

**CALL CENTER OUTSOURCING CONTRACT ANALYSIS
AND CHOICE**

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Abstract

This paper considers a call center outsourcing contract analysis and choice problem, faced by a contractor and a service provider. The service provider receives an uncertain call volume over multiple-periods, and is considering outsourcing all or part of these calls to a contractor. Each call brings in a fixed revenue to the service provider. Answering calls requires having service capacity, thus implicit in the outsourcing decision is a capacity decision. Insufficient capacity implies that calls cannot be answered, which in turn means there will be a revenue loss. Faced with a choice between a volume-based and a capacity-based contract offered by a contractor who has pricing power, the service provider determines optimal capacity levels. The optimal price and capacity of the contractor together with the optimal capacity of the service provider determine optimal profits of each party under the two contracts being considered. Each party will prefer the contract that leads to higher profits. The paper characterizes optimal capacity levels, and partially characterizes optimal pricing decisions under each contract. The impact of demand variability and economic parameters on contract choice are explored through numerical examples. It is shown that no contract type is universally preferred, and that operating environments as well as cost-revenue structures have an important effect in outsourcing contract design and choice.

Keywords: call center; outsourcing; subcontracting; contract choice; capacity investment; exogeneous and endogeneous price.

1 Introduction

A growing number of companies outsource their call center operations. According to International Data Corporation (1999), the worldwide call center services market totalled \$23 billion in revenues in 1998, and is estimated to double to \$8.6 billion by 2003. Outsourcing constitutes 74% of this market and is projected to be \$42 billion in 2003. Datamonitor (1999) expects call center out-

sourcing to boom in Europe, where the \$7 billion market in 1999 is expected to grow to \$15 billion by 2004. In terms of outsourced agent positions, this constitutes a growth from 74,000 in 1999 to 126,500 in 2003.

While for some companies outsourcing the entire call center operation constitutes the best option, many are hesitant to hand over their most important source of customer contact to another firm. In-sourcing to a shared services organization (Akşin and Masini, 2005) is one alternative approach. Another practice known as co-sourcing (Fuhrman, 1999), is one where some calls are kept in-house while others are outsourced. Co-sourcing may be preferred because of the additional safety it provides in case of a disaster, or because only the strategically less important calls are outsourced to a third party. The decision of how to share calls in a co-sourcing setting is an important one. In practice, this can take many forms where for example certain types of calls are outsourced and others are kept in-house, or overflow calls in all functions are outsourced. In the former type, the decision is driven by strategic considerations while in the latter it is mostly an economic decision. In this paper we will focus on the latter type of sharing where only economical issues are taken into consideration. Whatever the chosen form of sharing, detailed outsourcing contracts that specify requirements, service levels, and price are deemed necessary for success.

This paper is motivated by the call center outsourcing problem of a major mobile telecommunications service provider (operator) in Europe. The overall objective is to evaluate two types of contracts made available to this company by a contractor. In making this comparison, detailed contracts that specify capacity and price within each type of contract will be considered. Like in most call center outsourcing situations, the contractor operates a much larger call center operation compared to the one by the operator. This allows the contractor to have an advantageous cost structure and power in negotiating prices. The telecommunications service provider opted for a co-sourcing solution. Accordingly, some call types would be kept in-house while others would be shared with a contractor. The calls that are kept in-house are those of the most valuable customers, and strategic considerations have led to the in-sourcing decision for these calls. This paper is only concerned with the sharing of the remaining calls. The economic implications of the contract will determine what proportion of these calls will be co-sourced, if at all. In other words, it is possible that under economic rationality all of these calls or none of them can be outsourced. The contractor proposed two forms of sharing. The first type, which we label as *Contract 1* or as *subcontracting the base*, is a form of capacity reservation whereby the company reserves enough capacity for a steady level of calls at the contractor for a given fee. All calls in excess of this level are considered for

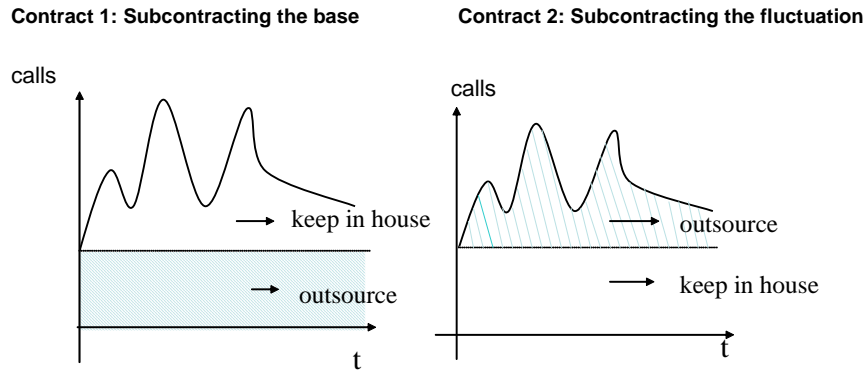


Figure 1: The two contracts offered by the contractor

treatment in-house. One could also name this as a *pay for capacity* contract. The second type of contract, labeled as *Contract 2* or as *subcontracting the fluctuation*, stipulates that the telecommunication service provider answers all calls up to a specified level in-house, beyond which calls are diverted to the contractor. In other words, in this type of contract overflow calls are outsourced. The contractor charges a fee per call treated. These two cases are illustrated in Figure 1. This contract could be labeled as a *pay for job* contract. In both settings, the contractor has pricing power. Both parties try to maximize their own profits.

While we are motivated by this particular instance, we note that the problem is common to call center outsourcing in general. As also noted by Gans and Zhou (2004), call center outsourcing contracts are typically volume-based or capacity-based. Some contracts involve payment for capacity that is utilized, as in our Contract 2. Others have payment for capacity irrespective of whether it is utilized or not, as in our Contract 1. In order to model the contract choice problem, we specify the contracts in more detail, using our motivating instance to do this.

For the service outsourcing problem, we analyze the optimal capacity and pricing decisions under each contract type. We then explore the telecommunication service provider's basic question, namely which contract type they should prefer. Current practice at the company is to co-source with a contract of the form *subcontracting the base*. While basic operations management intuition would suggest that keeping the less variable portion of the demand in-house and outsourcing the high variation overflow would be more beneficial, we illustrate that both contract types may be preferred, depending on economic parameters and demand characteristics. Our analysis further highlights the importance of modeling outsourcing contracts as multiple period problems. While subcontracting the base results in complete outsourcing in the single period case, we find that

co-sourcing can be optimal once demand fluctuations across multiple time periods are considered. This difference also impacts the resulting contract choice. In the following section, we provide a brief literature review. Section 3 formulates the model. This is then analyzed in order to determine optimal service capacities in Section 4 and the optimal prices in Section 5. In Section 6 we discuss some model extensions. Section 7 presents a numerical study to illustrate the relationships between contract parameters, demand characteristics, and contract preferences of each party. We provide concluding remarks in Section 8.

2 Literature Review

The problem of outsourcing or subcontracting has been studied in the economics literature in the context of vertical integration. This literature does not consider capacity constraints. Kamien et al. (1989), Kamien and Li (1990) first model capacity constraints, either implicitly or explicitly, in the context of subcontracting production. Kamien and Li (1990) consider two firms that are in the same market and that can subcontract from each other. In this regard, both of their firms resemble our operator firm. The firms consider a production planning problem with a subcontracting option. Rather than analyzing the performance of specific contracts like we do, they consider coordinating contracts. One of their most important results is that optimal production and inventory quantities are less variable in the presence of subcontracting. In our setting, the possibility of outsourcing or co-sourcing allows the operator firm to smooth its capacity levels between periods.

The supply chain literature provides a rich set of models that address supply contract design and analysis, where capacity is explicitly taken into account as a decision variable. See for example Tsay et al. (1998), Lariviere (1998), Anupindi and Bassok (1998), and references therein. As also noted by Van Mieghem (1999), these models typically consider only one party's capacity investment decision. Similar to Van Mieghem (1999), the capacity investment of both the call center and the contractor are decision variables in our setting. We further consider the price as a decision variable for the contractor.

Van Mieghem (1999) and many of the papers in the supply contracts literature consider the outsourcing problem in a single-period setting. The nature of the call center outsourcing problem requires a multi-period analysis, as is the case in this paper. A continuous time analysis of subcontracting, in a manufacturing context, can be found in Tan (2002), which addresses the demand variability inherent in a multi-period problem. Subcontracting is considered as a capacity option

whose value is evaluated in the presence of demand uncertainty. Contract parameters are assumed to be exogenously specified, and the problem is analyzed from the standpoint of a single decision maker. Atamturk and Hochbaum (2001) provide a multi-period treatment of subcontracting, and use call centers as one of their motivating examples. In their paper, demand is deterministic, price is determined exogenously, and the capacity investment decision is once again only considered for a single party.

We consider a multi-period model of outsourcing, with multiple decision makers, in the presence of uncertain demand. Capacity investment levels of each firm and the outsourcing price are the decision variables. As such, our model combines various features found separately in previous subcontracting models in the literature. Our analysis demonstrates that for *Contract 1*, in a single period setting, the optimal decision is one where the call center outsources all calls to the contractor. Given this result, in such a setting, our model can be analyzed as a single party capacity investment decision problem, like in Atamturk and Hochbaum (2001) and Lariviere and Porteus (2001).

Cachon and Harker (2003), Ren and Zhou (2004), Allon and Federgruen (2005) also consider outsourcing contracts in a service setting. Cachon and Harker (2003) analyze a queueing game between two service providers. The option of outsourcing to a contractor is one of the alternatives considered in comparing different supply chain designs for the service providers. In their outsourcing contract, the contractor charges a price per customer and ensures a service level, while the service providers guarantee a minimum demand rate to the contractor. This resembles Contract 2, though service levels and demand sharing is implicitly determined through each party's optimal capacity decisions in our setting. Cachon and Harker (2003) and Allon and Federgruen (2005) are concerned with the competition between service providers, while we are interested in the specifics of the relationship between a single service provider and contractor. Ren and Zhou (2004) analyze call center outsourcing contracts that can coordinate staffing and service quality decisions. The fluid approximation of a queueing system is used to model the call centers. The analysis only considers a single period outsourcing problem. Both Ren and Zhou (2004) and de Véricourt and Zhou (2005) consider the implications of service quality. This aspect of the problem is not treated herein.

