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Suggested Citation: Caner Canyakmaz, Fikri Karaesmen and Süleyman Özekici (2017), "Minimum-Variance Hedging for Managing Risks in Inventory Models with Price Fluctuations", Foundations and Trends® in Technology, Information and Operations Management: Vol. XX, No. XX, pp 1–13. DOI: XXX.

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Minimum-Variance Hedging for Managing Risks in Inventory Models with Price Fluctuations

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ABSTRACT

We consider the financial hedging of a random operational cash flow that arises in inventory operations with price and demand uncertainty. We use a variance minimization approach to find a financial portfolio that would minimize the total variance of operational and financial returns. For inventory models that involve continuous price fluctuations and price-dependent demand that arrives in continuous time, we characterize the minimum-variance hedging policies and numerically illustrate their effectiveness.

0.1 Introduction

We consider a firm that is running inventory operations in an environment with input and/or output price uncertainty in addition to uncertain demand. There are significant risks in such an environment and effective risk management is essential.

Financial hedging is one way of managing inventory operations risks under price and demand volatility. The objective is to manage downside operational risks by keeping a portfolio of financial instruments to

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exploit possible correlations with the operational cash flows. We focus on such a formulation where the random operational cash flow is given and the goal is to find a an appropriate financial portfolio that would enable better controlling downside risks. In particular, we investigate a minimum-variance approach to determine an optimal financial portfolio that minimizes the variance of the total (operational + financial) returns. Although there are other approaches for risk management, the minimum-variance approach leads to intuitive and explicit solutions even for fairly complicated operational cash flows.

Financial hedging of operational risks is a rapidly growing topic in the operations literature. We will not attempt a full review of this literature here but briefly introduce some recent papers. Gaur and Seshadri (2005) consider hedging demand risk in a newsvendor problem. They characterize optimal hedges in case of both perfect and partial correlations. Caldentey and Haugh (2006) introduce a continuous-time modeling framework for dynamically hedging operational risks. In a multi-period setting, Kouvelis *et al.* (2013) investigate an inventory system where a volatile spot market is used for immediate procurement and liquidation alongside with long-term procurement contracts. The authors characterize optimal procurement and portfolio decisions with and objective of maximizing inter-period mean-variance utility by investing in financial securities written on the commodity price.

Our operational focus is on price fluctuations and their effects on inventory operations. This is a well-investigated problem in operations management literature. To mention a few papers, Kalymon (1971) investigates a periodic-review inventory system where the purchase price follows a Markov process. Berling and Martınez-de-Albéniz (2011) consider a continuous-review system with Poisson demand arrivals and input prices following a two-factor price process. Matching each unit bought with a demand arrival, they characterize base-stock levels as a series of two thresholds and analyze the effect of price evolution on optimal policy. Haksöz and Seshadri (2007) review supply chain systems that involve spot market procurements where spot prices fluctuate randomly. Inderfurth and Kelle (2011) analyze the management of two alternative procurement sources, capacity reservation and spot markets.

In Section 0.2, starting from more simpler models, we analyze minimum-variance hedging applications on different types of cash flows with a consideration of continuous price fluctuations at the end. In Section 0.4, we present a numerical study that illustrates the effects of variance-minimizing investments on risk reduction. Lastly, in Section 0.5, we give some concluding remarks and perspectives.

0.2 Minimum-Variance Hedging

In this section, we demonstrate the general minimum variance approach in a case where a firm invests in an operational project at time $t = 0$ which has random return X at the end of the investment horizon $t = T$. The firm can also choose to invest in a financial asset whose return S at the end of the horizon is random and proportional to the invested amount α. The total return from the asset at time T is therefore αS. To simplify the investment problem, let us further assume that $E[S] = 0$. Therefore, $E[X + \alpha S] = E[X]$ and investing in the asset in addition to the operational project does not improve expected total returns. On the other hand, investing in the asset S does alter the variance of the total return $Var(X + \alpha S)$. We use this simple setup to benefit from the investment in the financial asset to minimize the variance of the return if X and S have non-zero correlation. Let us denote by $Cov(X, S)$ the covariance between X and S and by $\rho_{X,S}$ the corresponding correlation coefficient. Then, we solve the following optimization problem:

