$\bar{k}_{NIS} = -h\left(\frac{\mu(1-\rho)}{\log(\rho)} + \lambda\right) l.$$

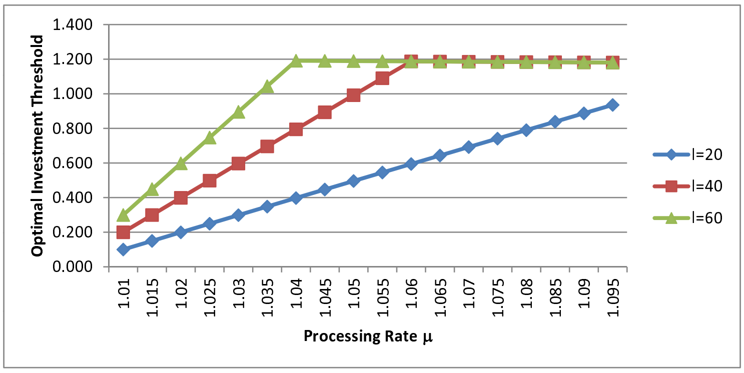

Fig. 2 Effect of processing rate on the optimal investment threshold varying service rates and for three different demand leadtimes

We can summarize the investment threshold result as follows:

$$\bar{k}_{NIS} = \begin{cases} -h\left(\frac{\mu(1-\rho)}{\log(\rho)} + \lambda\right) l & \text{if } l < \frac{-\log(1-\alpha')}{\mu(1-\rho)} \\ -h\log(1-\alpha')\left(\frac{\rho}{1-\rho} + \frac{1}{\log(\rho)}\right) & \text{otherwise} \end{cases}$$

The effects of various parameters on the investment threshold are less clear than the case of uncapacitated systems. To gain some insights, we take a numerical example with $h = 1$, $b = 10$, $\lambda = 1$ and vary the processing rate μ for three different lead times $l = 20, 40, 60$. Figure 2 depicts the optimal investment thresholds. We see that the investment threshold is non-decreasing in l and in μ. At the same time, systems with higher demand lead times reach the optimal investment threshold for lower values of μ. Additional processing capacity does not change the value of ADI if demand lead times are large.

6.2 With Inventory and Capacity Sharing

Let us now assume that capacity and inventory can be pooled such that all locations can share the same joint capacity and inventory. Capacity pooling can take place in different ways but for simplicity we assume that the pooled capacity is modeled by a single processor that can process items at rate $N\mu$.

We then perform a similar analysis to Section 6.1. Without ADI, we have a single server make-to-stock queue with $\rho = N\lambda/N\mu = \lambda/\mu$ as before. The optimal base stock level for this system is equal to the optimal base stock level of a single location without capacity sharing given in (9). Plugging in this optimal base stock level in the expected profit function, we find that

$$E[\Pi_{IS}] = N\lambda(r-w) - h\left\lfloor \frac{\log(1-\alpha')}{\log(\rho)} \right\rfloor.$$

With ADI, let us first consider the case with long demand lead times ($l > \tau^c$). The partial investment case appears difficult to analyze using existing results but we can investigate the effect of full ADI investment (at all N locations). In this case, the optimal expected profit only depends on the financial parameters of the problem and on the average utilization ρ but does not depend on l. The optimal expected profit is therefore equal to:

$$E[\Pi_{N,IS}] = N(r-w)\lambda + h\log\left(1-\alpha'\right)\frac{\rho}{1-\rho} - Nk.$$

We then obtain the critical investment threshold for full investment as:

$$\bar{k}_{IS} = \frac{-h\log\left(1-\alpha'\right)\left(\frac{\rho}{1-\rho} + \frac{1}{\log(\rho)}\right)}{N}.$$

The above threshold is equal to the corresponding threshold in (12) divided by N. Inventory and capacity sharing significantly reduce the investment threshold for ADI. In addition, there is a second difference in the system with no capacity sharing and the one with capacity sharing. Under capacity sharing, the critical lead time is:

$$\tau^c_{IS} = \frac{-\log(1-\alpha')}{N\mu(1-\rho)}.$$

The above is also N times smaller than the corresponding critical lead time without capacity sharing given in (11). Therefore, under capacity sharing the system achieves its maximum profit for much shorter demand lead times.