In addition to differences in modeling assumptions, we also ask a different question compared to earlier papers in the operations literature. Our ultimate objective is to answer the contract choice problem faced by the telecommunications service provider, as described in the Introduction. The issue of contract choice is one which has been dealt with extensively within contract theory in economics. Bajari and Tadelis (2001) explore the choice between fixed price and cost-plus

procurement contracts in the construction industry. In a fixed price contract, the seller is offered a fixed price for project completion. This resembles Contract 1 where a fixed capacity reservation fee is paid. In a cost-plus contract, the seller is reimbursed for costs. Uncertainty affects the project completion cost, thereby affecting what the buyer will pay. A cost-plus contract resembles Contract 2 in that the actual payment is determined by the volume of calls that are eventually outsourced. The source of uncertainty is the randomness in demand. However if call volumes exceed total capacity, the operator loses revenue. As such, in this case the revenue of the operator is also affected. Different types of demand uncertainty, the fact that both revenue and cost are affected by these, and the assumption that the contractor has pricing power are the major differences of our analysis from theirs. We next point out some recent papers that deal with IT outsourcing problems. Gopal et al. (2003) perform an empirical analysis of offshore software development, where the choice between fixed price and time-and-materials type contracts are explored. It is shown that among other things, this choice is driven by requirements uncertainty. This resembles the demand uncertainty in our case. Kalnins (2004) explores empirically the role of firm relationships in choosing between these types of contracts. In our analysis, we will explore the role of demand uncertainty and economic parameters on such a contract choice, using a modeling approach.

Unlike most call center models, as reviewed in detail by Gans et. al. (2002), we do not model the call center as a queue in this paper. This choice is in part driven by tractability concerns, since embedding a queueing model in the games we analyze would not allow us to pursue the contract design and choice problem fully. However, we feel that this choice is not inappropriate since these types of outsourcing contract decisions are strategic decisions which would not be affected fundamentally by waiting or abandonment behavior of customers. This is illustrated in the model extensions section where we show that our results are robust to the no queueing assumption, making use of a fluid approximation as in Ren and Zhou (2004). Queueing models become necessary to analyze detailed implementations of call center outsourcing contracts (Gans and Zhou, 2004; Milner and Lennon, 2005). Viewing the call center outsourcing problem at the operational level, Gans and Zhou (2004) analyze the routing problem faced between a service provider and contractor. We do not consider this aspect of the problem herein.

3 Formulation of the Model

We consider a call center operator (also referred to as a service provider) (B) which can send part or all of the calls it receives to a contractor (A). Calls arrive to the operator during N different periods, which represent parts of a typical day, like half-hour intervals. We assume that the number of calls in period t is a real random variable D_t , characterized by the density $f_t(\cdot)$, for $t \in \{1, \dots, N\}$. $F_t(\cdot)$ denotes the corresponding cumulative function, and has an inverse $F_t^{-1}(\cdot)$. We will also use $G_t(\cdot) := 1 - F_t(\cdot)$ and $\tilde{G}_t(\cdot) := \sum_{\tau=1}^t G_\tau(\cdot)$, where $t \in \{0 \dots N\}$ with $\tilde{G}_0(x) := -\infty$.

A contract specifies how the arriving calls are distributed between the operator and the contractor. Calls that are not answered by either the operator or the contractor are lost. On the other hand, an answered call brings a revenue of r per unit to the operator, even when that call was handled by the contractor. In the following, we analyze two types of contracts.

The operator chooses a service capacity level K_t^B for each period t . Similarly K_t^A denotes the service capacity of the contractor for period t . We define c^B and c^A , the unit costs of the service capacity level per period for the operator and the contractor respectively. It is assumed throughout that $c^A < c^B$. In the first type of contract, all the K_t^A 's, $t \in \{1, \dots, N\}$ are equal to a unique capacity K^A which is fixed by the operator, while in the second one, these levels are chosen by the contractor. We denote by $K_t := K^A + K_t^B$, the total service capacity of the system in period t . All parameters are common knowledge. In general outsourcing settings, true cost information may be difficult to obtain. However in call centers where around seventy percent of costs are personnel costs, we assume that knowledge of the local labor market will enable an estimation of true costs, even if this information is not directly shared.

3.1 Contract 1: Subcontracting the Base

In this contract, the operator specifies the capacity level K^A of the contractor. This capacity level will remain constant during the day, namely $K_t^A = K^A$ for $t \in \{1, \dots, n\}$. In turn, the contractor charges a capacity reservation price γ per unit of capacity and per period. This contract is only attractive to the operator if $c^B > \gamma$. Otherwise the operator keeps all capacity in-house.

The operator chooses K^A and K_t^B , $t \in \{1, \dots, N\}$ in order to maximize its total profit π^B , which is equal to,

$$\pi^B = \sum_{t=1}^N \pi_t^B \tag{1}$$

where π_t^B is the profit for period t given by:

$$\pi_t^B = rE[\min(D_t, K_t)] - c^B K_t^B - \gamma K^A. \quad (2)$$

The corresponding total profit of the contractor is equal to,

$$\pi^A = N(\gamma - c^A)K^A. \quad (3)$$

For the time being γ can be regarded as an exogenously determined contract parameter. Section 5 focuses on the case where the contractor sets γ in order to maximize its profit π^A .

3.2 Contract 2: Subcontracting the fluctuation

In this contract, the operator sends all calls it cannot answer to the contractor. The contractor charges a unit price p per answered call. Calls that are not handled by the contractor do not incur any payment. This contract will lead to the outsourcing of some calls by the operator only if $r > p$ since the firm has no interest to outsource calls otherwise.

Hence, in period t , the operator first tries to saturate his service capacity K_t^B , before sending calls to the contractor. In other words, the number of calls D_t^B received by the operator is equal to $\min(D_t, K_t^B)$. The corresponding number D_t^A received by the contractor is then equal to $(D_t - K_t^B)^+$ where $(x)^+ = \max\{0, x\}$.

The operator chooses K_t^B , $t \in \{1, \dots, N\}$ in order to maximize its total profit, which is equal to,

$$\pi^B = \sum_{t=1}^N \pi_t^B \quad (4)$$

where π_t^B is the given by:

$$\pi_t^B = rE[\min(D_t, K_t)] - c^B K_t^B - pE[\min(D_t^A, K_t^A)]. \quad (5)$$

For this type of contract, the contractor chooses K_t^A , $t \in \{1, \dots, N\}$ in order to maximize its total profit,

$$\pi^A = \sum_{t=1}^N \pi_t^A \quad (6)$$

where π_t^A is given by:

$$\pi_t^A = pE[\min(D_t^A, K_t^A)] - c^A K_t^A. \quad (7)$$

Once again, as far as the capacity decisions are considered p can be viewed as an exogenously determined parameter. Section 5 focuses on the more complicated pricing problem where the contractor selects p in order to maximize π^A .

4 Optimal Service Capacities

In this section, we derive the optimal service capacity levels for both contracts. For the time being, we assume an exogenously set price γ or p .

4.1 Contract 1: Subcontracting the Base

The following proposition provides the optimal capacity levels that the operator should set.

Proposition 1 *The capacity levels K^{A*} and K_t^{B*} , $t \in \{1, \dots, N\}$, which maximize the operator's profit function can be characterized as follows:*

1. Determine ϕ_t for $t \in \{1, \dots, N\}$, where $\phi_t := F_t^{-1}(1 - c^B/r)$,
2. Re-index the periods such that $\phi_1 \leq \dots \leq \phi_t \leq \dots \leq \phi_N$,
3. Compute $\kappa_t := \tilde{G}_t^{-1}(\frac{tc^B - N(c^B - \gamma)}{r})$ for $t \in \{1, \dots, N\}$ and set $\kappa_0 = +\infty$,
4. Define t^* to be the time period such that, $t^* = \max(t, t \in \{1, \dots, N\}; \phi_t \leq \kappa_{t-1})$.
5. Compute K^{A*} and K_t^{B*} as follows,

$$\begin{aligned} K^{A*} &= \kappa_{t^*} \\ K_t^{B*} &= [\phi_t - K^{A*}]^+, \text{ for } t \in \{1, \dots, N\} \end{aligned}$$

Proof: All proofs can be found in the Appendix.

Since the objective function (1) is separable in the time periods, the capacity decision is independent of the pattern of F_t s over time. That is, different orderings of t will lead to the same optimal capacity levels.

For the single period case, since $c^A < c^B$, co-sourcing is never optimal. For the multi-period case however, the operator may outsource some, but not all calls to the contractor. To illustrate this point, consider a two period problem ($N = 2$) where demand for the second period stochastically dominates demand in the first one (i.e. $F_1(\cdot) \geq F_2(\cdot)$). The next proposition establishes that if γ is large enough, then co-sourcing is optimal.

Proposition 2 *Assume $D_1 \leq_{st} D_2$. Define the critical price $\tilde{\gamma}$ as*

$$\tilde{\gamma} = \frac{rG_1 \left(F_2^{-1} \left(1 - \frac{c^B}{r} \right) \right) + c^B}{2}$$

If $\gamma > \tilde{\gamma}$ then co-sourcing is optimal and

$$\begin{aligned} K^{A*} &= F_1^{-1} \left(1 - \frac{2\gamma - c^B}{r} \right) \\ K_1^{B*} &= 0 \\ K_2^{B*} &= F_2^{-1} \left(1 - \frac{c^B}{r} \right) - F_1^{-1} \left(1 - \frac{2\gamma - c^B}{r} \right) \end{aligned}$$

If $\gamma \leq \tilde{\gamma}$ then outsourcing all the calls is optimal and

$$\begin{aligned} K^{A*} &= \tilde{G}_2^{-1} \left(\frac{2\gamma}{r} \right) \\ K_1^{B*} &= K_2^{B*} = 0 \end{aligned}$$

Note that in Proposition 2 when D_1 and D_2 have the same probability distribution ($F_1(\cdot) = F_2(\cdot) = F(\cdot)$), $\tilde{\gamma} = c^B$ and outsourcing all the calls to the contractor is optimal with $K_1^{B*} = K_2^{B*} = 0$ and $K^{A*} = F^{-1}(1 - \gamma/r)$. The problem becomes then equivalent to the single period case. This suggests that as the between period variability increases, more co-sourcing is desirable. More formally, consider a series of two-period problems where $D_1 = A_{-\theta}$ and $D_2 = A_\theta$ for $\theta \geq 0$ where $\{A_\theta\}_{\theta \in \Theta}]-\infty, +\infty[$ is a series of i.i.d. random variables with $F_\theta(\cdot)$ and $f_\theta(\cdot)$ the corresponding cumulative probability and density functions. We are interested in studying the impact of the parameter $\theta \geq 0$ on the service capacity levels when the variability between periods increases with θ while the total demand average remains constant, i.e. when $E[A_{-\theta}] + E[A_\theta] = \mu$ does not depend on θ . To that end, we further assume that the series $\{A_\theta\}_{\theta \in \Theta}$ are ordered according to the first order stochastic dominance, $A_\theta \geq_{st} A_{\theta'}$ for $\theta' \leq \theta$, which is equivalent to $F_\theta(\epsilon) \leq F_{\theta'}(\epsilon)$ for all $\epsilon \geq 0$. For instance, for a given random variable D with cumulative distribution $F(\cdot)$ and finite mean μ , the series $A_\theta = (1 + \theta)D$ for $\theta \in \Theta = [-1, 1]$ satisfies the previous conditions where $D_1 = (1 - \theta)D$ and $D_2 = (1 + \theta)D$ for $\theta \geq 0$ with $E[D_1] + E[D_2] = \mu$. The parameter θ is then a measure of the between period variability since the stochastic dominance between period becomes stronger as θ increases while the total demand average in the system remains constant. The following result states that more co-sourcing is needed (that is, the operator increases its service capacity) as the between period variability (i.e. θ) increases.