$$\min_{\alpha} \; Var(X + \alpha S).$$

This results in:

$$Var(X + \alpha S) = Var(X) + \alpha^2 Var(S) + \alpha Cov(X, S).$$

This is a convex function and the minimizing value of α is given by:

$$\alpha^* = -\frac{Cov(X, S)}{Var(S)}. \tag{1}$$

α^* is the optimum investment amount in the financial asset that enables the smallest variance of the total return. We can then characterize the reduction in variance between the unhedged operational cash flow X, and the optimal hedged cash flow $X + \alpha^* Y$:

$$\Delta = Var(X) - Var(X + \alpha^* S) = \rho^2_{X,S} Var(X)$$

and the relative reduction in variance with respect to the unhedged cash flow is

$$\Delta_R = \Delta / Var(X) = \rho^2_{X,S}. \tag{2}$$

Observing the characterization given in (1), it is clear that if there is a negative correlation between the random operational payoff and external investment yield, then one should buy α^* units of S. Similarly, in the case of positive correlation, one should shortsell α^* units of S, if possible, in order to minimize the cash flow variance. We also note from (2) that the relative reduction in variance is completely characterized by $\rho^2_{X,S}$. In the extreme case of perfect positive or negative correlation between X and S, i.e., $\rho_{X,S} = 1$ or $\rho_{X,S} = -1$, one can achieve a 100% reduction in variance.

The idea of minimum-variance hedging can also be extended to the case where there are multiple external investment opportunities. Let us assume that there are n of them available. Defining the net returns of these investments by the column vector $S = \{S_1, S_2, ..., S_n\}$ and investment amounts by the column vector $\alpha = \{\alpha_1, \alpha_2, ..., \alpha_n\}$, one can write the objective function as

$$\min_{\alpha} \; Var(X + \alpha^T Y) = Var(X) + \alpha^T C \alpha + 2\alpha^T Cov(X, S) \tag{3}$$

where C is the covariance matrix between net returns from investment options and $Cov(X, S)$ is the covariance vector of operational return X and investment returns. Note that objective function given in (3) is also convex in α (C is always semidefinite) and the variance-minimizing investment amounts are given by the vector

$$\alpha^* = -C^{-1} Cov(X, S). \tag{4}$$

It is clear from (4) that no other information than the covariance matrix of external investment returns and the covariance vector between operational and external returns is needed to calculate the hedge that minimizes the variance of the final cash flow.

0.3 Minimum-Variance Hedging in Inventory Operations

0.3.1 Newsvendor Model

The minimum-variance hedging framework can easily be applied to settings involving inventory operations. To demonstrate the application, we first consider the case of a newsvendor who buys items at c per unit at $t = 0$ and sells them at a per unit selling price of p. Demand D is realized at time T. The well-known newsvendor problem is to determine the order quantity in order to maximize the expected net cash flow (or possibly some other function related to the cash flow). For our purposes, we focus on the cash flow for a given order quantity. Assuming a unit penalty cost of b, the random return as a function of ordering decision y can be written as

$$CF(y, D) = -cy + p\min(D, y) - b(D - y)^+. \tag{5}$$

Rearranging the terms, one can write the covariance between the return from operations $CF(y, D)$ and return from ith alternative investment S_i as a function of operational decision y as

$$\mu_i(y) = (b + p)Cov(\min(D, y), S_i) - bCov(D, S_i).$$

Then, following (4), the minimum-variance hedge as a function of y can be written as

$$\alpha^*(y) = -C^{-1}\mu(y) \tag{6}$$

where $\mu(y) = \{\mu_1(y), ..., \mu_n(y)\}$ is a column vector.

The above expression characterizes the variance minimizing portfolio for a newsvendor. More details on the newsvendor application of minimum-variance hedging framework can be found in Okyay *et al.*, 2014 who also investigate additional cases of random supply.

