Problem Set 2 Solutions

1. Matt consumes only goods $x$ and $y$, and his demand functions are $x(p_x, p_y, I) = I/3p_x$ and $y(p_x, p_y, I) = 2I/3p_y$, where $p_x$ is the price of $x$, $p_y$ is the price of $y$, and $I$ is his income.

(a) Is $x$ normal or inferior? How about $y$? Find their income elasticities.

His demand function for $x$ is $x(p_x, p_y, I) = I/3p_x$. As income increases demand for $x$ increases. Therefore, normal good. His demand for $y$ is $y(p_x, p_y, I) = 2I/3p_y$. With the same reasoning we can conclude that $y$ is a normal good.

Income elasticities are as the following:

$$\epsilon_x = \frac{\Delta x}{\Delta I} \frac{I}{x} = \frac{1}{3p_x} \frac{I}{I} = \frac{1}{3p_x} > 0$$

$$\epsilon_y = \frac{\Delta y}{\Delta I} \frac{I}{y} = \frac{2}{3p_y} \frac{I}{I} = \frac{2}{3p_y} > 0$$

(b) Are $x$ and $y$ substitutes or complements, or what? Neither $p_x$ affects the demand for $y$ nor $p_y$ affects the demand for $x$. Therefore, they are neither complements nor substitutes.
(c) Carefully draw a graph with $x$ on the horizontal axis and $y$ on the vertical axis. Let $p_x = p_y = 1$, and draw the three budget lines corresponding to $I = 3, 6, 9$. Then draw the three points on these lines that Matt would demand given those budget lines. Then connect the three points with a line to show all the points he would demand as income shifts.

The budget line is the following $p_x x + p_y y = I$. When $p_x = p_y = 1$, we have $x + y = 1$. Therefore

$$x(1, 1, I) = \frac{I}{3} \text{ and } y(1, 1, I) = \frac{2I}{3}$$

<table>
<thead>
<tr>
<th>I</th>
<th>Budget Line</th>
<th>$x^*$</th>
<th>$y^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$x + y = 3$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>$x + y = 6$</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>$x + y = 9$</td>
<td>3</td>
<td>6</td>
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</tbody>
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Note: Whatever his income is Matt spends $1/3$ of it to $x$ and $2/3$ of it to $y$. The equation of the line we draw is $y = 2x$.

(d) Repeat c), except now hold $p_y = 1$ and $I = 6$ fixed and vary $p_x = .5, 1, 2$.

$p_y = 1$ and $I = 6$, then the budget constraint is $p_x x + y = 6$ and
demand is given as follows:

\[
x(p_x, 1, 6) = \frac{6}{3p_x} = \frac{2}{p_x}
\]

\[
y(p_x, 1, 6) = \frac{2.6}{3.1} = 4
\]

<table>
<thead>
<tr>
<th>(p_x)</th>
<th>Budget Line</th>
<th>(x^*)</th>
<th>(y^*)</th>
</tr>
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<tbody>
<tr>
<td>0.5</td>
<td>(0.5x + y = 6)</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>(x + y = 6)</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>(2x + y = 6)</td>
<td>1</td>
<td>4</td>
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As \(p_x\) increases the demand for \(y\) does not change, the demand for \(x\) decreases.

(e) Repeat c), except now hold \(p_x = 1\) and \(I = 6\) fixed and vary \(p_y = .5, 1, 2\).

\(p_x = 1\) and \(I = 6\), then the budget constraint is \(x + p_y y = 6\) and demand is given as follows:

\[
x(1, p_y, 6) = \frac{6}{3} = 2
\]

\[
y(1, p_y, 6) = \frac{2.6}{3p_y} = \frac{4}{p_y}
\]
2. Let $c_0$ (respectively, $c_1$) denote the consumption today (respectively, tomorrow). Suppose that Mert is willing to give up 4 units of consumption today for 5 units of consumption tomorrow. Moreover, assume that his endowment for today is $m_0 = 60$ and for tomorrow is $m_1 = 70$.

(a) Write down a utility function that represents Mert’s preferences over consumption for today and tomorrow.