Proposition 3 *Assume that the price γ is fixed. Consider the 2 period problem where $D_1 = A_{-\theta}$ and $D_2 = A_\theta$. If $A_\theta \geq_{st} A_{\theta'}$ for $\theta > \theta'$ and $E[A_{-\theta}] + E[A_\theta]$ is a constant for $\theta > 0$ then:*

1. *The capacity levels K^A and K_2^B are respectively non-increasing and non-decreasing in $\theta \geq 0$, while $K_1^{B*} = 0$,*

2. There exists a threshold $\tilde{\theta}$ such that co-sourcing is optimal if and only if $\theta > \tilde{\theta}$.

4.2 Contract 2: Subcontracting the Fluctuation

For this contract, the profit functions of both the operator and the contractor are separable into N profit functions π_t^B and π_t^A respectively. Each party specifies its service capacity for the entire horizon independently, but simultaneously. A and B act strategically, taking the other's decision into account. For each period, the contractor specifies its own service capacity, which impacts the operator's profit. Similarly, the operator's choice modifies the contractor's profit. This situation creates then a game between the operator and the contractor, whose final profits are determined by a Nash Equilibrium in each period. The following proposition specifies the capacity levels at the equilibrium for a given price p .

Proposition 4 *In Period t , the unique Nash equilibrium is reached for the following capacity levels:*

If $p > rc^A/c^B$,

$$\begin{aligned} K_t^{B*} &= F_t^{-1} \left[1 - \frac{c^A + c^B}{p} - \frac{rc^A}{p^2} \right] \\ K_t^{A*} &= F_t^{-1} \left[1 - \frac{c^A}{p} \right] - K_t^{B*}. \end{aligned}$$

If $p \leq rc^A/c^B$, $K_t^{B} = F_t^{-1}(1 - c^B/r)$ and $K_t^{A*} = 0$*

5 Pricing decision

So far, we have assumed that the prices of the different contracts (the capacity reservation price γ and the price per call p) are set exogenously by the market. In this section we assume that their values are determined by the contractor. By setting a price, the contractor may change the capacity level decisions of the operator, which may in turn impact the contractor's profit. Hence, when prices are endogenous, both contracts induce a game, in which the contractor is a Stackelberg leader. In Contract 1, given the price γ set by A, B optimizes K^A and K_t^B . In Contract 2, once A sets the price p , A and B play a Nash game to determine the equilibrium levels of K_t^A and K_t^B . The general analysis of these games is difficult. We first restrict our study to the single-period case. Analytical results are also presented with 2 periods for Contract 1. Then in Section 7, through numerical examples, we explore the pricing decision in multi-period settings.

5.1 Characterizing The Capacity Reservation Price

We start by defining a probability distribution with an increasing generalized failure rate (IGFR) as in Lariviere and Porteus (2001). A distribution is said to have an IGFR if

$$g(\epsilon) = \frac{\epsilon f(\epsilon)}{G(\epsilon)}, \quad (8)$$

is weakly increasing for all ϵ such that $F(\epsilon) < 1$. This property is satisfied by common distributions like the normal, uniform, gamma (Erlang), Weibull, etc. Given a random variable, we also define the function $J(\cdot)$ such that $J(\epsilon) = G(\epsilon)(1 - g(\epsilon))$. If the distribution has IGFR, then J is non-increasing for ϵ such that $J(\epsilon) \geq 0$.

Proposition 5 *Suppose that F has IGFR with a finite mean and support $[a, b)$. A Stackelberg equilibrium exists and the corresponding capacity levels K^{A*} , K^{B*} and the reservation price γ^* satisfy:*

$$\begin{aligned} K^{B*} &= 0 \\ J(K^{A*}) &= c^A/r \\ \gamma^* &= rG(K^{A*}). \end{aligned}$$

Given this equivalence result between the single-period case of Contract 1 and the problem in Lariviere and Porteus (2001), we can further draw on their Lemma 1 and conclude that γ^* will decrease as a function of the coefficient of variation of certain demand distributions (for example uniform, gamma, normal). This is demonstrated in the numerical analysis section.

The existence of an equilibrium can also be shown for the two period case as stated by the following result. Consider a two period problem such that $D_1 \leq_{st} D_2$. Denote by g_i , $i = 1, 2$ their respective general hazard rates, and by $J_i(\cdot)$ the corresponding functions $J_i(\epsilon) = G_i(\epsilon)(1 - g_i(\epsilon))$.

Proposition 6 *Suppose that both F_1 and F_2 have IGFR with finite means and supports $[a, b)$. A unique Stackelberg equilibrium exists and the corresponding capacity levels K^{A*} , K_1^{B*} , K_2^{B*} and the reservation price γ^* satisfy:*

If $J_1(\phi_2) < (2c^A - c^B)/r$ then

$$\begin{aligned} K_1^{B*} &= 0 \\ K_2^{B*} &= F_2^{-1} \left(1 - \frac{c^B}{r} \right) - F_1^{-1} \left(1 - \frac{2\gamma^* - c^B}{r} \right) \\ J_1(K^{A*}) &= (2c^A - c^B)/r \\ \gamma^* &= \frac{1}{2} \left(rG_1(K^{A*}) + c^B \right). \end{aligned}$$

If $(2c^A - c^B)/r \leq J_1(\phi_2) < 2c^A/r - J_2(\phi_2)$ then

$$\begin{aligned} K_1^{B*} &= K_2^{B*} = 0 \\ K^{A*} &= \phi_2 \\ \gamma^* &= \frac{1}{2} \left(rG_1(\phi_2) + c^B \right). \end{aligned}$$

If $2c^A/r - J_2(\phi_2) \leq J_1(\phi_2)$ then

$$\begin{aligned} K_1^{B*} &= K_2^{B*} = 0 \\ J_1(K^{A*}) + J_2(K^{A*}) &= 2c^A/r \\ \gamma^* &= \frac{r}{2} \tilde{G}_2(K^{A*}). \end{aligned}$$

with $\phi_2 = F_2^{-1}(1 - c^B/r)$.

For the single period case, we have seen that by Proposition 5, it is never optimal to co-source and no calls are kept in house. On the other hand for the two-period problem, the previous proposition shows that co-sourcing can be optimal. However, Proposition 3 does not hold in general when γ is not fixed and it is not clear how the service capacity levels at the equilibrium vary as the between period variability increases (i.e. the first order stochastic dominance increases while the total demand average remains constant). Nevertheless, we show in the following that stochastic dominance still plays a crucial impact on co-sourcing decisions at the equilibrium. More precisely, consider a series of two-period problems such that $D_1 = A_0$ and $D_2 = A_\theta$ where $\{A_\theta\}_{\theta \geq 0}$. We further assume that $A_\theta \geq_{hr} A_{\theta'}$ for $0 \leq \theta' \leq \theta$. The hr -stochastic dominance (that is simply referred to as the stochastic dominance in the following) $A_\theta \geq_{hr} A_{\theta'}$ is equivalent to $h_\theta(\epsilon) \leq h_{\theta'}(\epsilon)$ for all $\epsilon \geq 0$, where $h_\theta(\cdot)$ is the hazard rate of A_θ ($h_\theta(\epsilon) = f_\theta(\epsilon)/G_\theta(\epsilon)$). (For instance, $\{A_\theta\}_{\theta \geq 0}$ can be a series of exponentially distributed random variables with rate θ). Note that the stochastic dominance ' \geq_{hr} ' implies the first order stochastic dominance ' \geq_{st} '. When $\theta = 0$, demands in both periods have the same distribution $F_0(\cdot)$ and co-sourcing is not optimal (all the calls are outsourced). As θ increases the total average demand increases while the between period stochastic dominance becomes stronger. The following result states that the capacity decisions and the price at the equilibrium evolve monotonically in θ . In particular, K^{B*} is non-decreasing in θ .

Proposition 7 *Assume that $A_\theta, \theta \geq 0$ have IGFR with finite mean. Consider the 2 period problem where $D_1 = A_0$ and $D_2 = A_\theta$. If $A_\theta >_{hr} A_{\theta'}$ for $\theta > \theta'$ then:*

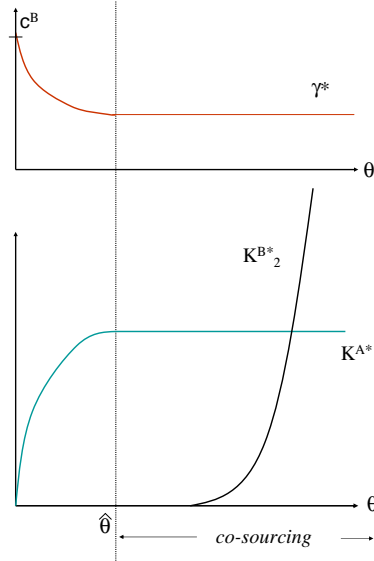


Figure 2: Impact of the between period stochastic dominance

1. The capacity levels at the equilibrium K^{A*} , K_2^{B*} are non-decreasing in $\theta \geq 0$, $K_1^{B*} = 0$, and the equilibrium price γ^* is non-increasing in θ ,
2. There exists a threshold $\hat{\theta}$ such that for $\theta > \hat{\theta}$, K^{A*} and the equilibrium price γ^* remain constant.

In other words, the operator manages a large increase in the between period stochastic dominance (i.e. $\theta > \hat{\theta}$) by handling more calls in-house while keeping the outsourced service capacity at a constant level. On the other hand, even for a significant surge of the total demand average, co-sourcing is not optimal as long as demands are not too unbalanced between period (with respect to the stochastic dominance). Figure 2 depicts this impact for a typical situation.

5.2 Characterizing The Price Per Call

A complete characterization of the optimal price is difficult in this case, however the following bounds can be established on the optimal value of p . Let $c_0 = (c^A + c^B)/2$.

Proposition 8 *The price per call at the equilibrium p^* verifies*

$$\frac{2c^A}{c^A + c^B}r \leq p^* \leq r$$

We next partially characterize the Stackelberg game for distributions with non-increasing failure rate (DFR). A distribution is said to have a DFR if $f(\epsilon)/G(\epsilon)$ is non-increasing. Such distributions

usually correspond to mixtures of different populations (the class of DFR distributions is closed under random mixtures). Characterizations of similar games have been provided before in Lariviere and Porteus (2001) for demand distributions having IGFR, and Dong and Rudi (2004) for normally distributed demand. Note that the problem being considered herein is significantly more difficult, due to the fact that the contractor's profit function (corresponding to the manufacturer in the mentioned papers) is not deterministic. As a result, the first order conditions one gets for p^* depend on both the density and the distribution of the demand (and functions thereof) in a complicated way, thus rendering the analysis less tractable. Changes of variables, as in the above papers, do not circumvent the problem because of the complex relationship between K^B and the price p .