Finally, let us focus on the case with demand lead times of l where $l < \tau^c_{IS}$. Once again, it is not easy to analyze the case with partial ADI investment (investment at n locations where $1 < n < N$). Therefore, we again consider the all-or-nothing investment strategy. If ADI investment takes place at all N locations, the optimal base stock level is given by:

$$S^* = \left\lfloor \frac{\log(1-\alpha')}{\log(\rho)} + \frac{N\mu(1-\rho)l}{\log(\rho)} \right\rfloor.$$

The expected profit per unit time then becomes:

$$E[\Pi_{N,IS}] = N(r-w)\lambda - h\left(\frac{\log(1-\alpha')}{\log(\rho)} + \left(\frac{N\mu(1-\rho)}{\log(\rho)} + N\lambda\right)\right) - Nk.$$

We then obtain the following investment threshold:

$$\bar{k}_{IS} = -h\left(\frac{\mu(1-\rho)}{\log(\rho)} + \lambda\right) l.$$

This threshold is identical to the corresponding threshold with no inventory sharing. However, the critical demand lead times without inventory sharing are smaller. To summarize:

$$\bar{k}_{IS} = \begin{cases} -h\left(\frac{\mu(1-\rho)}{\log(\rho)} + \lambda\right) l & \text{if } l < \frac{-\log(1-\alpha')}{N\mu(1-\rho)} \\ \frac{-h\log\left(1-\alpha'\right)\left(\frac{\rho}{1-\rho} + \frac{1}{\log(\rho)}\right)}{N} & \text{otherwise.} \end{cases}$$

7 Summary and Discussion

This section provides a summary of the results for the three different models covered in Sections 4, 5, and 6. We first investigate the benefits of inventory and capacity sharing and ADI assuming that ADI has no cost, therefore full investment can be made. In order to perform a comparison, we use the following three benchmarks:

$$\begin{aligned} \Delta_{IS} &= E[\Pi_{NIS}] - E[\Pi_{IS}] \\ \Delta_{ADI} &= E[\Pi_{NIS,ADI}] - E[\Pi_{NIS}] \\ \Delta_{IS,ADI} &= E[\Pi_{IS,ADI}] - E[\Pi_{NIS}]. \end{aligned}$$

Δ_{IS} measures the gains from inventory sharing alone and Δ_{ADI} the gains from ADI alone. Finally, $\Delta_{IS,ADI}$ reports the gains when both inventory sharing and full ADI investment take place. The results are reported in Table 1 for the case of perfect demand information where perfect information is taken to be the case of long demand lead times for systems with replenishment lead times.

Model	Δ_{IS}	Δ_{ADI}	$\Delta_{IS,ADI}$
Newsvendor	$(N-\sqrt{N})(r-s)\phi(z_\alpha)\sigma$	$N(r-s)\phi(z_\alpha)\sigma$	$N(r-s)\phi(z_\alpha)\sigma$
Exogenous Lead Time	$(N-\sqrt{N})(r-s)\phi(z_\alpha)\sqrt{\lambda L}$	$N(r-s)\phi(z_\alpha)\sqrt{\lambda L}$	$N(r-s)\phi(z_\alpha)\sqrt{\lambda L}$
Capacitated Supply	$(N-1)h\frac{\log(1-\alpha')}{\log\rho}$	$-Nh\log(1-\alpha')\left(\frac{\rho}{1-\rho} + \frac{1}{\log(\rho)}\right)$	$h\log(1-\alpha')\left(\frac{\rho}{1-\rho} + \frac{N}{\log(\rho)}\right)$

Table 1 Gains due to inventory sharing and ADI under perfect information

We observe from Table 1 that the newsvendor case and the case of exogenous lead times manifest similar behaviour. The benefits of ADI are more significant when demand variance for the newsvendor or the supply lead time for the exogenous lead time case increases. The benefits are also increasing in the number of customer locations. The case of production/inventory systems is different but for a fixed demand rate, the benefits are increasing in the processing rate and also in the number of locations N under ADI.

Next, we explore the investment thresholds for the three models considered. We first summarize the results under perfect ADI in Table 2.