0.3.2 Hedging Fluctuating Price and Demand Risks

One can also utilize minimum-variance hedging on more complicated operational settings that involve both demand and price risks. To illustrate this, assume now that the demand is not realized only at T, but rather it is generated by an arrival process that is affected by the random prices until time T. Let us assume that there is a continuous price process $P = \{P_t; t \in [0, T]\}$ which describes the purchase price for the item and affects sales prices. More specifically, assume that $c = P_0$ is the purchase price and the item is sold to nth arriving customer at a unit price of $f(P_{T_n})$ where f is a positive selling price function. Unit demand arrives continuously according to a Poisson process with rate $\lambda(P_t)$ and in case of shortage, the customers are backordered to be satisfied at time T and the selling price is set and paid at the time of customer arrival. Let us denote the arrival process as $N = \{N_t; t \in [0, T]\}$ where N_t denotes the number of customers that arrived until time t and let $\mathcal{T} = (T_{1,}.., T_{N_T})$ denote the corresponding arrival times. Assuming, without loss of generality, that there is no time value of money, one can write the cash flow accumulated until time T as

$$CF(y, N, P) = -cy + \sum_{j=1}^{N_T} f(P_{T_j}) - (b + P_T)(N_T - y)^+ \tag{7}$$

where the first term is the total purchase cost for y units, the second term is the total accumulated revenue from sales and the last term is the total backorder and repurchase cost. Note that the newsvendor cash flow given (5) is a special case of (7) where $P_t = p$ and $f(.) = p$.

The apparent risks involved in firm's cash flow in (7) arise from randomly fluctuating prices and random customer arrival times (i.e., random demand). In this setting, the risk that the firm bears spans the entire horizon and a minimum-variance hedge should also reflect this as it exploits all existing correlations. Here we assume that given the random prices during $[0, T]$, customer arrival process is conditionally independent from S, random return of investment alternatives. In this case, the minimum-variance hedge is given by (6) and the covariance of returns from inventory activities and ith investment alternative is

$$\mu_i(y) = \int_0^T Cov(f(P_u)\lambda(P_u), S_i)du - Cov\left((b + P_T)(N_T - y)^+, S_i\right). \tag{8}$$

Note that continuous price fluctuations and customer arrivals responding to these changes throughout the sales horizon is reflected in the result given in (8), which considers possible correlations at all points in $[0, T]$.

There are several advantages in utilizing the minimum-variance framework in general operational settings. Intrinsically, the financial hedge, quantified by α is dependent on the operational decision y but the operational decision which may depend on other longer term factors does not depend on the financial portfolio. This underlines the fact that the operation is the main focus and know-how of the firm and the financial hedge is a support to the operation and may be provided separately if operational parameters are shared. We can then further specialize to explore different trade-offs. In particular, one consistent benchmark is to take operational decisions that maximize the expected unhedged cash flows $E[CF(y, N, P)]$ and find the corresponding optimal financial hedge.

Next, we investigate a special application involving futures, one of the most extensively used financial contracts for risk hedging.

Special Case: Hedging with Futures

Assume that S is a fairly priced future on P_T with random return $S = P_T - P_0$ (Baxter and Rennie, 1996). For the operational setting, assume that customer arrival process N is independent from the price process P with $\lambda(P_t) = \lambda$. This, in turn, implies that N and S are independent which means that only price-related risks can be reduced by investing in the future while the demand risk remains unchanged. By (1) and (8), one can calculate the minimum-variance hedge in this particular case by

$$\alpha^*(y) = E\left[(N_T - y)^+\right] - \lambda \int_0^T \beta_t dt \tag{9}$$

where

$$\beta_t = \frac{Cov\left(f\left(P_t\right), P_T\right)}{Var\left(P_T\right)}.$$

Note that (9) characterizes the optimal number of future contracts to be bought or sold in order to minimize the variance of the final cash flow when the order-up-to level is y at time 0. The two components in α^* respectively hedge the random repurchase cost and the total revenue from sales in (7). The decision maker should take a $E\left[(N_T - y)^+\right]$ units of long position on the future in order to eliminate the price risk in the repurchase of backordered items. One can not fully eliminate the risk associated with $(N_T - y)^+$ by investing in the future, since N is independent of S. The last term, on the other hand, partially eliminates the price related risks in the operational cash flow as only a single future contingent on P_T is used for hedging, whereas the revenue term is affected by price changes during $[0, T]$. Moreover, it is observed from (9) that the higher the order level y, the lower α^* since total backorders, hence the repurchase risk decreases in order quantity.