In the following we provide sufficient conditions for which the equilibrium price is equal to r so that the operator never prefers outsourcing the fluctuation (Contract 2).

Proposition 9 *If F has DFR, the equilibrium p^* is always equal to r and the call center operator never prefers outsourcing the fluctuation (Contract 2) as a contract.*

Depending on their parameters, the gamma, Weibull and lognormal distributions can all have DFR. (The gamma and Weibull distributions always have IGFR so that Proposition 5 applies in these cases.) In particular the exponential distribution has a constant failure rate, and $p^* = r$ from Proposition 9. When the failure rate is increasing, the existence of p^* is not easy to show. Our numerical results suggest that π^A can be concave or convex depending on the distribution, that is

$$\frac{\partial^2 \pi^A}{\partial^2 p} = \frac{c^A}{p} \frac{\partial K}{\partial p} - \frac{c^A + c^B}{p} \frac{\partial K^B}{\partial p} + \frac{rc^A - c^B p}{p^2} \frac{\partial^2 K^B}{\partial^2 p} \quad (9)$$

is either negative or positive for $p \in [\rho c_0, r]$ (from Proposition (8)). As a consequence, when the distribution has an increasing failure rate, p^* can be less than r . For instance, the uniform distribution has an increasing failure rate and $p^* = 3rc^A/c^B$ as stated by the following proposition.

Proposition 10 *If demand is uniformly distributed, the equilibrium p^* is given by $p^* = \min(3rc^A/c^B, r)$.*

6 Model Extensions

We have modeled and analyzed two basic service outsourcing contracts in the preceding sections. Our assumptions were driven in part by the actual problem that motivated the analysis, and partially by our concern to keep the model as simple as possible. In this section, we discuss some extensions that generalize our analysis to other possible applications and that question the

simplifying assumption of no queueing that was made. We first consider the same contracts when a service level constraint is imposed. Then we reconsider the two basic contracts when the call center is modeled as a queue rather than a series of unlinked systems with finite capacity. Finally we discuss the possibility of having different types of contracts when constraints about fixed capacity levels across time periods are imposed or lifted.

6.1 Including a Service Level in the Contracts

In practice, call center outsourcing contracts may also have service level requirements. In our setting, this corresponds to a constraint on the percentage of lost calls. We analyze the impact of imposing such a service level constraint in the single period problem first. The numerical analysis section explores the effect of a service level constraint on contract performance in the multiple period setting. A service level of $1 - \alpha$ can be written as the following probability

$$P(D > K^A + K^B) \leq \alpha.$$

This is equivalent to

$$1 - F(K) \leq \alpha.$$

For Contract 1, the effect of such a constraint can be analyzed directly in the single-period setting. If the service level constraint is not binding, then the optimal price is determined by Proposition 5. If the constraint is binding, then the contractor can set γ right below c_B since the operator has the obligation to satisfy the constraint by reserving the minimum capacity required to meet the service level.

For Contract 2, making use of Proposition 4, a service level constraint can be expressed as, when $p \geq rc^A/c^B$

$$F(K) = \frac{p - c^A}{p} \geq 1 - \alpha, \quad (10)$$

Making use of this relationship, we can show the following result

Proposition 11

- If $c^B/r < \alpha$ (resp. $c^A/r > \alpha$), then B does not outsource any call and $F(K^B) = 1 - c^B/r$ (resp. $F(K^B) = 1 - \alpha$).
- If $c^A/r \leq \alpha \leq c^B/r$, then the price of the contract with service level agreement is equal to $\tilde{p} = \max(p^*, c^A/\alpha)$ and K , K^A and K^B are given by Proposition 4 evaluated in \tilde{p} .

Let us briefly discuss the implications of market-imposed service levels on the two contracts. First, a binding service level contract requires a higher total capacity investment than what is justified by the financial parameters. In Contract 1, this seems to give the contractor higher pricing power since the operator is obliged to meet a high service level. In general, the contractor benefits and the operator loses in this situation. Contract 2 presents a more complicated case. For very high and very low service levels the operator prefers not to outsource any calls. Co-sourcing only takes place for a certain range of service levels and even then the contractor does not obtain additional pricing power. For high service levels, we would then expect the contractor to prefer Contract 1 whereas the operator is indifferent between the two contracts. For lower service levels which result in co-sourcing in Contract 2, the operator may prefer this contract.

6.2 An Approximation when the Call Center is Modeled as a Queue

In this section, we illustrate how the model being analyzed in this paper can be seen as an approximation to a $G/G/s$ queueing model with abandonments as in Whitt (2006) and Ren and Zhou (2005). As noted before, embedding an exact formulation of such a queueing system in our contract analysis will yield an intractable problem. Instead we show the similarity of the model to a fluid approximation of the queueing system. Our aim is to illustrate the robustness of the basic model results to the no queueing assumption that was made.

Calls come in at a rate of λ . Different from Ren and Zhou (2004) and like Whitt (2006) we assume that the arrival rate of calls is itself a random variable. Thus λ corresponds to the single period demand D from before. Given the time horizon of service outsourcing contracts, it is highly appropriate to assume an uncertain demand rate. As emphasized by Whitt (2006) indeed the uncertainty in parameters (i.e. such as λ) tends to have a more dominant effect than stochastic process variability (such as the variability in the queueing process itself). The same property is used in the analysis of Harrison and Zeevi (2005), where it is argued that temporal and stochastic variability in the arrival rates dominates all other forms of variability. These observations support our use of a fluid approximation in place of an exact queueing analysis. As in Harrison and Zeevi (2005) our arrival rates λ_t (D_t) are both temporally and stochastically variable. Green and Kolesar (1991) show that in systems with large frequency of events and with large service rates, the pointwise stationary approximation which ignores links between periods performs remarkably well. Both of these characteristics are present in large call centers. Ignoring the link between periods may distort current results only in settings where capacity shortages in one period affect the demand

of subsequent periods. This occurs for example when abandoning or blocked customers retry in later periods, as analyzed in Aguir et al. (2004), or due to persisting queues during times of high congestion. For such environments, ignoring the link between periods may have an important effect on contract performance and choice. We leave the analysis of such cases to future research.

In the queueing setting, the capacity variables K will be replaced by μs where μ is the service rate and s the number of servers in a $G/G/s$ queue. Now, calls will be lost essentially in the form of customer abandonments. For the fluid model, borrowing notation from Whitt (2006), we let $T(s)$ denote the number of customers served in steady state and $L(s)$ be the abandonment in steady state. The fluid approximation then provides the following expressions:

$$T(s) = \min(\lambda, \mu s), \quad (11)$$

$$L(s) = (\lambda - \mu s)^+. \quad (12)$$

In the fluid approximation, all calls that exceed the capacity of the system will abandon. This is exactly like our original system where demand that exceeds capacity is lost. An abandonment cost as assumed by Whitt (2006) could easily be incorporated in our current framework if desired. To conclude we note that a fluid approximation of a $G/G/S$ queue with customer abandonments behaves qualitatively the same way as our basic deterministic capacity model as long as the uncertainty in the call arrival rate is significant. In addition, if the pointwise stationary approximation is appropriate, this similarity also extends to the multi-period setting. Queueing phenomena will then change the qualitative nature of our results only in settings where the link between periods cannot be ignored. In this case, a numerical approach as in Aguir et al. (2004) needs to be embedded in the contract analysis.

6.3 Having a Choice of Fixed or Variable Capacity

Contract 1 considers a setting where the operator reserves a fixed capacity level at the contractor for the entire time horizon, while adjusting its own capacity level every period. Under this contract, the contractor promises to dedicate servers in its call center to calls from the operator company. Even though the latter aspect of the contract is not modeled herein, this characteristic drives the constant capacity reservation assumption. We expect to see a preference for constant capacity at the contractor when contracts are of a pay for capacity type as in Contract 1, and dedicated capacity needs to be created. More generally, one could envisage an agreement as in Contract 1, where the operator reserves a different level of capacity at the contractor in each time period.

When this is the case, the problem decomposes into N single period problems. Based on our earlier results, we can then state that the operator will opt to outsource everything to the contractor in all time periods and co-sourcing will not occur.

Under Contract 2, both players adjust their capacity levels in each period. Since the operator does not guarantee a certain call volume to the contractor, it is natural for the latter to adapt its capacity in response to demand fluctuations. In practice, this contract also allows the contractor to pool calls from different clients using the same servers. Thus company specific training may not be as intense, and training some servers only for certain high volume periods may be justified. A different version of this problem would become relevant if the operator experienced a cost for adjusting its capacity level or for managing a variable capacity level. Then in a pay for volume environment as in Contract 2, the operator could opt to keep a fixed capacity level throughout. Analyzing Contract 2 when B's capacity is fixed over the entire time horizon is challenging, since the Nash equilibrium for capacity investment does not decompose into single period problems in this case. As a result, we leave the analysis of this version for future research. Note that a fixed capacity level by the operator could imply more volume being outsourced to the contractor under high between period variability, which could lead to a lowering of price by the contractor under this contract. Whether this conjecture is true would be worth exploring since such a difference would also affect the contract choice problem.

7 Numerical Analysis

This section will explore optimal prices, profits, and capacities for the two contracts under different environments. These environments will be described by the variability of demand and the economic parameters r, c^A, c^B that establish the margins for each party. We consider two types of demand variability: *within period variability*, which is determined by the demand distribution of a given time period and *between period variability*, which refers to the demand pattern across multiple periods, captured through the change in the parameters of a particular demand distribution. Our first objective is to develop a general understanding of each contract under steady demand (i.e. no between period variability). We then explore the impact between period variability has on contract behavior. Restricting our attention to a particular level of within period variability, we then explore the role economic parameters have on these contracts. Finally, we also briefly investigate the situation with a market-imposed service level constraint. For all settings, our ultimate objective

is to address the contract choice problem posed in the Introduction: under what conditions does each party prefer a particular contract?

7.1 The role of within period variability

We consider the simplest multi-period setting with three time periods. In order to isolate the effect of within period variability, this section considers an identical demand distribution in each time period. For the experiments we use the Erlang family of distributions (a subclass of Gamma distributions). The m -stage Erlang distribution (referred to as Erlang- m) is completely characterized by its mean and the number of stages m , and has the following probability density function:

$$f(x) = \frac{x^{m-1} e^{-x/\theta}}{\theta^m (m-1)!} \quad x \geq 0$$

where m is positive integer and θ is a positive real number. The mean of the distribution is $m\theta$. Note that the Erlang-1 distribution is the simple exponential distribution. Focusing on the Erlang family enables us to systematically investigate the effect of variability since an Erlang- m distribution is more *variable* than an Erlang- m' distribution with the same mean when $m' > m$ according to a convex stochastic order. This, naturally, implies that the variance is decreasing in m (for identical means). Finally, Erlang distributions possess the IGFR property required by Proposition 5 and the exponential distribution has the (weak) DFR property required by Proposition 9.