Model	NIS	IS
Newsvendor	$(r-s)\phi(z_\alpha)\sigma$	$\frac{(r-s)\phi(z_\alpha)\sigma}{\sqrt{N}}$
Exogenous Lead Time	$(h+b)\phi(z_\alpha)\sqrt{\lambda L}$	$\frac{(h+b)\phi(z_\alpha)\sqrt{\lambda L}}{\sqrt{N}}$
Capacitated Supply	$-h\log(\alpha')\left(\frac{\rho}{1-\rho}+\frac{1}{\log(\rho)}\right)$	$\frac{-h\log(\alpha')\left(\frac{\rho}{1-\rho}+\frac{1}{\log(\rho)}\right)}{N}$

Table 2 Investment thresholds under perfect ADI

It is observed from Table 2 that investment thresholds are smaller under inventory sharing and are decreasing in the number of customer locations for all three models. The rate of decrease in the third models is higher. Inventory and capacity pooling leave little additional benefit to be reaped by using ADI and make ADI investment less attractive.

Finally, we report a summary of the investment threshold results under imperfect information in Table 3.

Model	NIS Imperfect	IS Imperfect
Newsvendor	$(1-\sqrt{1-t})(r-s)\phi(z_\alpha)\sigma$	$\frac{(r-s)\phi(z_\alpha)\sqrt{(1-t)}\sigma}{\sqrt{N}}$
Exogenous Lead Time	$(h+b)\phi(z_\alpha)(\sqrt{\lambda L}-\sqrt{\lambda(L-l)})$	$\frac{(h+b)\phi(z_\alpha)(\sqrt{\lambda(L)}-\sqrt{\lambda(L-l)})}{\sqrt{N}}$
Capacitated Supply	$-h\left(\frac{\mu(1-\rho)}{\log(\rho)}+\lambda\right)l$	$-h\left(\frac{\mu(1-\rho)}{\log(\rho)}+\lambda\right)l$

Table 3 Investment thresholds under imperfect ADI

The results from Table 3 are similar to those from Table 2. The quality of demand information (or the demand lead time provided by the customers) has a direct effect on the investment threshold. For production/inventory systems the investment threshold is identical for small times but it was seen in Section 6.2 that the critical demand lead times depend on the number of locations. This again leads to the conclusion that inventory and capacity pooling lowers the need for ADI investment.

8 Conclusions

We analyzed the impacts of ADI on supply chain profits for structures that allow resource sharing at the supply stage or not. While each inventory model has its own specifics and critical parameters, some general principles emerge. ADI is more valuable when there is a lot of demand variability that can be removed using ADI. Resource sharing seems to make ADI relatively less valuable precisely for this reason. It enables considerable variability reduction and there is less uncertainty to be alleviated using ADI. Naturally, this makes ADI investment more likely in decentralized systems from an economic point of view. Nevertheless, ADI may have significant benefits for systems with shared resources.

There is some existing research on providing incentives in return for ADI. However, most of this research investigates simple supply chains. There still appears to be room for designing ADI incentive structures/contracts in multi-stage supply chains under realistic inventory dynamics. The potential cost savings for supply chains are enormous and more ADI is likely to be shared and used if its benefits can be shared in a fair manner.

References

1. AXSÄTER, S., Planning order releases for an assembly system with random operation times. *OR Spectrum*, **27** (2005), 459–470.
2. BENJAAFAR, S. AND COOPER, W.L. AND MARDAN, S., Production-inventory systems with imperfect advance demand information and updating. *Naval Research Logistics*,**58**, (2011), 88–106.
3. BOURLAND, K.E., S.G. POWELL AND D.F PYKE, Exploiting Timely Demand Information to Reduce Inventories. *European Journal of Operational Research*, **92** (1996), 239-253.
4. BUZACOTT, J.A. AND J.G. SHANTHIKUMAR, Stochastic Models of Single Stage MRP-Controlled Production Systems., *Working Paper*, (1989).
5. BUZACOTT, J.A. AND J.G. SHANTHIKUMAR, *Stochastic Models of Manufacturing Systems*, Prentice-Hall, (1993).
6. BUZACOTT, J.A. AND J.G. SHANTHIKUMAR, Safety Stock versus Safety Time in MRP Controlled Production Systems. *Management Science*, **40**, (1994), 1678-1689.
7. CHEN F., Market Segmentation, Advanced Demand Information, and Supply Chain Performance. *Manufacturing & Service Operations Management*, **3** (2001), 53-67.
8. CLAUDIO, D. AND KRISHNAMURTHY, A., Kanban-based pull systems with advance demand information. *International Journal of Production Research*, **47** (2009), 3139–3160.
9. DE CROIX G.A. AND V.S. MOOKERJEE, Purchasing Demand Information in a Stochastic-Demand Inventory System. *European Journal of Operational Research*, **102** (1997), 36-57.
10. DELLAERT, NP AND MELO, MT, Approximate solutions for a stochastic lot-sizing problem with partial customer-order information. *European Journal of Operational Research*, **150**, (2003), 163–180.
11. VAN DONSELAAR K., L.R. KOPCZAK, AND M. WOUTERS, The Use of Advance Demand Information in a Project-Based Supply Chain, *European Journal of Operational Research*. **130** 2001, 519-528.
12. GALLEGO G. AND Ö. ÖZER, Integrating Replenishment Decisions with Advance Order Information, *Management Science*, **47**, (2001), 1344-1360.