0.4 Numerical Analysis

In this section, we present some numerical illustrations that reveal the extent of risk reduction on the random operational cash flow when alternative correlated external investments are used. In the following numerical setup, we use a geometric Brownian motion process to model sudden and continuous price changes. For the investment alternatives, we use two different derivatives contingent on the value of P_T, which are a future and a European call option. A unit of future bought at time 0 with maturity date T gives a payoff of $S_1 = P_T - P_0$ at time T. On the other hand, a call option with strike price K and maturity date T yields the net payoff $S_2 = (P_T - K)^+ - E_{\mathcal{Q}}\left[(P_T - K)^+\right]$ where the former term is the payoff obtained from exercising the option and the latter is the fair-price of this call option at time 0. Note that $\mathcal{Q}$ is the equivalent martingale measure (risk-neutral measure) of price process P. To simplify, (and to isolate the effect of price volatilities), we assume that P is already a martingale, i.e., it does not yield a positive (or negative) payoff in expectation. In particular, we assume that

$$P_t = P_0 e^{-\frac{1}{2}\sigma^2 t + \sigma\sqrt{t} W_t}$$

where W is a Wiener process with $E[W_t] = 0$ and $Var(W_t) = t$. Note that $E(P_t) = P_0$ for all $t > 0$. We use the following operational and financial parameters. $P_0 = c = 20$, $f(p) = 2p$, $b = 4$, $K = 20$ and next we will use various volatility σ values. For the customer arrivals, we assume a piecewise linear arrival rate function $\lambda(x) = (A - Bx)^+$, where $A = 90$ and $B = 0.7$.

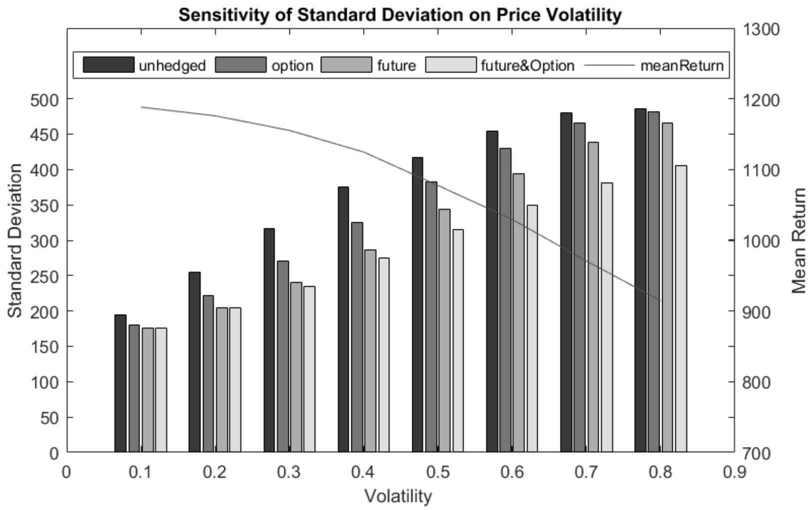

Figure 1: Effect of Price Volatility on Risk Reduction

With the above parameters, we analyze the effect of using a future and/or and option on cash flow risk with respect to magnitude of price volatility. For a consistent benchmark, we use ordering quantities that maximize the expected total unhedged cash flow, i.e., y^*. In Figure 1, it is observed that using the future yields a greater reduction on the standard deviation of the cash flow compared to using the call option with strike price $K = 20$ for all price volatility component values ranging from $\sigma = 0.1$ to $\sigma = 0.8$. It is also clear that the greatest risk reduction is observed when simultaneously investment takes place in both the future and the option. Note that since the underlying price process P is assumed to be a martingale, both derivatives are also martingales and fair-priced which means that they do not add anything on the expected value of the cash flow. Yet, we observe that the mean return of the cash flow is decreasing with respect to price volatility due to changes in the value of $E\left[P_t\lambda(P_t)\right]$ and the changes in the expectation-maximizing ordering quantity y^* with respect to the magnitude of price fluctuations. Note that the main driver of the decrease in mean return is that any price increase which makes $\lambda(P_t) = 0$ implies that demand arrivals halt and this is more likely to occur when prices fluctuate more.