We analyze four demand distributions, Exponential, Erlang-2, Erlang-10, and Erlang-100, going from high (H) within period variability, to low (L) within period variability. The exponential (Erlang-1) represents a highly variable demand, the Erlang-10 resembles a Normal Distribution (with a coefficient of variation of 0.32), and Erlang-100 is used as a test case for low variability (coefficient of variation=0.1).

When an identical demand distribution is assumed in each period, the multi-period problem is structurally equivalent to the single-period problem. In order to compute the optimal capacities we make use of Propositions 1 and 4. The optimal price for each contract is then calculated via a numerical search (using discrete intervals of 0.01). For Contract 2, we also make use of the bounds established in Proposition 8 to restrict the search region. Finally, for the exponential distribution Proposition 9 directly yields the optimal price for Contract 2.

Demand is steady (independent and identically distributed in each period) with a mean of 30. In these examples $c^A = 2$, $c^B = 5$, and $r = \{6, 10\}$. For each contract Tables 1, 2, and 3 tabulate optimal prices, profits, and capacities.

	$(r, c^A, c^B)=(6,2,5)$					$(r, c^A, c^B)=(10,2,5)$				
	γ^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}	γ^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}
Exp.	3.76	74.02	43.45	14.02	(0,0,0)	4.99	187.15	138.06	20.79	(0,0,0)
Erl-2	4.30	108.88	48.65	15.78	(0,0,0)	4.99	226.22	214.12	25.22	(0,0,0)
Erl-10	4.99	188.10	52.76	20.97	(0,0,0)	4.99	260.39	338.9	29.03	(0,0,0)
Erl-100	4.99	243.17	77.77	27.2	(0,0,0)	4.99	268.29	415.04	29.91	(0,0,0)

Table 1: Contract 1 for steady demand

Demand Dist.	p^*	K^{A^*}	K^{B^*}	π^{A^*}	π^{B^*}
Exp.	6	(27.49, 27.49, 27.49)	(5.47, 5.47, 5.47)	105.07	7.95
Erl-2	6	(23.37, 23.37, 23.37)	(10.97, 10.97, 10.97)	97.38	20.53
Erl-10	4	(29.00, 29.00, 29.00)	(0,0,0)	135.21	154.63
Erl-100	4	(29.90, 29.90, 29.90)	(0,0,0)	165.66	172.53

Table 2: Contract 2 for steady demand: $(r, c^B, c^A)=(6,5,2)$

The following observations can be made for Contract 1:

- γ^* can take values that are strictly less than c^B . When γ^* approaches c^B , B earns its participation profit. When γ^* is away from its upper limit c^B , B may earn more than its participation profit under this contract.
- Both π^{A^*} and π^{B^*} are decreasing in demand variability.
- γ^* is decreasing in demand variability. Figure 3 (left graph) illustrates that this happens as long as margins are low. Otherwise, γ^* remains at its boundary value even for the Exponential distribution with high variability. Thus, γ^* seems to be decreasing for decreasing values of r as well, ceteris paribus.

The above observations are consistent with Proposition 5 and the results in Lariviere and Porteus (2001).

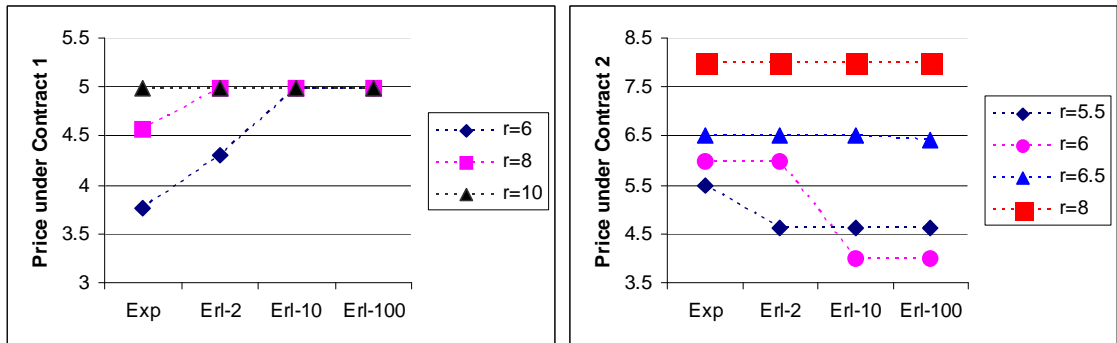


Figure 3: Optimal Prices for Contracts 1 (left) and 2 (right)

The following observations can be made for Contract 2 from Tables 2 and 3:

Demand Dist.	p^*	K^{A*}	K^{B*}	π^{A*}	π^{B*}
Exp.	10	(27.49, 27.49, 27.49)	(20.79, 20.79, 20.79)	105.07	138.08
Erl-2	10	(19.74, 19.74, 19.74)	(25.18, 25.18, 25.18)	78.04	213.37
Erl-10	10	(8.55, 8.55, 8.55)	(29.00, 29.00, 29.00)	35.16	338.03
Erl-100	10	(2.59, 2.59, 2.59)	(29.90, 29.90, 29.90)	10.88	414.15

Table 3: Contract 2 for steady demand $(r, c^B, c^A)=(10,5,2)$

- The optimal price p^* is equal to r except in two instances involving low variability (Erlang-10 and Erlang-100) distributions. Thus even for increasing failure rates, i.e. when these rates are low, we find the same result as shown in Proposition 9. As a result B can rarely earn more than its participation constraint. The impact of r on p^* is further explored in Figure 3 (right graph). We see that for lower r values, p^* remains below its boundary value r for higher levels of demand variability, thus allowing B to earn more than its participation constraint. As margins increase, p^* becomes equal to r even for the low variability settings like Erlang-100.
- π^{B*} is decreasing in demand variability. π^{A*} is increasing in demand variability when $p^* = r$ but decreasing in variability when $p^* < r$. This is also observed for the examples in Figure 3 (right graph). When $p^* = r$, A is used to treat the overflow, so as demand becomes less variable π^{A*} decreases. When $p^* < r$ most calls are outsourced to A, so less variability is beneficial for the contractor in these cases.
- p^* is non-decreasing in demand variability, until the boundary value of $p^* = r$ is reached.
- B invests in some capacity unless $p^* < c^B$.
- Total capacity investment by both parties is higher under Contract 2 in all cases (compared to Contract 1).
- The relationship between demand variability and total capacity investment is not monotonic under this Contract.

The first observation implies that Contract 2 will rarely be preferred by B. This will happen when demand variability is low and/or when margins are low, suggesting a commodity type of service. Contrasting this to the first observation for Contract 1, we expect Contract 1 to be preferred by B more frequently. Coupled with the differences in the response of π^{A*} and π^{B*} to variability, we anticipate contract choice to change as a function of demand variability as well as margins. This will be explored below.

7.2 The impact of between period variability

In this section, we drop the assumption that demand is steady and consider the effect of between period variability. The calculations are performed in a similar manner using the analytical results in combination with a numerical search for the optimal prices. Mean demands are assumed to be (15, 60, 15). Note that the total mean demand is the same as before, however the way it is distributed over time is different. The between period variability notion we use here is different from the slightly stronger between period stochastic dominance introduced before Proposition 7. This is so because we want to explore settings where total mean demand remains constant once multiple period fluctuations are introduced. All other parameters are the same. For each contract Tables 4, 5 and 6 tabulate optimal prices, profits, and capacities.

	$(r, c^A, c^B)=(6,2,5)$					$(r, c^A, c^B)=(10,2,5)$				
	γ^*	π^{A*}	π^{B*}	K^{A*}	K^{B*}	γ^*	π^{A*}	π^{B*}	K^{A*}	K^{B*}
Exp.	3.68	52.46	32.50	10.41	(0, 0.53, 0)	4.79	136.40	138.13	16.30	(0, 25.29, 0)
Erl-2	4	64.14	44.74	10.69	(0, 11.24, 0)	4.99	113.20	213.74	12.62	(0, 37.73, 0)
Erl-10	4.99	94.18	52.45	10.5	(0, 31.33, 0)	4.99	130.24	338.47	14.52	(0, 43.49, 0)
Erl-100	4.99	121.63	77.36	13.56	(0, 40.63, 0)	4.99	134.19	414.59	14.96	(0, 44.84, 0)

Table 4: Contract 1 under fluctuating demand

Demand Dist.	p^*	K^{A*}	K^{B*}	π^{A*}	π^{B*}
Exp.	6	(13.74, 54.98, 13.74)	(2.73, 10.94, 2.73)	105.07	7.95
Erl-2	6	(11.68, 46.74, 11.68)	(5.48, 21.93, 5.48)	97.38	20.53
Erl-10	4	(14.50, 58.01, 14.50)	(0, 0, 0)	135.21	154.63
Erl-100	4	(14.95, 59.80, 14.95)	(0, 0, 0)	165.66	172.53

Table 5: Contract 2 under fluctuating demand: $(r, c^B, c^A)=(6,5,2)$

Demand Dist.	p^*	K^{A*}	K^{B*}	π^{A*}	π^{B*}
Exp.	10	(13.74, 54.98, 13.74)	(10.40, 41.59, 10.40)	105.07	138.08
Erl-2	10	(9.87, 39.48, 9.87)	(12.59, 50.35, 12.59)	78.04	213.37
Erl-10	10	(4.28, 17.10, 4.28)	(14.50, 58.01, 14.50)	35.16	338.03
Erl-100	10	(1.29, 5.18, 1.29)	(14.95, 59.80, 14.95)	10.88	414.15

Table 6: Contract 2 under fluctuating demand: $(r, c^B, c^A)=(10,5,2)$

Comparing the results in this Section to the previous one, we note the following:

- γ^* is also decreasing as a function of between period demand variability.
- π^{A*} and π^{B*} under Contract 2 do not depend on how demand is allocated between periods. (This follows from the fact that for Erlang distributions the optimal capacity in each period can be written as the product of expected demand in that period and an identical safety factor that

depends on the financial parameters.)

- As predicted by Proposition 7 under stronger assumptions, co-sourcing can take place even under Contract 1.
- With between period variability, both parties' profits are down under Contract 1. Given that B is already close to its participation constraint it does not lose much. A suffers more since co-sourcing increases as between period variability is introduced.
- Comparing Tables 1 and 4 we note that total capacity can increase as in the case with Exponential demand and parameters (6, 5, 2) or decrease in the case with Exponential demand and parameters (10, 5, 2).

These observations imply that there is an important effect that between period variability has on contract characteristics. This in turn suggests that between period variability will play a role in determining contract choice.