Since Mert is willing to give up 4 units of consumption today to consume 5 units tomorrow, his marginal utility from tomorrow’s consumption is 4 and his marginal utility from today’s consumption is 5. Therefore the following would represent his preferences: $U(c_0, c_1) = 5c_0 + 4c_1$

(b) Draw his intertemporal budget line if there is an interest rate of $r = 75\%$.

Let us first assume that he does not consume anything today. Then the amount he would be able to consume tomorrow would
be

\[ c_1 = m_1 + m_0(1 + r) \]
\[ c_1 = 70 + 60(1 + 0.75) = 175 \]

Now, let us assume that he is not consuming tomorrow. Then the maximum amount that he would consume today is

\[ c_0 = m_0 + \frac{m_1}{1 + r} \]
\[ c_0 = 60 + \frac{70}{1.75} = 100 \]

(c) Find his optimal consumption for today and tomorrow. Would he save or borrow? How much?

The slope of his indifference curve is \( MRS = -\frac{MU_{c_0}}{MU_{c_1}} = -\frac{5}{4} \)

and the slope of his budget line is \(-(1 + r) = -1.75\). His budget line is steeper than his indifference curves. This means that the market rate of substitution between today and tomorrow is higher than his own marginal rate of substitution. 1 unit of consumption today worths 1.25 units of consumption tomorrow. If he gives up one unit of consumption today with the market interest rate of 75%, he would be able to get 1.75 units of consumption tomorrow.
Therefore, he would consume the maximum he can tomorrow and not consume anything today.

(d) Suppose that $r$ decreases to 25%. How would his consumption behavior change? Would he borrow or save?

Now the slope of the budget line has decreased to $-(1 + r) = -1.25$, which is equal to his $MRS$. Therefore, he will be indifferent between borrowing and lending.

(e) Now assume that he had a minor accident and his trade off between today’s and tomorrow’s consumption has changed perma-
nently. Now, he is willing to give up one unit of consumption today for two units of consumption tomorrow. How would his consumption behavior change with the same endowment levels and with an interest rate of \( r = 75\% \)?

Since his preferences have changed, we need to define a new utility function that represents his new preferences. Now, his marginal utility from today’s consumption is 2 and marginal utility from tomorrow’s consumption is 1. Therefore, the following would represent his preferences \( U(c_0, c_1) = 2c_0 + c_1 \). Now, the slope of the budget line becomes \(-2\). To give up one unit of consumption today, he needs 2 units of consumption tomorrow. However, with 75% interest rate, he would only get 1.75 units of consumption tomorrow if he gives up 1 unit of consumption today. Therefore, he will consume the maximum he can today.

3. Cansu has preferences over pasta \((P)\) and sushi \((S)\) represented by the utility function \( U(S, P) = 2P + 3S \). Pasta costs \( p_P = $10\) and sushi costs \( p_S = $5\). She is a graduate student and earns a monthly allowance of $600.

(a) Draw her budget line and a couple of her indifference curves. Write down the demand functions \( P(p_S, p_P, m) \) and \( S(p_S, p_P, m) \). Find her optimal consumption bundle.
Her budget constraint is $10P + 5S = 600$ and her demand functions are as follows:

$$P(p_S, p_P, m) = \begin{cases} 
0 & \text{if } p_S/p_P < 3/2 \\
[0, m/p_P] & \text{if } p_S/p_P = 3/2 \\
\frac{m}{p_P} & \text{if } p_S/p_P > 3/2 
\end{cases}$$

$$S(p_S, p_P, m) = \begin{cases} 
\frac{m}{p_S} & \text{if } p_S/p_P < 3/2 \\
[0, m/p_S] & \text{if } p_S/p_P = 3/2 \\
0 & \text{if } p_S/p_P > 3/2 
\end{cases}$$

She would not consume pasta, instead she would only consume sushi. Optimal bundle is $(120, 0)$.

(b) Assume that the price of sushi increased so that now it costs $p_S = $20. Find her new optimal bundle. Moreover, find income and substitution effects resulted by the change of her consumption.