13. GALLEGO, G. AND ÖZER, Ö. , Optimal replenishment policies for multiechelon inventory problems under advance demand information. *Manufacturing & Service Operations Management*, **5**, (2003), 157–175.
14. GAVIRNENI S., R. KAPUSCINSKI AND S. TAYUR, Value of Information in Capacitated Supply Chains. *Management Science*, **45** 1999, 16-24.
15. GAYON, J.P. AND BENJAAFAR, S. AND DE VÉRICOURT, F. , Using Imperfect Advance Demand Information in Production-Inventory Systems with Multiple Customer Classes. *Manufacturing & Service Operations Management*, **11** 2009, 128–143.
16. GILBERT S.M. AND R.H. BALLOU, Supply Chain Benefits from Advance Customer Commitments. *Journal of Operations Management*, **18** (1999), 63-71.
17. GRAVES, S.C. AND MEAL, H.C. AND DASU, S. AND QIU, Y. AND AXSÄTER, S. AND SCHNEEWEISS, C. AND SILVER, E., Two-stage production planning in a dynamic environment *Multi-Stage Production Planning and Inventory Control, v266, Springer-Verlag, Berlin*, **18** (1986), 9–43.
18. GRAVES, S.C. AND KLETTER, D.B. AND HETZEL, W.B., A dynamic model for requirements planning with application to supply chain optimization. *Operations Research*, **46** (1998), S35–S49.
19. GÜLLÜ R., On the Value of Information in Dynamic Production/Inventory Problems Under Forecast Evolution. *Naval Research Logistics*, **43**, (1996), 289-303.
20. GÜLLÜ R., A Two-Echelon Inventory Model and the Value of Information under Correlated Forecasts and Demands. *European Journal of Operational Research*, **99** (1997), 386-400.
21. HARIHARAN, R. AND P. ZIPKIN, Customer-order Information, Leadtimes and Inventories. *Management Science*, **41** (1995), 1599-1607.
22. HEATH, D.C. AND JACKSON, P.L., Modeling the evolution of demand forecasts with application to safety stock analysis in production/distribution systems. *IIE Transactions*, **26** (1994), 17–30.
23. IRAVANI, S.M.R. AND LIU, T. AND LUANGKESORN, KL AND SIMCHI-LEVI, D., A produce-to-stock system with advance demand information and secondary customers. *Naval Research Logistics* , **54**, (2007), 331–345.
24. KAMINSKY, P. AND SWAMINATHAN, J.M., Utilizing forecast band refinement for capacitated production planning. *Manufacturing & Service Operations Management*, **3** (2001), 68–81.
25. KAMINSKY, P. AND SWAMINATHAN, J.M., Effective heuristics for capacitated production planning with multiperiod production and demand with forecast band refinement. *Manufacturing & Service Operations Management*, **6** (2004), 184–194.
26. KARAESMEN, F., J.A. BUZACOTT AND Y. DALLERY, Integrating Advance Order Information in Production Control. *IIE Transactions*, **34** (2002), 649-662.
27. KARAESMEN, F. AND LIBEROPOULOS, G. AND DALLERY, Y., The value of advance demand information in production/inventory systems, *Annals of Operations Research*, **126**, 2004, 135–157.
28. KARAESMEN, F. AND LIBEROPOULOS, G. AND DALLERY, Y., Production/inventory control with advance demand information. In: *Stochastic Modeling and Optimization of Manufacturing Systems and Supply Chains*, D.D. Yao, J.G. Shanthikumar and W.H.M Zijm (eds.), Kluwer Academic Publishers, 2003.
29. KARAESMEN, F., Inventory systems with advance demand information and random replenishment times. *Proceedings of the 4nd Aegean International Conference on the Analysis of Manufacturing Systems*, (2003), 1–4.
30. KIM, J. AND AHN, H. AND RIGHTER, R., Optimal production policies with multistage stochastic demand lead times, *Probability in the Engineering and Informational Sciences*, **23**, (2009), 515–543.
31. KOÇAĞA, Y.L. AND ŞEN, A.,Spare parts inventory management with demand lead times and rationing, *IIE Transactions*, **39**, (2007), 879–898.
32. KUNNUMKAL, S. AND TOPALOGLU, H., *International Journal of Production Economics*, **111** (2008), 543–561.