7.3 Contract choice

Based on the profits tabulated for the previous set of examples, Table 7 summarizes contract choice by each party. Whenever B appears in both the columns for Contract 1 and Contract 2, this means that both contracts force B to its participation constraint, and it is assumed that B is indifferent between these contracts. The following observations can be made:

- Focusing on the steady demand cases in the first two columns, we observe that within period variability can change preferences for contracts. This is due to the opposite effect within period variability has on the optimal price of the two contracts. However as noted in the steady demand examples, as r increases (while costs remain the same) optimal price reaches its boundary under both contracts, thus making B indifferent between the two contracts (rows 5-8) irrespective of within period variability.
- Rows two through four show that between period variability can change only one party's preference (A in this instance), thus enabling a clear preference for one of the contracts by both parties. The switch in preference from Contract 1 to Contract 2 occurs for A in these cases, because its profits under Contract 1 become less attractive compared to the steady demand case. γ^* is lower in one case, and B opts to invest in capacity during the peak period, thus decreasing A's volume compared to the steady demand case. Thus, with a lower (or same) price and lower demand volumes A earns less under Contract 1 once between period variability is introduced. Between period variability does not affect A's profits under Contract 2, so the preference for this contract is purely a function

of the change in profits under Contract 1.

- Contract preferences differ as a function of margins, as demonstrated by the cases for $r = 6$ and $r = 10$.
- In these examples, the margin effect seems to dominate the earlier mentioned variability effects. For the high margin cases ($r = 10$), B is forced to its participation constraint in both contracts under almost all demand scenarios. A prefers Contract 1 even when there is between period variability, because B invests much more in internal capacity when margins, and as a result optimal prices are high under Contract 2, thus outsourcing very little to A compared to Contract 1. Assuming B is indifferent, we can state that Contract 1 is preferred in all these cases. Supposing that high margins represent a complex and/or valuable service, we can state that for such environments B would be inclined to do things in house irrespective of demand structure.
- In the low margin cases, joint contract preferences are more difficult to obtain.
- Contract 2 is preferred by both parties only in cases that combine low margins, low within period variability, and high between period variability.

		(30,30,30)		(15,60,15)	
(r, c^B, c^A)		Contract 1	Contract 2	Contract 1	Contract 2
(6,5,2)	Exp.	B	A	B	A
	Erl-2	A,B	-	B	A
	Erl-10	A	B	-	A,B
	Erl-100	A	B	-	A,B
(10,5,2)	Exp.	A,B	B	A,B	-
	Erl-2	A,B	B	A,B	B
	Erl-10	A,B	B	A,B	B
	Erl-100	A,B	B	A,B	B

Table 7: Contract choice by A and B

7.4 The impact of economic parameters

In this section we focus on the Exponential demand distribution case, and explore the role of cost parameters c^A , c^B and revenue r further. As shown before, $p^* = r$ in all cases. Rather than reporting detailed profit and capacity results, we tabulate contract choice for these examples.

The examples in rows 1-3 and 4-6 have r and c^A fixed for different values of c^B . A comparison of each party's choice demonstrates that the relative value of c^B , through its impact on both A's and B's margins, can change contract preferences. These examples also show that c^B needs to be high enough relative to r and c^A (like in 6,5,2 or 10,8,2) to make outsourcing worth pursuing for B. Rows 1 and 6 as well as 5 and 7 keep the cost structure the same while varying r . These

	(30,30,30)		(15,60,15)	
(r, c^B, c^A)	Contract 1	Contract 2	Contract 1	Contract 2
(6,5,2)	B	A	B	A
(6,4,2)	A,B	B	B	A,B
(6,3,2)	A,B	B	A,B	B
(10,8,2)	B	A	B	A
(10,6,2)	A,B	-	B	A
(10,5,2)	A,B	B	A,B	B
(20,6,2)	A,B	-	A,B	B

Table 8: Contract choice under an Exponential demand distribution

examples confirm earlier observations that changing the value of r impacts contract preferences. Finally, rows 2 and 5 demonstrate how A's preference can change as a function of between period variability. Based on these examples, we can state that all economic parameters have an important role in determining contract choice by defining profit margins available to each party. It is difficult to generalize these choices independent of demand variability effects.

7.5 The Effect of Service Levels

In this section, we briefly focus on the extension with service levels described in Section 6.1 and present a set of numerical results for both contracts with service level constraints. As before, we focus on three-period examples. Demand is assumed to be steady and two different demand distributions are used: exponential with mean 30 and Erlang-10 with mean 30. There are two different sets of financial parameters: $(r, c^A, c^B)=(6,2,5)$ and $(r, c^A, c^B)=(10,2,5)$. In Tables 9 and 10, for Contract 1, we report the optimal capacities and profits for both the contractor and the operator for three different service level requirements: 95%, 90%, and 80% (i.e. $\alpha=0.05, 0.1,$ and 0.2).

	Exp.					Erl-10				
α	γ^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}	γ^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}
0.05	4.99	805.51	-832.39	89.87	0	4.99	422.49	-170.45	47.12	0
0.1	4.99	618.93	-548.09	69.08	0	4.99	382.12	-108.99	42.6	0
0.2	4.99	432.35	-292.25	48.28	0	4.99	336.38	-46.47	37.56	0

Table 9: Contract 1 with service levels for steady demand $(r, c^A, c^B)=(6,2,5)$

Let us summarize the main observations from Tables 9 and 10. Recall that whenever the service level constraint is binding, the contractor can set γ^* just below c_B and leave the operator indifferent between outsourcing everything and not outsourcing at all. Because of this, the contractor is better off as service levels are increased (i.e. as α shrinks) since it can sell more capacity at the same

α	Exp.					Erl-10				
	γ^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}	γ^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}
0.05	4.99	805.51	-490.39	89.87	0	4.99	422.49	186.14	47.12	0
0.1	4.99	618.93	-224	69.08	0	4.99	382.12	243.68	42.6	0
0.2	4.99	432.35	-2.80	48.28	0	4.99	336.38	297.36	37.56	0

Table 10: Contract 1 with service levels for steady demand $(r, c^A, c^B)=(10,2,5)$

price. On the other hand, the operator's profit improves as the service level constraint is relaxed. Another observation from Tables 9 and 10 is that, as expected, the contractor prefers higher demand variability which requires a higher capacity as long as the service level constraint is binding, and the operator prefers lower demand variability.

The corresponding results for Contract 2 are presented in Tables 11 and 12. Since this contract results in more complicated cases, we report results for a larger set of service level requirements: from 95% to 40%.

α	Exp.					Erl-10				
	p^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}	p^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}
0.05	-	0	-835.08	0	89.87	-	0	-171.86	0	47.12
0.1	6	0	-550.16	0	69.08	-	0	-110.27	0	42.62
0.2	6	0	-292.25	0	48.28	-	0	-47.60	0	37.56
0.3	-	0	-163.79	0	36.12	-	0	-11.81	0	34.16
0.4	6	105.07	7.95	27.49	5.47	5	57.14	77.31	13.60	17.83
0.5	6	105.07	7.95	27.49	5.47	4	135.21	154.63	29	0
0.6	6	105.07	7.95	27.49	5.47	4	135.21	154.63	29	0

Table 11: Contract 2 with service levels for steady demand $(r, c^A, c^B)=(6,2,5)$

α	Exp.					Erl-10				
	p^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}	p^*	π^{A^*}	π^{B^*}	K^{A^*}	K^{B^*}
0.05	-	0	-493.08	0	89.87	-	0	184.72	0	47.12
0.1	-	0	-226.16	0	69.08	-	0	242.40	0	42.62
0.2	10	105.07	138.08	27.49	20.79	10	35.16	338.03	8.55	29.00
0.3	10	105.07	138.08	27.49	20.79	10	35.16	338.03	8.55	29.00
0.4	10	105.07	138.08	27.49	20.79	10	35.16	338.03	8.55	29.00
0.5	10	105.07	138.08	27.49	20.79	10	35.16	338.03	8.55	29.00
0.6	10	0	138.08	0	27.49	10	0	338.03	0	29.00

Table 12: Contract 2 with service levels for steady demand $(r, c^A, c^B)=(10,2,5)$

The main observations from Tables 11 and 12 can be summarized as follows. As established in Proposition 11, if the service level is too high (i.e. α is small) B does not outsource any calls despite the high cost of meeting the service level. As the desired service level falls, B outsources some calls and both parties make a positive profit. In a more extreme situation, if the desired

service level is very low (the row corresponding to $\alpha = 0.6$ in Table 12), it becomes optimal for B not to outsource any calls, this time making a positive profit. In addition B, is observed to benefit from reduced demand variability whereas A may or may not experience a gain depending on the equilibrium price.

Finally, as for the contract choice problem, in our limited set of examples, the operator is indifferent between the two contracts for high service levels but prefers Contract 2 for lower service levels and the contractor always prefers Contract 1. This should not imply that different choices cannot be observed for other parameters. For instance, if the service levels are not binding, then the arguments outlined in Section 7.3 prevail.

7.6 Summary

The numerical analysis has explored the contract choice problem under different settings. In doing this, contracts were compared based on optimal capacity choices by both players and optimal price when analyzed separately. In practice, the contractor may have the freedom of offering the two contracts to the operator first, and then asking the operator to choose one. In that case it may be optimal for the contractor to offer a non-optimal price under one of the contracts if this choice implies the selection of the contract that provides higher profits for the contractor by the operator. Formally, this contract choice problem would result in a different Stackelberg game, not analyzed herein. The statements about contract choice in this section should be interpreted with this in mind. They can be viewed as guidelines about contract choice, to be used in a negotiation process with the contractor.

In these examples, the within period demand variability is observed to have an impact on the optimal price. Thus, under Contract 1, as also in Proposition 5, lower within period demand variability implies higher γ values in equilibrium, approaching the c^B upper limit. The optimal price p under Contract 2 on the other hand, approaches its upper value r for environments with high within period demand variability. Whenever $p = r$ or γ approaches c^B the operator earns profits equal to the case when everything is performed in-house.

By construction, the contractor offers to absorb variability in demand under Contract 2, thus allowing it to charge a high price per call (for example $p = r$). Under Contract 1, the contractor is essentially offering a low price ($\gamma \leq c^B$) and not absorbing any of the operator's demand variability. It is then not surprising to observe that as r increases, Contract 1 with its price capped by c^B , becomes more attractive for the operator. The operator's choice of Contract 1 turns out to be

also the contractor's choice in high margin settings, or under particular variability conditions when margins are low.

The contractor's preference also depends on the between period variability effect. Indeed we find both in some low within period, low revenue cases and some high within period, high revenue cases that the contractor's preference switches from one contract to the other as a function of the between period demand variability. This underlines the importance of considering the multi-period effect in such outsourcing contract choice problems. For some cases the switch occurs because an increase in between period variability may decrease outsourcing by the operator under Contract 1. As is also evident from Proposition 1, in the multi-period setting with fluctuating demand, the operator may choose to invest in capacity under both contracts. Unless call volumes are steady across periods (under Contract 1) or within period variability and margins are low (under Contract 2), B will not prefer to outsource all calls to A.