We know that her optimal bundle after the price change is $C = (60, 0)$ (See the demand functions on part a). So the total effect for demand on $P$ is an increase of 60 units and total effect for demand on $S$ is a decrease of 120 units. To decompose the total effect into income and substitution effect, we will first get rid of the income effect. Since now Cansu is relatively poor, we will give
her money just as much to make her afford her original bundle, $A = (120, 0)$ without changing the prices, $p_P = 10$ and $p_S = 20$.

$$10 \times 0 + 20 \times 120 = 2400$$

With the new prices, she needs $2400, therefore we give her an extra $1800 (Her income was $600). Since we get rid of the income effect by giving her money, the next step is to find her optimal bundle on the new budget line (red line on the graph below). The optimal bundle on this budget line is $B = (0, 120)$. No, we know that any change from bundle $A$ to bundle $B$ is due to substitution effect. Any change from bundle $B$ to bundle $C$ is due to income effect. Therefore, we conclude as follows:

- Substitution effect on $S$ is $-120$ and on $P$ is $+120$,
- Income effect on $S$ is $0$ and on $P$ is $-60$.

4. Ned has a utility function given by $U(x, y) = \sqrt{x} + \sqrt{y}$ with an income of $m = 8$. Moreover, prices are given as follows $p_x = $1 and $p_y = $1.
(a) Find his optimal bundle.

Let us first write down the budget constraint:

\[ p_x x + p_y y = 8 \]
\[ x + y = 8 \]

Then we should calculate the marginal rate of substitution,

\[ MRS = \frac{MU_1}{MU_2} \]
\[ = \frac{1/2 x^{-1/2}}{1/2 y^{-1/2}} = \frac{y^{1/2}}{x^{1/2}} \]

From the optimality condition, we have \( x^* = y^* \). When we substitute this into the budget constraint, we find the optimal bundle as \((x^*, y^*) = (4, 4)\).

(b) Find his optimal bundle when \( p_{x_2} \) rises to $2. Show the total effect of the price change and then decompose it into income and substitution effects.

Since the price of \( y \) increased, the budget constraint will change:

\[ p_x x + p_y y = 8 \Rightarrow x + 2y = 8 \]

The marginal rate of the substitution is the same since preferences are the same. The optimality condition implies that \( x^* = 4y^* \). When we substitute this into the budget constraint we have \((x^*, y^*) = (16/3, 4/3)\). The total effect is a decrease in demand for \( y \) by \( 8/3 \) units and an increase in demand for \( x \) by \( 4/3 \) units.

We know that substitution effect and income effect together add up to total effect. To decompose, let’s first get rid of the income effect. Since Ned is poorer (not in real terms but relatively), we will give him money so that he can afford the original bundle \((4, 4)\).

\[ p_x x + p_y y = 1 \cdot 4 + 2 \cdot 4 = 12 \]

Now, we will find the optimal bundle on the new budget line. From the optimality condition we have \( x^* = 4y^* \). Therefore, the optimal bundle is \((8, 2)\). Then
• Substitution effect: $-2$ units for $x$ and $+4$ units for $y$
• Income effect: $-2/3$ units for $x$ and $-8/3$ units for $y$

5. Tina’s utility function for $x$ and $y$ is $U(x, y) = 4(x^{1/2}) + y - 55$

(a) Find her demand functions for $x$ and $y$.

The optimality condition is as follows:

$$MRS = 2x^{-1/2} = \frac{p_x}{p_y}$$

Then demand functions are,

$$x(p_x, p_y, I) = \begin{cases} 4 \frac{p_y^2}{p_x^2} & \text{if } I \geq 4 \frac{p_y^2}{p_x} \\ \frac{I}{p_x} & \text{otherwise} \end{cases}$$

$$y(p_x, p_y, I) = \begin{cases} I - 4 \frac{p_y^2}{p_x^2} & \text{if } I \geq 4 \frac{p_y^2}{p_x} \\ 0 & \text{otherwise} \end{cases}$$
(b) Let \( p_y = 2 \) and \( I = 20 \). Suppose that price of \( x \) increases from \( \$1 \) to \( \$2 \). Find the income and substitution effects of this change on consumption of \( x \) and \( y \).