33. LIBEROPOULOS, G. AND KOUKOUMIALOS, S., Tradeoffs between base stock levels, numbers of kanbans, and planned supply lead times in production/inventory systems with advance demand information, *International Journal of Production Economics*, **96** (2005), 213–232.
34. LIBEROPOULOS, G. AND TSIKIS, I., Multi-Stage Control with Lot Sizing and Advance Demand Information. In: *Stochastic Modeling and Optimization of Manufacturing Systems and Supply Chains*, D.D. Yao, J.G. Shanthikumar and W.H.M Zijm (eds.), Kluwer Academic Publishers, 2003.
35. LIBEROPOULOS, G., On the tradeoff between optimal order-base-stock levels and demand lead-times, *European Journal of Operational Research*, **190** (2008), 136–155.
36. LU, Y. AND SONG, J.S. AND YAO, D.D., Order fill rate, leadtime variability, and advance demand information in an assemble-to-order system, *Operations Research*, **190**, (2003), 292–308.
37. MARKLUND, J., Controlling inventories in divergent supply chains with advance-order information, *Operations Research*, (2006), 988–1010.
38. MILGROM P. AND J. ROBERTS, Communication and Inventory as Substitutes in Organizing Production, *Scandanivian Journal of Economics*, **90** (1988), 275-289.
39. ÖZER, Ö, Replenishment strategies for distribution systems under advance demand information, *Management Science*, **49**, (2003), 255–272.
40. ÖZER, Ö. AND WEI, W., Inventory control with limited capacity and advance demand information, *Operations Research*, **58**, (2004), 988–1000.
41. SCHOENMEYR, T. AND GRAVES, S.C., Strategic safety stocks in supply chains with evolving forecasts, *Manufacturing & Service Operations Management*, **11** (2009), 657–673.
42. TAN, T. AND GÜLLÜ, R. AND ERKIP, N., Modelling imperfect advance demand information and analysis of optimal inventory policies , *European Journal of Operational Research*, **117** (2007), 897–923.
43. TAN, T., Using imperfect advance demand information in forecasting, *IMA Journal of Management Mathematics*, **19** (2008), 897–923.
44. TAN, T., Using imperfect advance demand information in ordering and rationing decisions, *International Journal of Production Economics*, **121** (2009), 665–677.
45. THONEMANN, U.W., Improving supply-chain performance by sharing advance demand information, *European Journal of Operational Research*, **142** (2002), 81–107.
46. TOKTAY, L.B. AND L.M. WEIN, Analysis of a Forecasting-Production-Inventory System with Stationary Demand. *Management Science*, **47** (2001), 1268-1281.
47. UÇKUN, C. AND KARAESMEN, F. AND SAVAŞ, S., Investment in improved inventory accuracy in a decentralized supply chain. *International Journal of Production Economics*, **113** (2008), 546–566.
48. WANG, T. AND TOKTAY, B.L., Inventory management with advance demand information and flexible delivery. *Management Science*, **54** (2008), 716–732.
49. WIJNGAARD, J., The effect of foreknowledge of demand in case of a restricted capacity: The single-stage, single-product case. *European Journal of Operational Research*, **159** (2004), 95–109.
50. WIJNGAARD, J. AND KARAESMEN, F., Advance demand information and a restricted production capacity: on the optimality of order base-stock policies. *OR Spectrum*, **29** (2007), 643–660.
51. ZHU, K. AND THONEMANN, U.W., Modeling the benefits of sharing future demand information. *Operations Research*, **20**, (2004), 136–147.
52. ZIPKIN, P.H., *Foundations of Inventory Management*, McGraw-Hill, (2000).