By allowing the outsourcing of individual calls, Contract 2 offers additional flexibility compared to Contract 1. As observed in the examples, total capacity investment by the operator and contractor in each period is also higher under Contract 2. Thus customer service is better when this Contract is preferred by both parties. Despite its attractiveness in terms of flexibility and customer service, conditions that ensure a joint choice of Contract 2 are found to be quite restrictive: settings with commoditized services with seasonalities in their demand. Low within period demand variability and low revenues make the price under Contract 2 more affordable for the operator, while high between period fluctuations make the flexibility offered by this contract more attractive thus resulting in the outsourcing of a higher volume of calls.

Market-imposed service levels have an effect on the contract choice only if the market's service level requirement is higher than what is achieved by purely economic considerations. In this case, our brief investigation reveals that for these examples A always prefers Contract 1 while B prefers Contract 2 whenever cosourcing is optimal in this contract and is indifferent otherwise.

In summary, we find that none of the contracts are universally preferred, and that preferences change as a function of within period variability, between period variability, and profit margins. Our results further demonstrate that the pricing decision plays an important role in this choice, and that taking price as an exogenously given parameter could be misleading. The analysis points to the need for a good understanding of the operating environment of service companies, before contract decisions are made for outsourcing.

8 Concluding Remarks

This paper is among the first to model call center outsourcing contracts, and to explore these in terms of design and contract choice. While the motivating example came from a call center, the results would also be applicable to other types of service outsourcing like the outsourcing of back-office functions. Key distinguishing features of the model are the presence of multiple decision makers, uncertain demand, endogenous pricing decisions, and most importantly a multi-period decision horizon. We find that all of these features have an important impact on the contract choice problem, and that the qualitative nature of the results change as a function of these features. This points to the importance of taking them into account in answering questions about service outsourcing contracts. From a modeling standpoint, one can conclude that the endogenous pricing feature is the one that complicates the analysis the most. Whenever the contractor is a price taker, our analysis fully characterizes both contracts.

For managers who face these types of contract design and choice problems, our analysis demonstrates that in addition to a knowledge of economic parameters like costs, managers need to have a very good understanding of the underlying demand uncertainties. Evaluation of different contract choices should not be simplified to a cost per transaction basis.

There are several modeling extensions some of which have been discussed in Section 6. While all of these extensions merit deeper investigation, we are able to provide some basic insights. The consideration of market-imposed service-level constraints is observed to have an effect on the contract choice problem. On the other hand, if service levels are negotiated as part of the contract, the modeling and analysis become much more challenging. Another important extension is to address the queueing dimension of the problem. Our approach, with some modifications, can be employed to obtain basic qualitative insights provided that main cause of uncertainty in the queueing model is in the mean arrival rate of calls as is argued in several recent call center papers. Other types of contracts can be investigated. Some of these can be analyzed using a similar approach while others require a completely different investigation. Finally, imposing a penalty for lost calls in these contracts will influence the results relating to contract comparisons, given that the two contracts differ in the service level they provide.

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A Appendix

Proof of Proposition 1: Assume that the first four steps have been completed. The derivative of π^B with respect to K_t^B is equal to,

$$\frac{\partial \pi^B}{\partial K_t^B} = rG_t(K_t) - c^B \quad (13)$$

with $K_t = K_t^A + K_t^B \geq 0$. Hence π^B is concave in K_t^B and the optimal capacity of the operator is equal to $[\phi_t - K^A]^+$ when all other capacity levels are fixed. It remains to compute the value of K^A which maximizes $\tilde{\pi}^B(K^A) := \pi^B(K^A, K_1^B(K^A), \dots, K_N^B(K^A)) = \pi^B(K^A, [\phi_1 - K^A]^+, \dots, [\phi_N - K^A]^+)$.

For a given $\tau \in [0, \dots, N+1]$, consider values of K^A in the interval $[\phi_\tau, \phi_{\tau+1}]$ (with $\phi_0 = 0, \phi_{N+1} = +\infty$). For all $t \leq \tau$, $K_t^B = [\phi_t - K^A]^+ = 0$ and $K_t = K^A$. For all $t \geq \tau + 1$, K_t^B and K_t are respectively equal to $\phi_t - K^A$ and ϕ_t . Hence the profit function is equal to:

$$\begin{aligned} \tilde{\pi}^B(K^A) &= \sum_{t=1}^{\tau} \left(r \int_0^{K^A} x f_t(x) dx + (rG_t(K^A) - \gamma)K^A \right) \\ &\quad + \sum_{\tau+1}^N \left(r \int_0^{\phi_t} x f_t(x) dx + (rG_t(\phi_t) - c^B)\phi_t \right) \\ &\quad + (N - \tau)(c^B - \gamma)K^A. \end{aligned} \quad (14)$$

This function is twice differentiable for $\phi_\tau \leq K^A < \phi_{\tau+1}$, and we have for the first and second order derivatives, (we assume in the rest of the proof that derivatives computed on the lower-bound (resp. upper-bound) of an interval correspond to the right (resp. left) hand derivatives)

$$\tilde{\pi}^{B'}(K^A) = r\tilde{G}_\tau(K^A) - c^B\tau + N(c^B - \gamma) \quad (15)$$

$$\tilde{\pi}^{B''}(K^A) = -r \sum_{t=1}^{\tau} f_t(K^A). \quad (16)$$

Hence, over the interval $[\phi_\tau, \phi_{\tau+1}]$, $\tilde{\pi}^B$ is strictly concave, increasing if and only if $K^A \leq \kappa_\tau$. Furthermore, noting that $r\tilde{G}_{\tau+1}(\phi_{\tau+1}) = r\tilde{G}_\tau(\phi_{\tau+1}) + c^B$, one can easily check using (15) that $\lim_{x \rightarrow \phi_{\tau+1}^+} \tilde{\pi}^{B'}(x) = \lim_{x \rightarrow \phi_{\tau+1}^-} \tilde{\pi}^{B'}(x)$. Thus, $\tilde{\pi}^{B'}$ is continuous and $\tilde{\pi}^B$ is strictly concave. Since furthermore $\tilde{\pi}^{B'}(0) = (r - c^B) + N(c^B - \gamma) > 0$, the first order condition $r\tilde{G}_\tau(K^A) = c^B\tau - N(c^B - \gamma)$ subject to $K^A \in [\phi_\tau, \phi_{\tau+1}]$ has a unique solution which corresponds to the optimal capacity level. In other words, there is a unique τ^* such that $\kappa_{\tau^*} \in [\phi_{\tau^*}, \phi_{\tau^*+1}]$ and $K^{A*} = \tau^*$. Note finally that κ_t is non-increasing in t and ϕ_t is non-decreasing in t so that τ^* is also the largest t such that $\phi_t \leq \kappa_{t-1}$ (or equivalently, the smallest t such that $\phi_{t+1} > \kappa_t$). \square

Proof of Proposition 2: Note first that $D_1 \leq_{st} D_2$ implies that $\phi_2 \geq \phi_1$. In order to apply Proposition 1, we need to compare ϕ_2 with κ_1 . We have

$$\begin{aligned}\phi_2 > \kappa_1 &\Leftrightarrow F_2^{-1}(1 - c^B/r) > G_1^{-1}[(2\gamma - c^B)/r] \\ &\Leftrightarrow \gamma > \tilde{\gamma}\end{aligned}$$

It follows then that $t^* = 1$ if and only if $\gamma > \tilde{\gamma}$ and the result is obtained from the direct application of Proposition 1. \square

Proof of Proposition 3: Note first that $F_\theta^{-1}(\epsilon)$ and $1 - F_{-\theta}(\epsilon)$ are respectively non-decreasing and non-increasing in $\theta \geq 0$ from the first order stochastic dominance $A_\theta \geq_{st} A_{\theta'}$ for $\theta \geq \theta'$. It follows that $1 - F_{-\theta}(F_\theta(\epsilon))$ and hence $\tilde{\gamma}$ are non-increasing in $\theta > 0$. The second part of the proposition follows from Proposition 2 by defining $\tilde{\theta} := \min_{\theta \geq 0} \{\gamma > \tilde{\gamma}\}$. A similar approach shows the first part. \square

Proof of Proposition 4: In each period, the game between the operator and the contractor is equivalent to the capacity game between a manufacturer and a subcontractor described by Van Mieghem (1999), where the market demand for the subcontractor is zero. More precisely, consider the production-subcontracting subgame in Section 3.1 of Van Mieghem (1999) and the notations within. When the market demand for the subcontractor is zero ($D_S = 0$), the supplier does not produce goods for this market ($x_S = 0$) but produces at capacity for the manufacturer ($x_t^S = K_S$). It follows that the manufacturer outsources the surplus of his market demands ($x_t = \min([D_M - K_M]^+, K_S)$) and the capacity investment game described in Section 3.2 of Van Mieghem (1999) (the choice of the capacities (K_M, K_S)) is equivalent to our operator-contractor game (determination of (K_t^B, K_t^A)). Van Mieghem (1999) shows that a unique Nash equilibrium exists for the capacity investment game. We can hence deduce a similar result for the operator-contractor capacity game. Furthermore, noting that

$$\pi_t^A = p \int_{K_t^B}^{K_t} (x - K_t^B) f(x) dx + p K_t^A [1 - F(K_t)] - c^A K_t^A$$

the best-response curve of the contractor is given by the positive solution of the following first order condition

$$\frac{\partial \pi_t^A}{\partial K_t^A} = p(1 - F(K_t)) - c^A = 0 \tag{17}$$

for a given K_t^B (where $K_t = K_t^A + K_t^B$), and with $K_t^A = 0$ when $p < c^A$ that is when (17) does not admit a solution.

Similarly, the best-response curve of the operator is given by the following first order condition, after some simplifications,

$$\frac{\partial \pi_t^B}{\partial K_t^B} = r[1 - F(K_t)] - c^B + p[F(K_t) - F(K_t^B)] = 0 \quad (18)$$

subject to $K_t^B \leq K_t$. When $p > rc^A/c^B > c^A$, plugging (17) in (18), we obtain

$$F(K_t^B) = 1 - \frac{c^A + c^B}{p} + \frac{rc^A}{p^2}. \quad (19)$$

Note that the right hand-side of (19) is negative or null if and only if $rc^A \leq c_0^2$, with $c_0 := (c^A + c^B)/2$, and $p \in [p_1, p_2]$ where $p_1 = c_0 \left(1 - \sqrt{1 - rc^A/c_0^2}\right)$ and $p_2 = c_0 \left(1 + \sqrt{1 - rc^A/c_0^2}\right)$. Noting that $p_2 < rc^A/c^B$, we can then deduce K_t^B and K_t^A of Proposition 4 from (17) and (19) when $p > rc^A/c^B$, which also implies that the constraint $K_t^B \leq K_t$ is satisfied. On the other hand when $p < rc^A/c^B$, the constraint $K_t^B \leq K_t$ is binding and $K_t^A = 0$ so that $K_t^{B*} = F_t^{-1}(1 - c^B/r)$.