Notice that both before and after the change, the solution is interior.

- Before the change \( x(1,2,20) = 16 \) and \( y(1,2,20) = 2 \)
- After the change \( x(2,2,20) = 4 \) and \( y(2,2,20) = 6 \)

then the total effect is \(-14\) units of decrease for \( x \) and \( 4 \) units of increase of \( y \).

To be able to afford the first bundle \((16,2)\) after the price change, he needs an income of \( 2 \times 16 + 2 \times 2 = 36 \). With this income he would demand 4 units of \( x \), and 16 units of \( y \). Therefore,

- Substitution effect is \(-12\) units for \( x \) and \(+10\) units for \( y \).
- Income effect is \( 0 \) unit for \( x \) and \(-14\) units for \( y \).


- (10.3)
  
  (a) Present value of his endowment is

  \[
  M_1 + \frac{M_2}{(1 + r)} = 2000 + \frac{1100}{1 + 0.1} = 3000,
  \]

  and the future value of his endowment is

  \[
  M_1(1 + r) + M_2 = 2000(1.1) + 1100 = 3300.
  \]

![Graph showing consumption next year in thousands](image)
(b) \[ MRS = \frac{MU_{c_1}}{MU_{c_2}} = \frac{C_2}{C_1} \]

(c) Slope of the budget line is \(-(1 + r) = -1.1\). The optimality condition is as follows:
\[ MRS = -(1 + r) \]
\[ \frac{C_2}{C_1} = 1.1, \]
and the budget line is given by
\[ C_1 + \frac{C_2}{1 + r} = M_1 + \frac{M_2}{(1 + r)} \]
\[ C_1 + \frac{C_2}{1.1} = 3000 \]

(d) From the optimality condition we have \(C^*_2 = 1.1C^*_1\) when we substitute this into the budget constraint we find the optimal bundle as \((C^*_1, C^*_2) = (1500, 1650)\).

(e) He will save $500 in the first period since he has an endowment of $2000, but he only consumes $1500 in the first period.

(f) If interest rate increases and becomes 20% then the budget line becomes
\[ C_1 + \frac{C_2}{1 + r} = M_1 + \frac{M_2}{(1 + r)} \]
\[ C_1 + \frac{C_2}{1.2} = 3000 \]
Moreover the present value of his endowment will become \(2000 + \frac{1100}{1.2} \approx 2917\) and the future value of his income is \(2000(1.2) + 1100 = 3500\). If Nickleby is choosing A over E, then with the new interest rate, he should prefer any point above E to any point below E by WARP.

(g) Now the optimality condition becomes \(C^*_2 = 1.2C^*_1\). When we substitute this into the budget constraint, we get \((C^*_1, C^*_2) = (1458.5, 1750.2)\).
(h) He is borrowing $541.5 since he is consuming less than his first period income.

• (10.8)

(a) With 10% interest rate the present value of his endowment is 50000 since he does not have any endowment in the second period, he can not borrow. Optimality condition is $c_2^* = 1.1c_1^*$, when we substitute this into the budget constraint we have $c_1^* = 25000$ and $c_2^* = 26500$. Since he is a saver when interest rate is $r = 10\%$ without looking at his preferences, we can say that he will keep on saving when interest rate increases. Since the present value of his income is same for any interest rate, he will keep on consuming the same amount.

(b) The optimality condition will become $c_2^* = rc_1^*$ where $r > 1.1$. Since $c_1^*$ is same as before, $c_2^*$ will increase due to an increase in the interest rate.

(c) If his income in the first period is 0 then the present value of his income becomes $0 + \frac{55000}{1.1} = 50000$. The optimality condition holds $c_2^* = 1.1c_1^*$, therefore he would consume 25000 in the first period. If interest rate increases, now the present value of his income changes. It becomes $0 + \frac{55000}{r} > 50000$ where $r > 1.1$. Therefore he would consume more in period 1 when interest rate increases.