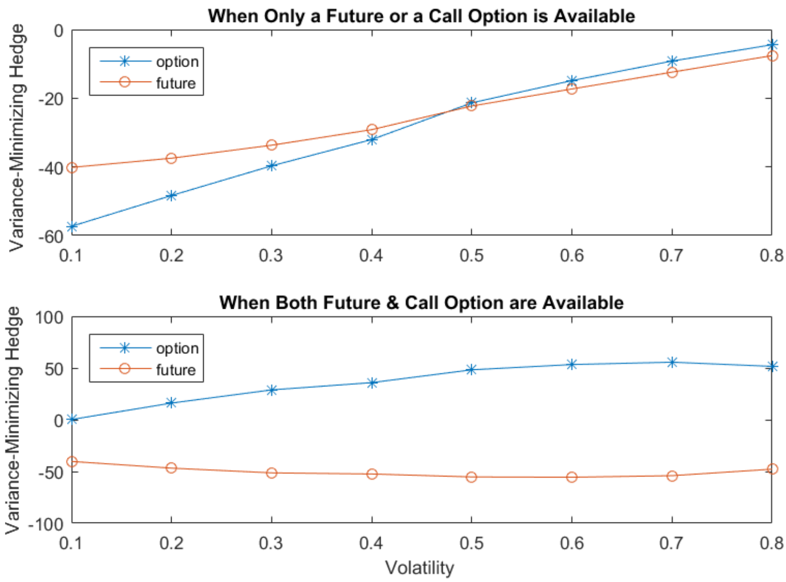

Figure 2: Sensitivity of Minimum-Variance Hedges to Price Volatility

Figure 2 shows the optimal amount securities to buy (or shortsell) to minimize the cash flow variance for both cases where there is simultaneous use of the future and the call option or not. When either of them is used, it is observed that it is optimal to shortsell a specific amount of these securities where the optimal number of transactions increase as prices get more volatile. When investing in both of them at the same time, we observe that opposite financial positions are taken on these two investment alternatives. In particular, it is optimal to shortsell a specific amount of future while buying a specific amount of the call option at the same time. The number of transactions made on both of these securities increase as price volatility increases.

0.5 Conclusion and Perspectives

In this paper, we formulated minimum-variance hedging problems and characterized optimal hedges for demand and price risks in inventory problems. Optimal hedges reflect the effect of price changes on the cash flow throughout the sales horizon. In a numerical experiment, it is observed that using a future is a more effective tool than using a call option and amount of the optimal hedges decrease as price volatility increases. If both securities are available at the same time, then greatest reduction in risk is observed by taking opposite financial positions.

This discussion also reveals that a nice feature of the minimum-variance formulation is that it leads to a structured optimization problem whose solution can be obtained explicitly even for complicated price and demand processes. More precisely, it provides a tractable characterization that works for any operational decision (i.e., ordering quantity). This idea can also be extended to the case where the decision maker has dynamic hedging opportunities to revise his initial investment over time by observing newly acquired information on price and demand. This case is presented in Canyakmaz *et al.*, 2017 where the authors investigate dynamic financial hedging for both single and multi-period inventory systems involving continuous price fluctuations.

It is also useful to contrast the minimum-variance approach with the well known mean-variance optimization objective that investigates the trade-offs between the expected payoff and its variance. For a complete understanding of the mean-variance type risk trade-off, one needs to trace the efficient frontier of non-dominated solutions. By definition, the minimum variance approach yields minimum variances for each operational policy. If the operational policy space is small and structured (such as order-up-to or base-stock policies), one can computationally explore non-dominated mean-variance policies and trace the efficient frontier by searching over this space for all minimum variance policies.

This line of research can be also be extended to operational settings where sales prices are also affected by the firm's decisions. An interesting case is the consideration of both financial hedging and sales markup decisions on top of fluctuating prices for different objective functions. Another possible direction is to consider the case where the decision maker has a budget constraint when investing in external alternatives.

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