Proof of Proposition 5: For any given γ , Proposition 1 implies that the corresponding optimal capacity levels K^A and K^B are equal to κ_1 and 0 respectively, since $\kappa_1 > \phi^1$ from $c^B > \gamma$. But when K^B is zero, the contract is equivalent to the supply chain wholesale contract for a supplier-vendor in Lariviere and Porteus (2001), where the operator is a retailer, the contractor a manufacturer, and K^A a quantity of products. Applying Theorem 1 of Lariviere and Porteus (2001) leads then to the result. \square

Proof of Proposition 6: We show in the following the first part of the result. The proof for the two last parts is similar. Assume then that $J_1(\phi_2) < (2c^A - c^B)/r$.

For $\gamma > \tilde{\gamma}$, the optimal capacity verifies $\gamma = (rG_1(K^A) + c^B)/2$ from Proposition 2. The profit of the contractor is then equal to $\pi^A(K^A) = (\gamma - c^A)K^A = (rG_1(K^A) + c^B - 2c^A)K^A$. The first order condition yields

$$J_1(K^A) = \frac{2c^A - c^B}{r}. \quad (20)$$

Since J_1 is non-increasing when it is positive and since $J_1(0) = 1 \geq (2c^A - c^B)/r$, Equation (20) admits a unique solution such that $J_1(K^{A*}) = 2c^A - c^B/r > J_1(\phi_2)$. It follows that $K^{A*} < \phi_2$ which is equivalent to $\gamma^* > \tilde{\gamma}$. In other words there exists a Stackelberg equilibrium such that $\gamma^* > \tilde{\gamma}$.

Consider now prices such that $\gamma \leq \tilde{\gamma}$. Following a similar approach using the second part of Proposition 2, we have $\gamma = r/2\tilde{G}_2(K^A)$ and we can deduce the following first order condition in K^A ,

$$J_1(K^A) + J_2(K^A) = \frac{2c^A}{r}. \quad (21)$$

But if $J_1(\phi_2) < (2c^A - c^B)/r$, then $J_1(\phi_2) + J_2(\phi_2) < 2c^A/r$ since $J_2(\phi_2) = G_2(\phi_2) - \phi_2 f_2(\phi_2) < c^B/r$ from the definition of ϕ_2 . Thus, the optimal capacity is in this case the smallest K^A such that $\gamma = r/2\tilde{G}_2(K^A) \leq \tilde{\gamma}$, which is equal to ϕ_2 from the definition of $\tilde{\gamma}$.

It follows that the Stackelberg game has a unique equilibrium such that K^{A*} satisfies Equation (20) with

$$\gamma^* = (rG_1(K^{A*}) + c^B)/2 > \tilde{\gamma}.$$

K_1^{B*} and K_2^{B*} are then given by the first part of Proposition 2. \square

Proof of Proposition 7: Consider $\phi_{2,\theta} = F_\theta^{-1}(1 - c^B/r)$ as defined in Proposition 2. (In the following, we sometimes add the subscript θ to the quantities that depend on this parameter, for the sake of clarity.). Since the distribution of $D_1 = A_0$ has an IGFR, $J_1(\epsilon)$ is non-increasing in ϵ when it is positive. Furthermore, from the first order stochastic dominance $A_\theta >_{st} A_{\theta'}$ for $\theta > \theta'$, $\phi_{2,\theta}$ is non-decreasing in θ and $J_1(\phi_{2,\theta})$ is non-increasing in θ as long as $J_1(\phi_{2,\theta}) > 0$. Hence there exist a unique threshold $\hat{\theta}$ such that $\hat{\theta}$ is the minimum value of θ satisfying $J_1(\phi_{2,\theta}) \leq (2c^A - c^B)/r$. For $\theta > \hat{\theta}$, $J_1(\phi_{2,\theta}) < (2c^A - c^B)/r$ so that at the equilibrium, K^{A*} and γ^* do not depend on θ according to the first part of Proposition 6. Furthermore, $K_1^{B*} = 0$ and K_2^{B*} is non-decreasing in θ since $F_\theta(\epsilon)$ decreases in θ from the first order stochastic dominance $A_\theta >_{st} A_{\theta'}$, $\theta > \theta'$.

It remains to show the first part of the proposition for $\theta < \hat{\theta}$ which corresponds to the two last cases of Proposition 6. Since K_θ^{A*} , $K_{2,\theta}^{B*}$ and γ_θ^* are continuous in θ , it suffices to show the result for each case. The second case is similar to the first one. For the last case we have

$$K_1^{B*} = K_2^{B*} = 0 \tag{22}$$

$$J_1(K_\theta^{A*}) + J_{2,\theta}(K_\theta^{A*}) = 2c^A/r \tag{23}$$

$$\gamma^* = \frac{r}{2}\tilde{G}_{2,\theta}(K^{A*}\theta). \tag{24}$$

From the stochastic dominance $A_\theta >_{hr} A_{\theta'}$ (and hence $A_\theta >_{st} A_{\theta'}$) with $\theta > \theta'$, for any ϵ such that $J_1(\epsilon) \geq 0$, we have $J_{2,\theta}(\epsilon) > J_{2,\theta'}(\epsilon) \geq J_{2,0}(\epsilon) = J_1(\epsilon) \geq 0$ and $J_{2,\theta}(\epsilon)$ is non-increasing in ϵ . It follows that $J_1(\epsilon) + J_{2,\theta}(\epsilon)$ is non-increasing in ϵ and non-decreasing in θ so that K_θ^{A*} is non-decreasing in θ as long as $J_1(K_\theta^{A*}) \geq 0$. Note however that $J_1(\phi_{2,\hat{\theta}}) + J_{2,\hat{\theta}}(\phi_{2,\hat{\theta}}) = 2c^A/r$ from the definitions of $\phi_{2,\hat{\theta}}$ and $\hat{\theta}$. Hence, for $\theta \leq \hat{\theta}$, $K_\theta^{A*} \leq \phi_{2,\hat{\theta}}$ and $J_1(K_\theta^{A*}) \geq J_1(\phi_{2,\hat{\theta}}) = (2c^A - c^B)/r > 0$. \square

Proof of Proposition 8: p^* is the maxima of $\pi^A = p \min([D - K^B(p)]^+, K^A(p)) - c^A K^A(p)$, subject to $\pi^B(p)$ being larger than the profit of the call center operator when it does not outsource

any calls. We can then restrict p such that $p \geq rc^A/c^B$, since K^A and then π^A are zero otherwise from the expression of K^A of Proposition 4.

Note then that

$$\begin{aligned} \frac{\partial \pi^A}{\partial p} &= \int_{K^B}^K G(x)dx + p \left(G(K) \frac{\partial K}{\partial p} - G(K^B) \frac{\partial K^B}{\partial p} \right) \\ &\quad - c^A \frac{\partial (K - K^B)}{\partial p} \\ &= \int_{K^B}^K G(x)dx + p \left(F(K^B) - F(K) \right) \frac{\partial K^B}{\partial p} \end{aligned} \quad (25)$$

where the last equality is true since $pG(K) = c^A$ from Proposition 4. Since $p \geq rc^A/c^B$, we have $F(K^B) - F(K) \leq 0$. As a result, if $\partial K^B/\partial p$ is negative for a given p then $\partial \pi^A/\partial p$ is positive, and p cannot be an equilibrium. A direct computation leads then to

$$\frac{\partial K^B}{\partial p} = 2 \left(\frac{c_0}{p^2} - \frac{rc^A}{p^3} \right) \frac{1}{f(K^B)} \quad (26)$$

which is negative or zero if and only if $p \leq rc^A/c_0$. It follows that $p^* \geq rc^A/c_0$.

Finally, let us show that $p^* \leq r$. From the definition of π^B and Proposition 4, we have

$$\frac{\partial \pi^B}{\partial p} = (r - p)G(K) \frac{\partial K}{\partial p} + \frac{p - r}{p} c^A \frac{\partial K^B}{\partial p} - \int_{K^B}^K G(x)dx. \quad (27)$$

which is negative for $p \geq r$ ($\partial K^B/\partial p$ is negative since $p \geq r \geq rc^A/c_0$). In this case, $\pi^B(p)$ is then less or equal to $\pi^B(r)$ which is also the profit of the call center operator when it does not outsource calls. Hence $p^* \leq r$. \square

Proof of Proposition 9: If F has DFR then for all $K^B \geq x$, $G(K^B)/f(K^B) \leq G(x)/f(x)$. As a result,

$$\int_{K^B}^K G(x)dx = \int_{K^B}^K \frac{G(x)}{f(x)} f(x)dx \geq \frac{G(K^B)}{f(K^B)} \left(F(K) - F(K^B) \right) \quad (28)$$

From Equation (25) and Proposition 4 we have

$$\frac{\partial \pi^A}{\partial p} \geq \frac{F(K) - F(K^B)}{f(K^B)} \frac{rc^A}{p^2} > 0 \quad (29)$$

Hence π^A is strictly increasing and $p^* = r$, which is also the operator's profit when it does not outsource calls and the operator never prefers outsourcing the fluctuation (Contract 2). \square

Proof of Proposition 10: Assume that demand has an uniform distribution over $[a, b]$. It follows that when $x \in [a, b]$, $G(x) = (b - x)/d$ with $d = b - a$, $\int_0^x G(u)du = (1 - x/2)x/d$ and $f(G^{-1}(x)) = dx + a$. Using (25), (26) and Proposition 4, we compute

$$\frac{\partial \pi^A}{\partial p} = \frac{F(K) - F(K^B)}{2ap^2} (3rc^A - c^B p)$$

which is positive if and only if $p < 3rc^A/c^B$. From Proposition 8, $p^* \in [2rc^A/(c^A + c^B), r]$ where $3rc^A/c^B > 2rc^A/(c^A + c^B)$ and the result directly follows. \square

Proof of Proposition 11: Denote by p^* the equilibrium price of the unconstrained problem. When $c^A/c^B > p$, B does not outsource any call from Proposition 4. Combined with the constraint (10) this implies then that if $c^B/r < \alpha$, $F(K^B) = F(K) = 1 - c^B/r$ (with $F(K) > 1 - \alpha$). Similarly, when $c^A/r > \alpha$, the price of the contract with service level agreement should be equal to c^A/α which is larger than r . But from Proposition 8, $p^* \leq r$ and the equilibrium price does not achieve the required capacity level. It follows that for the constrained problem, the profit of B is always higher when going solo and we have $F(K) = F(K^B) = 1 - \alpha$ (since $1 - \alpha > 1 - c^B/r$). Finally, when $c^A/r \leq \alpha \leq c^B/r$, the constraint is binding if and only if the equilibrium price cannot achieve the service constraint, that is if and only if $p^* \leq c^A/\alpha$, from which we obtain the result directly. \square